Let X be a curve over a number field. We will explain the relation between invariants of X such as the Belyi degree $d(X)$ and the stable Faltings height $h(X)$. For instance, we prove an explicit inequality relating these invariants: $h(X) < 10^9 d(X)^6$. As a first application we prove a conjecture of Edixhoven-de Jong-Schepers, related to computational aspects of Galois representations associated to surfaces over $\mathbb{Q}$, on the Faltings height of a cover of curves. Then we will discuss applications to computing integral points of certain moduli spaces, e.g., the space of hyperelliptic curves and the space of GL_2 abelian varieties. We will explain some aspects of the proof which require the construction of certain models of finite covers of curves.