# PROPORTIONATE NLMS FOR ADAPTIVE FEEDBACK CONTROL IN HEARING AIDS

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#### ABSTRACT

The proportionate normalized least-mean-squares (PNLMS) algorithm is commonly used in acoustic echo cancellation (AEC) context. It provides faster initial convergence and tracking rates compared to the NLMS algorithm for the case of sparse echo impulse responses. The improved PNLMS algorithm (IPNLMS) has been proven to be more powerful than PNLMS by exploiting new rules for computing the weight of each step-size corresponding to each adaptive filter coefficient. However, the application of the PNLMS and the IPNLMS algorithms for adaptive feedback control (AFC) in hearing aids (HAs) is still limited due to high correlation between the loudspeaker and incoming signals. This paper proposes implementations of the PNLMS/IPNLMS algorithms for AFC using the prediction error method (PEM) for hearing aids. The proposed methods have been evaluated for both speech and music incoming signals. Simulation shows that the proposed methods have faster initial convergence and tracking than the PEM using the NLMS algorithm (PEM-NLMS).

*Index Terms*— Adaptive feedback control, prediction error methods, hearing aids, PNLMS, IPNLMS.

#### 1. INTRODUCTION

Hearing aids (HAs) usually suffer from acoustic feedback problems due to a coupling between the loudspeaker and microphone. This acoustic feedback does not only limit the achievable stable gain, but also often produces howling. The most commonly used method to cancel the negative effects of the acoustic feedback is adaptive feedback control (AFC) [1, 2, 3, 4]. In this method an adaptive filter is used to estimate the acoustic feedback path, then the estimated feedback signal is subtracted from the microphone signal. However, the estimated feedback path often includes a bias due to the correlation between the loudspeaker and incoming signals. This correlation is present due to the closed loop feature of hearing aids.

Recently, the prediction error method (PEM) has been proposed as an efficient method to reduce this bias for speech incoming signals [3, 5, 6]. The PEM models the incoming signal by filtering a white gaussian-noise sequence via an all-pole filter. The inverse model of the incoming signal is estimated and then used to pre-whiten the inputs of an adaptive filter (in the feedback canceller path). As a result, an unbiased solution for the feedback path estimate may be achieved.

Commonly, the least mean squares (LMS) or normalized LMS (NLMS) are employed to estimate the acoustic feedback path due to their simplicity and efficiency. Unfortunately, these algorithms degrade the convergence rate of an AFC system for the case of spectrally coloured incoming signals. To improve the convergence as well as to achieve a low steady-state error several solutions have been introduced in the literatures, e.g., affine projection algorithms (APA) [7, 8, 9, 10], or variable step-size algorithms [11, 12, 13, 14, 15], or affine combination of two adaptive filters using different step-sizes [16, 17, 18], or PNLMS/IPNLMS algorithms [19, 20, 21].

The PNLMS/IPNLMS algorithms are popular in the acoustic echo cancellation (AEC) context. They provide faster initial convergence and tracking compared to the NLMS algorithm by using a tap-dependent step-size for updating the adaptive filter coefficients. However, their application for AFC in HAs is still limited due to the high correlation between the loudspeaker and incoming signals. In [22] a variant of the PNLMS algorithm called Levenberg-Marquardt regularized NLMS was introduced. However, this algorithm was only applied to the AEC and AFC in public address systems (PAs), not to HAs. Moreover, this algorithm required prior knowledge of the unknown feedback paths as well as incoming signal powers. Note that the length of the feedback paths and the distance between the loudspeaker and the microphone in HAs are much shorter than those in PAs. Thus the acoustic feedback problem in HAs is more severe than that in PAs. In [23] the PNLMS algorithm was employed for two-microphone AFC.

In this paper we propose to implement the PNLMS/IPNLMS algorithms for the AFC using PEM (PEM-PNLMS/PEM-IPNLMS) in HAs. Simulation results show that the proposed methods provide better initial convegence as well as tracking while maintaining lower steady-state error than the PEM-NLMS for both speech and music incoming signals. Furthermore, our proposed methods require no prior knowledge of the feedback paths as well as incoming signal power.

#### 2. PROPOSED AFC SYSTEM

The proposed AFC system is depicted in Fig. 1. This system is designed based on the PEM, but uses different algorithms (NLMS/PNLMS/IPNLMS) to estimate the impulse response (IR) of the feedback path. In the proposed system the microphone signal is defined by an addition of the feedback signal  $v(k) = \mathbf{f}^T(k) \mathbf{y}(k)$ 

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and the incoming signal u(k), i.e.,

$$x(k) = u(k) + v(k),$$
 (1)

where  $\mathbf{f}(k)$  is the  $L_f$ -dimensional IR vector of the true feedback path and  $\mathbf{y}(k)$  is the  $L_f$ -dimensional vector of the loudspeaker signal,

$$\mathbf{f}(k) = \left[f_0(k), f_1(k), \dots, f_{L_f-1}(k)\right]^T,$$
(2)

$$\mathbf{y}(k) = [y(k), y(k-1), \dots, y(k-L_f+1)]^T.$$
(3)

The error signal e(k) is computed as follows

$$e(k) = x(k) - \hat{\mathbf{f}}^{T}(k) \mathbf{y}(k), \qquad (4)$$

where the  $L_{\hat{f}}$ -dimensional vector  $\hat{\mathbf{f}}(k)$  is an estimate of  $\mathbf{f}(k)$ ,  $\hat{\mathbf{f}}(k) = \left[\hat{f}_0(k), \hat{f}_1(k), \dots, \hat{f}_{L_{\hat{f}}-1}(k)\right]^T$  and  $\mathbf{y}(k)$  is the  $L_{\hat{f}}$ -dimensional vector of the loudspeaker signal defined in a manner analogous to (3). Assuming that the forward path K(q,k) includes a delay  $d_k$  and an amplifier with broadband gain |K|, i.e.,  $K(q,k) = |K|q^{-d_k}$ . The delay is selected such that  $d_k \ge 1$ . The loudspeaker signal y(k) is formed by delaying and amplifying the error signal e(k) in the forward path, i.e.,

$$y(k) = K(q,k) e(k).$$
<sup>(5)</sup>

We assume that the incoming signal u(k) can be modeled using an all-pole filter  $G^{-1}(q, k)$  to filter a white Gaussian noise sequence w(k), i.e.,

$$u(k) = G^{-1}(q,k) w(k).$$
(6)

Then an estimate of G(q, k) can be utilized to pre-whiten the inputs of the adaptive filter  $\hat{F}(q, k)$ , i.e.,  $x_p(k) = \hat{G}(q, k) x(k)$  and  $y_p(k) = \hat{G}(q, k) y(k)$ , where  $\hat{G}(q, k)$  is an estimate of G(q, k)and G(q, k) is the inverse of the incoming signal model. The prewhitened error signal is denoted as

$$e_{p}\left(k\right) = x_{p}\left(k\right) - \hat{\mathbf{f}}^{T}\left(k\right)\mathbf{y}_{p}\left(k\right), \qquad (7)$$

where  $\mathbf{y}_{p}(k) = \left[y_{p}(k), y_{p}(k-1), \dots, y_{p}\left(k-L_{\hat{f}}+1\right)\right]^{t}$ . The Levinson-Durbin algorithm is used to estimate the coefficients of  $\hat{G}(q,k)$  from the error signal e(k) [24]. In the PEM-NLMS, the IR of the feedback path is estimated as

$$\hat{\mathbf{f}}(k) = \hat{\mathbf{f}}(k-1) + \frac{\mu}{(||\mathbf{y}_{p}(k)||_{2}^{2} + \delta_{NLMS})} \mathbf{y}_{p}(k) e_{p}(k), \quad (8)$$

where  $\delta_{NLMS}$  is a regularization parameter and  $\mu$  is a fixed stepsize. In the following the PEM with different algorithms for estimating the feedback path is analysed. These algorithms are the PNLMS and IPNLMS using  $l_1$ -norm or  $l_0$ -norm.

## 2.1. PEM-PNLMS

The PNLMS algorithm was firstly introduced by Duttweiler for the AEC context [19]. This algorithm uses an adaptive step-size in proportion to the estimated filter coefficient to update each coefficient of the adaptive filter. Therefore, a significant improvement in adaptation speed compared to the NLMS algorithm can be achieved. Note that unlike the AEC systems, the AFC systems suffer from correlation between the loudspeaker signal and the incoming signal.

In our proposed PEM-PNLMS for AFC applications, the PEM is used to decorrelate the loudspeaker signal and the incoming signal, resulting in a lower bias in the estimate of the feedback path. Then



Fig. 1: The proposed AFC system.

the pre-whitened signals are used in the PNLMS algorithm to further improve the initial convergence and tracking. The proposed PEM-PNLMS is described as

$$\hat{\mathbf{f}}(k) = \hat{\mathbf{f}}(k-1) + \frac{\mu \mathbf{A}(k-1) \mathbf{y}_p(k) e_p(k)}{\mathbf{y}_p^T(k) \mathbf{A}(k-1) \mathbf{y}_p(k) + \delta_{PNLMS}}, \quad (9)$$

$$\mathbf{A}(k-1) = diag\left\{a_0\left\{k-1\right\}, \dots, a_{L_f}-1\left\{k-1\right\}\right\}, \quad (10)$$

where  $\delta_{PNLMS}$  is a regularization parameter and  $\mathbf{A}(k-1)$  is a diagonal matrix. This diagonal matrix is used to allocate a step-size to each filter coefficient such that a larger coefficient receives a larger increment and vice versa. Hence, an increase in the convergence of that coefficient is achieved.

In the original PNLMS algorithm [19], the diagonal elements  $a_l \{k\}$  are computed as

$$a_{l}(k) = \frac{\lambda_{l}(k)}{\sum_{i=1}^{L_{f}-1} \lambda_{i}(k)},$$
(11)

$$\lambda_{l}'(k) = \max\left[\zeta, \left|\hat{f}_{0}\left(k\right)\right|, \dots, \left|\hat{f}_{L_{f}-1}\left(k\right)\right|\right], \qquad (12)$$

$$\lambda_{l}\left(k\right) = \max\left\{\rho\lambda_{l}'\left(k\right), \left|\hat{f}_{l}\left(k\right)\right|\right\},\tag{13}$$

where  $\rho$ ,  $\zeta$  are positive parameters with typical values  $\rho = 5/L_{\hat{f}}$ ,  $\zeta = 0.01$ . The constant  $\rho$  prevents  $\hat{f}_l(k)$  from stalling when it is very small and  $\zeta$  is a regularization parameter.

#### 2.2. PEM-IPNLMS

To further improve the convergence of the PNLMS, the improved PNLMS [20, 21] modified the PNLMS by providing new rules to better exploit the shape of the estimated feedback path for calculating the tap-dependent step-size. In [20] the  $l_1$ -norm of the adaptive filter is exploited, whereas the  $l_0$ -norm is used in [21]. In this subsection we propose to employ the IPNLMS with  $l_1$ -norm or  $l_0$ -norm for the PEM in HAs. The proposed PEM-IPNLMS ( $l_1$ -norm) method uses a smoother choice for the elements in (13), i.e.,

$$\lambda_{l}\left(k\right) = (1-\alpha) \frac{\left\|\hat{\mathbf{f}}\left(k\right)\right\|_{l_{1}}}{L_{\hat{f}}} + (1+\alpha)\left|\hat{f}_{l}\left(k\right)\right|, \qquad (14)$$

where  $-1 \leq \alpha < 1$  and  $\left\|\hat{\mathbf{f}}(k)\right\|_{l_1} = \sum_{l=0}^{L_f-1} \left|\hat{f}_l(k)\right|, l = 0, 1, \ldots, L_f - 1$ . Thus the diagonal elements of  $\mathbf{A}(k-1)$  in (9) can be recalculated as

$$a_{l}\left(k\right) = \frac{1-\alpha}{2L_{\hat{f}}} + (1+\alpha) \frac{\left|\hat{f}_{l}\left(k\right)\right|}{2\left\|\hat{\mathbf{f}}\left(k\right)\right\|_{l_{1}} + \varepsilon},\tag{15}$$

where  $\varepsilon$  is a small positive value added to avoid division by zero. If  $\alpha = -1$ , the IPNLMS becomes the NLMS algorithm. If  $\alpha \approx 1$ , the IPNLMS is similar to the PNLMS. In practice, good choices for  $\alpha$  are -0.5 or 0. The relation among the regularization parameters of the mentioned algorithms are  $\delta_{IPNLMS} = (1 - \alpha) \delta_{NLMS}/2L_{\hat{f}}$  and  $\delta_{PNLMS} = \delta_{NLMS}/L_{\hat{f}}$  [20].

For the IPNLMS algorithm the  $l_0$ -norm can be a good alternative to the  $l_1$ -norm [21]. The  $l_0$ -norm of a vector is the number of its non-zero components. The  $l_0$ -norm of a vector  $\mathbf{b} = [b_0, b_1, \dots, b_{L-1}]$  is defined as

$$\|\mathbf{b}\|_{l_0} = \sum_{l=0}^{L-1} f(b_l), \qquad (16)$$

$$f(b_l) = \begin{cases} 1, & b_l \neq 0\\ 0, & b_l = 0. \end{cases}$$
(17)

The function  $f(b_l)$  can be approximated as [25]

$$f(b_l) \approx 1 - \exp\left(-\gamma \left|b_l\right|\right),\tag{18}$$

where  $\gamma$  is a large positive value. Hence,

$$\|\mathbf{b}\|_{l_0} \approx \sum_{l=0}^{L-1} \left[1 - \exp\left(-\gamma \left|b_l\right|\right)\right].$$
(19)

In the PEM-IPNLMS using the  $l_0$ -norm the equations (13) and (15) are rewritten as follows

$$\lambda_{l}(k) = (1 - \alpha) \frac{\left\| \hat{\mathbf{f}}(k) \right\|_{l_{0}}}{L_{\hat{f}}} + (1 + \alpha) \left[ 1 - e^{\left(-\gamma \left| \hat{f}_{l}(k) \right| \right)} \right], \quad (20)$$

$$a_{l}(k) = \frac{1-\alpha}{2L_{\hat{f}}} + (1+\alpha) \frac{1-\exp\left(-\gamma \left|\hat{f}_{l}(k)\right|\right)}{2\left\|\hat{\mathbf{f}}(k)\right\|_{l_{0}} + \varepsilon}.$$
 (21)

## 3. SIMULATION RESULTS

Fig. 2 shows the IRs of measured acoustic feedback paths of length  $L_f = 100$  with a two-microphone behind-the-ear hearing aid [26] used for the simulations. The first feedback path ( $\mathbf{f}_1$ ) and the second feedback path ( $\mathbf{f}_2$ ) were measured in free-field and with a telephone receiver close to the ear, respectively. The sampling frequency was  $f_s = 16$ kHz. The proposed methods PEM-PNLMS and PEM-IPNLMS are evaluated for both speech and music incoming signals. The speech incoming signal is the same concatenated real male and female speech as in [4], while the music incoming signal is John Lennon's Imagine. All simulations are run for 80s with a sudden change from the free-field to the telephone-near feedback path after 40s. For the evaluation of all mentioned AFC methods we use the normalized misalignment (MIS) and the added stable gain (ASG) [3, 27] which are defined as

$$MIS = 10 \log_{10}(\frac{||\mathbf{f} - \hat{\mathbf{f}}||_2^2}{||\mathbf{f}||_2^2}),$$
(22)

ASG = 
$$10 \log_{10} \frac{1}{\max_{\Omega} |F(\Omega) - \hat{F}(\Omega)|^2}$$
  
-  $10 \log_{10} \frac{1}{\max_{\Omega} |F(\Omega)|^2}$ , (23)



Fig. 2: Impulse responses of measured feedback paths.

where  $\hat{F}(\Omega)$  and  $F(\Omega)$  are the frequency responses of estimated and measured acoustic feedback paths at the normalized frequency  $\Omega$ , respectively. We set the following parameters for all simulations. The adaptive filter of length  $L_f = 64$ , step-size  $\mu = 0.001$ ,  $\delta_{NLMS} = \varepsilon = 10^{-10}$  and  $\alpha = 0$ ,  $\gamma = 50$  were chosen. The prediction-error filter  $\hat{G}(q, k)$  of order 20 was updated using the Levinson-Durbin algorithm every 10 ms. The delay in the feedback canceller path was 1 sample, whereas the delay and gain in the forward path were  $d_k = 96$  samples and  $|K| = 30 \, dB$ , respectively.

Fig. 3 compares the performance of all metioned methods for the speech incoming signal. The proposed PEM-PNLMS outperforms the PEM-NLMS for both initial convergence and tracking rates while remaining a similar steady-state error. The PEM-IPNLMS provides a further improvement in initial convergence and tracking compared to the PEM-PNLMS. The performance of the PEM-IPNLMS using  $l_0$ -norm is slightly better than the PEM-IPNLMS using  $l_1$ -norm.

Fig. 4 illustrates the performance of the proposed methods for the music incoming signal. It can be seen that the proposed PEM-PNLMS and PEM-IPNLMS have slightly faster initial convergence as well as lower misalignment and ASG than the PEM-NLMS for the free-field feedback path. The powerful characteristic of the proposed methods is exposed when the feedback path suddenly changes after 40s, where the tracking of the PEM-PNLMS and PEM-IPNLMS (with  $l_1$ -norm/ $l_0$ -norm) is quicker than that of the PEM-NLMS.

 
 Table 1: Average misalignment and average added stable gain for the PEM using NLMS, PNLMS, IPNLMS algorithms with speech incoming signal.

AFC methods	$\overline{MIS}$	$\overline{ASG}$
PEM-NLMS	-19.1896	20.5104
PEM-PNLMS	-19.9289	21.1166
PEM-IPNLMS (l1-norm)	-20.1112	21.4114
PEM-IPNLMS (l <sub>0</sub> -norm)	-20.1920	21.4603



**Fig. 3**: (a) Misalignment, (b) ASG of the PEM using NLMS, PNLMS, IPNLMS algorithms with speech incoming signal.

**Table 2**: Average misalignment and average added stable gain for the PEM using NLMS, PNLMS, IPNLMS algorithms with music incoming signal.

AFC methods	$\overline{MIS}$	$\overline{ASG}$
PEM-NLMS	-14.6301	16.4178
PEM-PNLMS	-14.9743	17.0056
PEM-IPNLMS ( $l_1$ -norm)	-15.0450	16.8715
PEM-IPNLMS (l <sub>0</sub> -norm)	-15.0675	16.8815

Table 1- 2 present evaluations of the PEM-NLMS, PEM-PNLMS and PEM-IPNLM for the speech and music incoming signals, respectively. It can be seen that the PEM-PNLMS and PEM-IPNLM using  $l_1$ -norm or  $l_0$ -norm yield better average misalignment ( $\overline{MIS}$ ) as well as average added stable gain ( $\overline{ASG}$ ) compared to



Fig. 4: (a) Misalignment, (b) ASG of the PEM using NLMS, PNLMS, IPNLMS algorithms with music incoming signal.

the PEM-NLMS for both cases of the speech and music incoming signals. These results confirm the benefit of the proposed methods when using the tap-dependent step-size for estimating the adaptive filter coefficients for acoustic feedback control in HAs.

## 4. CONCLUSION

In the paper we have implemented and evaluated different adaptive algorithms exploiting the shape of the estimated feedback paths to calculate the tap-dependent step-size which is used for updating the adaptive filter coeficients for AFC using PEM in HAs. The proposed methods significantly improve the tracking rate compared to the PEM-NLMS for both speech and music incoming signals. Their initial convergence outperforms the PEM-NLMS for the case of speech incoming signal and also has slightly improvement for the case of music incoming signal. Moreover, the proposed methods do not require prior knowledge of the feedback paths as well as incoming signal power.

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