ESTIMATION OF THE COMMON PART OF ACOUSTIC FEEDBACK PATHS IN HEARING AIDS USING ITERATIVE QUADRATIC PROGRAMMING

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ABSTRACT

In adaptive feedback cancellation the convergence speed and the computational complexity depend on the number of adaptive parameters used to model the acoustic feedback path. To improve the convergence speed and reduce the computational complexity, it has been proposed to model the acoustic feedback path as the convolution of a time-invariant common pole-zero part and a time-varying variable part. Previous approaches to estimate all the coefficients minimized the so-called equation-error which possibly suffers from poor estimation accuracy in the vicinity of prominent spectral regions, e.g., spectral peaks. In this paper we therefore propose to minimize the so-called output-error by using a Steiglitz-McBride-like iteration scheme. To ensure the stability of the estimated pole-zero filter a frequency domain constraint is used leading to a quadratic programming problem. Experimental results using measured impulse responses from a two-microphone behind-the-ear hearing aid show that the proposed estimation scheme outperforms the existing estimation scheme in terms of modeling accuracy.

Index Terms— acoustic feedback cancellation, common part modeling, hearing aids, weighted least-squares, quadratic programming, Steiglitz-McBride iteration

1. INTRODUCTION

In recent years more and more hearing impaired persons have been supplied with open-fitting hearing aids. While largely alleviating problems related to the occlusion effect (e.g. the perception of one's own voice), open-fitting hearing aids are especially prone to acoustic feedback. This requires robust and fast-adapting acoustic feedback cancellation algorithms.

Although several different approaches exist to solve the problem of acoustic feedback in hearing aids (see e.g. [1] and references therein), adaptive feedback cancellation (AFC) seems the most promising, as it theoretically allows for perfect feedback cancellation. In AFC an adaptive filter is used to model the acoustic impulse response (IR) between the receiver and the microphone of the hearing aid. It is known that in general the computational complexity and the convergence speed of an adaptive filter depend on the number of its adaptive parameters [2]. To reduce the number of adaptive parameters in AFC it was hence proposed [3, 4] to model the feedback path as the convolution of two filters: (1) a fixed filter accounting for invariant or slowly varying parts of the feedback path, and (2) an adaptive filter enabling to track fast changes in the feedback path. The fixed filter can be thought to account for, e.g., fixed mechanical couplings, and fixed transducer and microphone characteristics [4] as well as individual characteristics of a particular ear [5]. Including such a fixed filter aims at reducing the number of adaptive parameters and thereby increasing the speed of convergence of the adaptive filter. The fixed filter may be estimated, e.g., from the IRs of multiple microphones, with the aim of modeling the common part of these IRs. Hence, in the remainder of the paper the fixed filter will be called *common part*, while the time-varying filter that is assumed to be different for each microphone will be called *variable part*.

Different models can be assumed for the common part, e.g., an all-zero filter [4], an all-pole filter [6] or a more general pole-zero filter [5]. An alternating least-squares (ALS) procedure was presented in [5] to estimate the coefficients of the common part using a pole-zero filter and the variable parts using all-zero filters. Similar to [6], where a common all-pole filter was assumed, the cost function that minimizes the so-called output-error was modified to yield an equation-error minimization, thereby simplifying the optimization procedure. However, pole-zero filters estimated using an equation-error minimization may suffer from poor estimation accuracy in the vicinity of prominent spectral regions, e.g., large spectral peaks [7]. One well-known approach to circumvent this problem is the Steiglitz-McBride scheme [8], which iteratively approximates the desired minimization of the output-error. In this paper we propose to use the Steiglitz-McBride scheme in the ALS procedure presented in [5] to estimate all the coefficients of the common part and the variable parts. Thus, we extend the ALS procedure of [5] to approximate the output-error and extend the Steiglitz-McBride scheme [8] to incorporate a common pole-zero model. Since the Steiglitz-McBride scheme in general does not guarantee stability of estimated poles (see e.g. [9]), their location has to be constrained to yield stable IRs. We propose to use a frequency-domain constraint previously used in [10], leading to a quadratic programming problem. Experimental results using measured acoustic feedback paths indicate that the proposed estimation scheme enables to achieve a better modeling accuracy compared to the previously proposed estimation scheme of [5]. Note that this paper is exclusively about the modeling of acoustic feedback paths and may not be confused with acoustic feedback cancellation algorithms.

2. SCENARIO AND NOTATION

Consider a single-input-multiple-output (SIMO) system with M outputs as depicted in Figure 1(a). Such a SIMO system arises, e.g., in a single-loudspeaker multiple-microphone setup in a multi-microphone hearing aid. The m-th output signal $Y_m(z)$ is related to the input signal $X(z)$ by the m-th acoustic transfer function (ATF)

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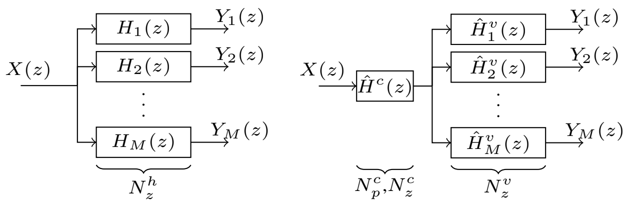

(a) SIMO system (b) Approximation of SIMO system

Fig. 1. System models.

$H_m(z)$, i.e.,

$$Y_m(z) = H_m(z)X(z). \tag{1}$$

We assume that the true ATFs $H_m(z)$ are causal all-zero filters of finite order N_z^h each. To reduce the number of coefficients required to model the M ATFs, we will approximate these ATFs as depicted in Figure 1(b) using

$$\boxed{\begin{bmatrix} \hat{H}_1(z) \\ \vdots \\ \hat{H}_M(z) \end{bmatrix} = \hat{H}^c(z) \begin{bmatrix} \hat{H}_1^v(z) \\ \vdots \\ \hat{H}_M^v(z) \end{bmatrix},} \tag{2}$$

where $\hat{H}_m(z)$ is split into two parts: a common (microphone-independent) part $\hat{H}^c(z)$ and a variable (microphone-dependent) part $\hat{H}_m^v(z)$. We assume that $\hat{H}^c(z)$ is a pole-zero filter with N_p^c poles and N_z^c zeros and $\hat{H}_m^v(z)$ is an all-zero filter with N_z^v zeros for each of the M microphones, i.e.,

$$\hat{H}^c(z) = \frac{B^c(z)}{A^c(z)} = \frac{\sum_{k=0}^{N_z^c} b^c[k]z^{-k}}{1 + \sum_{k=1}^{N_p^c} a^c[k]z^{-k}} \tag{3}$$

$$\hat{H}_m^v(z) = B_m^v(z) = \sum_{k=0}^{N_z^v} b_m^v[k]z^{-k} \tag{4}$$

where $a^c[k]$, $b^c[k]$, and $b_m^v[k]$ are the coefficients associated with the poles of the common part, the zeros of the common part and the zeros of the variable parts, respectively. The coefficient vectors are defined as

$$\mathbf{h}_m = [\ h_m[0] \quad h_m[1] \quad \dots \quad h_m[N_z^h]\ ]^T, \tag{5}$$

$$\mathbf{a}^c = [\ a^c[1] \quad a^c[2] \quad \dots \quad a^c[N_p^c]\ ]^T, \tag{6}$$

$$\mathbf{b}^c = [\ b^c[0] \quad b^c[1] \quad \dots \quad b^c[N_z^c]\ ]^T, \tag{7}$$

$$\mathbf{b}_m^v = [\ b_m^v[0] \quad b_m^v[1] \quad \dots \quad b_m^v[N_z^v]\ ]^T. \tag{8}$$

In addition we also define the zero-padded vectors

$$\tilde{\mathbf{h}}_m = [\ \mathbf{h}_m^T \quad \mathbf{0}^T\ ]^T, \tag{9}$$

$$\tilde{\mathbf{b}}^c = [\ (\mathbf{b}^c)^T \quad \mathbf{0}^T\ ]^T, \tag{10}$$

$$\tilde{\mathbf{b}}_m^v = [\ (\mathbf{b}_m^v)^T \quad \mathbf{0}^T\ ]^T, \tag{11}$$

where $\mathbf{0}$ is a vector containing zeros such that the length of $\tilde{\mathbf{h}}_m$, $\tilde{\mathbf{b}}^c$ and $\tilde{\mathbf{b}}_m^v$ is equal to $\tilde{N}_z^h + N_p^c + 1$, where $\tilde{N}_z^h = \max\{N_z^h, N_z^c + N_z^v\}$. We also define the concatenated vectors

$$\tilde{\mathbf{h}} = [\ \tilde{\mathbf{h}}_1^T \quad \tilde{\mathbf{h}}_2^T \quad \dots \quad \tilde{\mathbf{h}}_M^T\ ]^T, \tag{12}$$

$$\mathbf{b}^v = [\ (\mathbf{b}_1^v)^T \quad (\mathbf{b}_2^v)^T \quad \dots \quad (\mathbf{b}_M^v)^T\ ]^T, \tag{13}$$

$$\tilde{\mathbf{b}}^v = [\ (\tilde{\mathbf{b}}_1^v)^T \quad (\tilde{\mathbf{b}}_2^v)^T \quad \dots \quad (\tilde{\mathbf{b}}_M^v)^T\ ]^T. \tag{14}$$

In the following an iterative scheme is presented to minimize the weighted least-squares cost function using the Steiglitz-McBride scheme estimating all the required coefficients to model the M ATFs using the approximate SIMO system.

3. LEAST-SQUARES ESTIMATION

The objective is to compute the coefficient vectors $\mathbf{a}^c$, $\mathbf{b}^c$ and $\mathbf{b}^v$ that minimize the least-squared error between the true ATFs $H_m(z)$ and the approximated ATF model $\hat{H}_m(z)$ in (2), (3) and (4), i.e., minimizing the cost function

$$\bar{J}_{OE}(\mathbf{a}^c, \mathbf{b}^c, \mathbf{b}^v) = \sum_{m=1}^{M} \|\mathcal{Z}^{-1}\{\underbrace{H_m(z) - \frac{B^c(z)}{A^c(z)} B_m^v(z)}_{E_m^{OE}(z)}\}\|_2^2, \tag{15}$$

where $\mathcal{Z}^{-1}\{\cdot\}$ denotes the inverse z-transform. The error term $E_m^{OE}(z)$ corresponds to the so-called output-error and is in general known to be difficult to minimize [11]. To overcome this difficulty it has been proposed in e.g. [5, 6] to minimize the so-called equation-error $E_m^{EE}(z)$ instead, i.e.,

$$\bar{J}_{EE}(\mathbf{a}^c, \mathbf{b}^c, \mathbf{b}^v) = \sum_{m=1}^{M} \|\mathcal{Z}^{-1}\{\underbrace{A^c(z)H_m(z) - B^c(z)B_m^v(z)}_{E_m^{EE}(z) = A^c(z)E_m^{OE}(z)}\}\|_2^2. \tag{16}$$

The equation-error is easier to minimize and for the problem at hand can be solved using e.g. an ALS procedure in the time-domain [5]. As can been seen from (16) minimizing the equation-error essentially corresponds to minimizing the output-error $E_m^{OE}(z)$ prefiltered with $A^c(z)$ corresponding to the inverse of the all-pole transfer function $\frac{1}{A^c(z)}$ to be estimated. Although being easier to optimize, the minimization of $\bar{J}_{EE}$ in (16) may thus lead to poor estimation accuracy in the vicinity of prominent regions, e.g., spectral peaks, in the frequency response $H_m(e^{j\Omega})$ of $H_m(z)$ (see e.g. [7]). To circumvent this undesired property, in this paper we propose to incorporate the well-known iterative Steiglitz-McBride scheme [8] into the ALS procedure of [5] yielding a novel estimation scheme for the approximate SIMO system in Fig. 1(b). Thus, we aim to minimize the following cost function at each iteration step i:

$$\bar{J}_{WLS}(\mathbf{a}_i^c, \mathbf{b}_i^c, \mathbf{b}_i^v) = \sum_{m=1}^{M} \|\mathcal{Z}^{-1}\{\underbrace{\frac{1}{A_{i-1}^c(z)} E_{m,i}^{EE}(z)}_{E_{m,i}^{WLS}(z)}\}\|_2^2, \tag{17}$$

where $E_{m,i}^{EE}(z)$ is the equation-error at iteration i. As can be seen from (17) in each iteration the equation-error $E_{m,i}^{EE}(z)$ is prefiltered with the all-pole filter $\frac{1}{A_{i-1}^c(z)}$ from the previous iteration. Thus, ideally, at convergence $\lim_{i\to\infty} A_i^c(z) \approx A_{i-1}^c(z)$ and $\lim_{i\to\infty} E_{m,i}^{WLS}(z) \approx E_m^{OE}(z)$ approximating the output-error.

In the following first the minimization of the equation-error cost function in the time-domain using the ALS procedure proposed in [5] is briefly discussed and then the Steiglitz-McBride iteration of (17) is incorporated yielding a novel estimation scheme to approximate the minimization of the output-error cost function for the considered approximate SIMO system.

3.1. Equation-error based estimation scheme [5]

In the time-domain, the equivalent cost function to $\bar{J}_{EE}$ in (16) is given by [5]

$$\boxed{J_{EE}(\mathbf{a}^c, \mathbf{b}^c, \mathbf{b}^v) = \|\tilde{\mathbf{h}} + \tilde{\mathbf{H}}\mathbf{a}^c - \tilde{\mathbf{B}}^v\mathbf{b}^c\|_2^2,} \tag{18}$$

where the concatenated vector of the zero-padded true IRs $\tilde{\mathbf{h}}$ is defined in (12) and

$$\tilde{\mathbf{H}} = [\ \tilde{\mathbf{H}}_1^T \quad \dots \quad \tilde{\mathbf{H}}_M^T\ ]^T \tag{19}$$

is the $M(\tilde{N}_z^h + N_p^c + 1) \times N_p^c$-dimensional matrix of concatenated convolution matrices $\tilde{\mathbf{H}}_m$ of delayed versions of $\tilde{\mathbf{h}}_m$, i.e.,

$$\tilde{\mathbf{H}}_m = \begin{bmatrix} 0 & \dots & \dots & 0 \\ h_m[0] & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ h_m[N_p^c - 1] & \ddots & \ddots & h_m[0] \\ \vdots & \ddots & \ddots & \vdots \\ h_m[N_z^h] & \ddots & \ddots & \vdots \\ 0 & h_m[N_z^h] & \ddots & \vdots \\ \vdots & \ddots & \ddots & h_m[N_z^h] \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 \end{bmatrix}. \tag{20}$$

Similarly, $\tilde{\mathbf{B}}^v$ is the $M(\tilde{N}_z^h+N_p^c+1)\times(N_z^c+1)$-dimensional matrix constructed by concatenating the M convolution matrices $\tilde{\mathbf{B}}_m^v$ of the zero-padded variable part coefficient vectors $\tilde{\mathbf{b}}_m^v$ in (11), i.e.,

$$\tilde{\mathbf{B}}^v = [\ (\tilde{\mathbf{B}}_1^v)^T \quad (\tilde{\mathbf{B}}_2^v)^T \quad \dots \quad (\tilde{\mathbf{B}}_M^v)^T\ ]^T. \tag{21}$$

To minimize the non-linear cost function J_{EE} in (18) an ALS procedure was proposed in [5], thus at each iteration i J_{EE} is split into the following least-squares problems

$$\begin{cases} J_{EE}^v(\mathbf{b}_i^v) = \|\tilde{\mathbf{h}} + \tilde{\mathbf{H}}\mathbf{a}_{i-1}^c - \check{\mathbf{B}}_{i-1}^c\mathbf{b}_i^v\|_2^2, & \text{(22a)} \\ J_{EE}^c(\mathbf{a}_i^c, \mathbf{b}_i^c) = \|\tilde{\mathbf{h}} + \tilde{\mathbf{H}}\mathbf{a}_i^c - \tilde{\mathbf{B}}_i^v\mathbf{b}_i^c\|_2^2, & \text{(22b)} \end{cases}$$

where

$$\check{\mathbf{B}}_{i-1}^c = \begin{bmatrix} \tilde{\mathbf{B}}_{i-1}^c & & \\ & \ddots & \\ & & \tilde{\mathbf{B}}_{i-1}^c \end{bmatrix} \tag{23}$$

and $\tilde{\mathbf{B}}_{i-1}^c$ is the $(\tilde{N}_z^h+N_p^c+1)\times(N_z^v+1)$-dimensional convolution matrix of the zero-padded common part zero coefficient vector $\tilde{\mathbf{b}}_{i-1}^c$ defined in (10). The cost functions in (22) are then minimized alternatingly until convergence is achieved. Due to the convolution of the common all-zero filter $\mathbf{b}_i^c$ and the variable all-zero filters $\mathbf{b}_{m,i}^v$ both filters can only be uniquely identified up to a constant scalar. Therefore, to achieve a unique solution, prior to each iteration i $\mathbf{b}_{i-1}^c$ is normalized to unit-norm [5]. The complete estimation scheme of [5] is summarized in Table 1.

Table 1. Overview of ALS procedure [5]

input N_p^c, N_z^c, N_z^v, $\mathbf{h}_m$ $\forall m \in [1, M]$
initialize $\mathbf{a}_{i-1}^c$, $\mathbf{b}_{i-1}^c$
repeat
normalize common part zero coefficients for uniqueness
$\mathbf{b}_{i-1}^c \leftarrow \mathbf{b}_{i-1}^c / \|\mathbf{b}_{i-1}^c\|_2$,
estimate variable part coefficients
$\mathbf{b}_i^v \leftarrow$ minimize (22a),
estimate common part coefficients
$\mathbf{a}_i^c, \mathbf{b}_i^c \leftarrow$ minimize (22b),
$i \leftarrow i + 1$
until convergence

3.2. Proposed estimation scheme

It is known that the equation-error based optimization procedures may suffer from poor estimation accuracy in the vicinity of prominent spectral regions [7]. Therefore, we aim to approximate the desired minimization of the output-error in (15) by incorporating the Steiglitz-McBride scheme [8], i.e., the iterative prefiltering of the equation-error using $\frac{1}{A_{i-1}^c(z)}$, into the ALS procedure minimizing the equation-error presented in [5]. Thus, the goal of the proposed estimation scheme is to estimate the coefficient vectors that minimize the equivalent cost function to (17) in the time-domain, i.e.,

$$\boxed{\begin{aligned} &J_{WLS}(\mathbf{a}_i^c, \mathbf{b}_i^c, \mathbf{b}_i^v) \\ &\quad = \sum_{m=1}^{M} \| \frac{1}{A_{i-1}^c(q^{-1})} (\tilde{\mathbf{h}}_m + \tilde{\mathbf{H}}_m\mathbf{a}_i^c - \tilde{\mathbf{B}}_{m,i}^v\mathbf{b}_i^c)\|_2^2, \end{aligned}} \tag{24}$$

where q^{-1} is the unit-delay operator, i.e., $q^{-1}h[k] = h[k-1]$. As can be seen be comparing the cost functions J_{EE} in (18) and J_{WLS} in (24), the equation-error being minimized in (18) is prefiltered with the all-pole filter $\frac{1}{A_{i-1}^c(q^{-1})}$ in (24) aiming to approximate the output-error. Similar to the minimization of J_{EE} in (18) the minimization of J_{WLS} in (24) can be carried out by using an alternating scheme similar to (22) with the following least-squares problems

$$\begin{cases} J_{WLS}^v(\mathbf{b}_i^v) = \|\tilde{\mathbf{h}}_i^p + \tilde{\mathbf{H}}_i^p\mathbf{a}_{i-1}^c - \check{\mathbf{B}}_{i-1}^{c,p}\mathbf{b}_i^v\|_2^2, & \text{(25a)} \\ J_{WLS}^c(\mathbf{a}_i^c, \mathbf{b}_i^c) = \|\tilde{\mathbf{h}}_i^p + \tilde{\mathbf{H}}_i^p\mathbf{a}_i^c - \tilde{\mathbf{B}}_i^{v,p}\mathbf{b}_i^c\|_2^2. & \text{(25b)} \end{cases}$$

The superscript p indicates prefiltered quantities, i.e., by filtering the corresponding quantities in (22) by $\frac{1}{A_{i-1}^c(q^{-1}))}$, i.e.,

$$\tilde{\mathbf{h}}_{m,i}^p = (\hat{A}_{i-1}^c(q^{-1}))^{-1}\tilde{\mathbf{h}}_m, \tag{26}$$

$$\tilde{\mathbf{b}}_{i-1}^{c,p} = (\hat{A}_{i-1}^c(q^{-1}))^{-1}\tilde{\mathbf{b}}_{i-1}^c, \tag{27}$$

$$\tilde{\mathbf{b}}_{m,i}^{v,p} = (\hat{A}_{i-1}^c(q^{-1}))^{-1}\tilde{\mathbf{b}}_{m,i}^v. \tag{28}$$

The matrices $\tilde{\mathbf{H}}_{i-1}^p$, $\tilde{\mathbf{B}}_{i-1}^{v,p}$ and $\check{\mathbf{B}}_i^{c,p}$ in (25) are similarly defined as $\tilde{\mathbf{H}}_{i-1}$, $\tilde{\mathbf{B}}_{i-1}^v$ and $\check{\mathbf{B}}_i^c$ in (19), (21) and (23) but using the prefiltered quantities $\tilde{\mathbf{h}}_{m,i}^p$, $\tilde{\mathbf{b}}_{i-1}^{c,p}$ and $\tilde{\mathbf{b}}_{m,i}^{v,p}$ in (26) - (28). Again, due to the convolution of $\mathbf{b}_{m,i}^v$ and $\mathbf{b}_i^c$ the individual filters can only be determined uniquely up to a constant scalar, therefore, similar as in [5], $\mathbf{b}_i^c$ is normalized to unit-norm prior to each iteration.

3.3. Stability constraint

Since it is known that in general the iterative scheme in (25) does not guarantee the stability of the estimated filter (see e.g. [9]), the poles,

Table 2. Overview of proposed estimation scheme

input N_p^c, N_z^c, N_z^v, $\mathbf{h}_m \; \forall m \in [1, M]$
initialize $\mathbf{a}_{i-1}^c$, $\mathbf{b}_{i-1}^c$
repeat
prefilter true IRs
$\tilde{\mathbf{h}}_{m,i}^p \leftarrow (\hat{A}_{i-1}^c(q^{-1}))^{-1}\tilde{\mathbf{h}}_m \; \forall m \in [1, M]$,
normalize common part zero coefficients for uniqueness
$\mathbf{b}_{i-1}^c \leftarrow \mathbf{b}_{i-1}^c / \|\mathbf{b}_{i-1}^c\|_2$,
prefilter common part zero coefficients
$\tilde{\mathbf{b}}_{i-1}^{c,p} \leftarrow (\hat{A}_{i-1}^c(q^{-1}))^{-1}\tilde{\mathbf{b}}_{i-1}^c$,
estimate variable part coefficients
$\mathbf{b}_i^v \leftarrow$ minimize (25a),
prefilter variable part zero coefficients
$\tilde{\mathbf{b}}_{m,i}^{v,p} \leftarrow (\hat{A}_{i-1}^c(q^{-1}))^{-1}\tilde{\mathbf{b}}_{m,i}^v \; \forall m \in [1, M]$,
estimate common part coefficients
$\mathbf{a}_i^c, \mathbf{b}_i^c \leftarrow$ minimize (25b) s.t. (30),
$i \leftarrow i + 1$
until convergence

i.e., the roots of $A^c(z)$, need to be constrained to strictly lie inside the unit circle. In [10] it has been shown, that a sufficient (but not necessary) condition for the stability of $\frac{1}{A^c(z)}$ is that the real part of the frequency response $A^c(e^{j\Omega})$ is strictly positive, i.e.,

$$\mathcal{R}\{A^c(e^{j\Omega})\} > 0 \quad \forall \Omega \tag{29}$$

where $\mathcal{R}\{\cdot\}$ denotes the real part and Ω is the normalized frequency. To control the strength of the constraint a small positive constant $\delta > 0$ is typically introduced [10], i.e.

$$\mathcal{R}\{A^c(e^{j\Omega})\} \geq \delta \quad \forall \Omega. \tag{30}$$

This constraint can then be implemented by evaluating (30) over a dense grid of frequencies. The minimization of (25b) subject to the constraint (30) corresponds to a quadratic programming problem [12] and can be efficiently solved using, e.g., the MATLAB function *quadprog.m*. An overview summarizing all steps of the proposed estimation scheme is given in Table 2.

4. EXPERIMENTAL VALIDATION

In this section the proposed estimation scheme minimizing the weighted least-squares error using the iterative Steiglitz-McBride scheme (cf. Table 2) is evaluated and compared to the estimation scheme minimizing the equation-error proposed in [5]. Two acoustic feedback paths ($M = 2$) were measured using a two-microphone behind-the-ear hearing aid with open-fitting earmolds on a dummy head with adjustable ear canals similar to the ear canals presented in [13]. The IRs were sampled at $f_s = 16$ kHz and truncated to order $N_z^h = 99$. The measured feedback paths are depicted in Figure 2. As a performance measure for modeling accuracy the average normalized misalignment ε between the true (measured) IRs $\mathbf{h}_m$ and the estimated IRs $\hat{\mathbf{h}}_m$ is used, i.e.,

$$\varepsilon = 10\log_{10}\frac{1}{2}\sum_{m=1}^{2}\frac{\|\mathbf{h}_m - \hat{\mathbf{h}}_m\|_2^2}{\|\mathbf{h}_m\|_2^2}. \tag{31}$$

The coefficient vectors were initialized as $\hat{\mathbf{a}}_0^c = \mathbf{0}$ and $\hat{\mathbf{b}}_0^c = [\,1 \quad 0 \quad \ldots \quad 0\,]$. Although in general different initializations are possible, e.g., random or all-one sequences, it is beyond the scope of this paper to compare different initializations. The constraint in (30) was evaluated over a grid of 1024 equally spaced frequencies in the range $[0 - 8]$ kHz and $\delta = 10^{-8}$ was used. Convergence was assumed when the relative change of each of the coefficients of the common part and the variable parts as well as the relative change of J_{WLS}^c in (25b) was smaller than $5 \cdot 10^{-4}$.

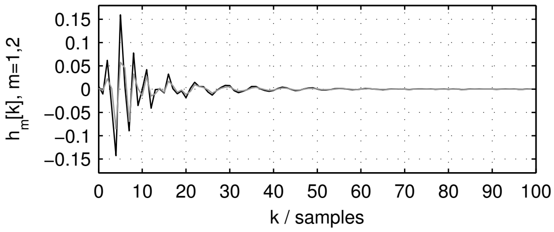

Fig. 2. Two measured feedback paths used in the experiment. The black line and gray line indicate $h_1[k]$ and $h_2[k]$, respectively.

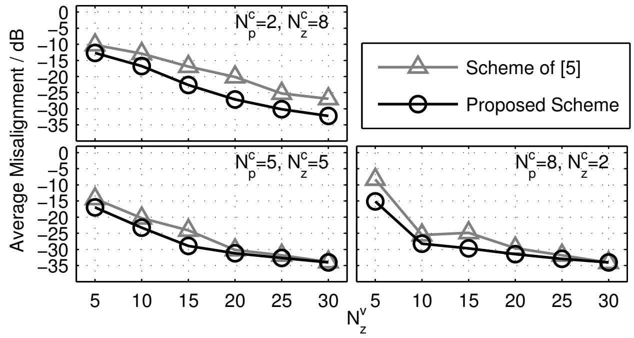

Fig. 3. Average misalignment as a function of N_z^v for different combinations that yield $N_p^c + N_z^c = 10$.

To compare the impact of different numbers of common poles and zeros, results for three different combinations of N_p^c and N_z^c leading to $N_p^c + N_z^c = 10$ as a function of N_z^v are depicted in Figure 3. This allows for direct comparison of the three combinations. It can be seen that an improvement over the estimation scheme proposed in [5] in almost all conditions is achieved. For the combination of $N_p^c = 2$, $N_z^c = 8$ the improvements as large as 7 dB for $N_z^v = 20$ can be achieved. In comparison, for both other combinations ($N_p^c = 5$, $N_z^c = 5$ and $N_p^c = 8$, $N_z^c = 2$) improvements tend to be smaller but decreases in normalized misalignment of as large as 5 dB can be observed for the proposed estimation scheme. In general, results indicate that the proposed estimation scheme shows largest improvements for low to medium number of variable part coefficients, i.e., for the presented data $N_z^v < 25$. Similar performance is observed for other sets of feedback paths. Thus, by using the proposed estimation scheme to estimate a common pole-zero model an increase in modeling accuracy can be achieved compared to the previously proposed estimation scheme of [5].

5. CONCLUSION

In this paper an iterative weighted least-squares scheme for estimating a common part and M variable parts from a set of M impulse-responses based on the well-known Steiglitz-McBride scheme was proposed. To guarantee the stability of the estimated common pole-zero model a frequency-domain constraint was incorporated yielding a quadratic programming problem. Experimental results using measured acoustical feedback paths indicate that the proposed estimation scheme outperforms the estimation scheme minimizing the equation-error in terms of modeling accuracy.

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