Black Holes with Gauge and Higgs Fields

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Outline

1. Acknowledgement VW
2. Introduction to Black Holes
   - General Relativity
   - Schwarzschild Black Holes
   - Kerr Black Holes
3. Microscopic Black Holes
   - Maxwell Theory
   - Kerr–Newman Black Holes
4. Non-Abelian Black Holes
   - Einstein-Yang-Mills Black Holes
   - Einstein-Yang-Mills-Higgs Black Holes
5. Outlook

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Thanks to

VolkswagenStiftung

Proposal for International Co-operation:
Between Europe and the Orient – A Focus on Research and Higher Education in/on Central Asia and the Caucasus

Outline

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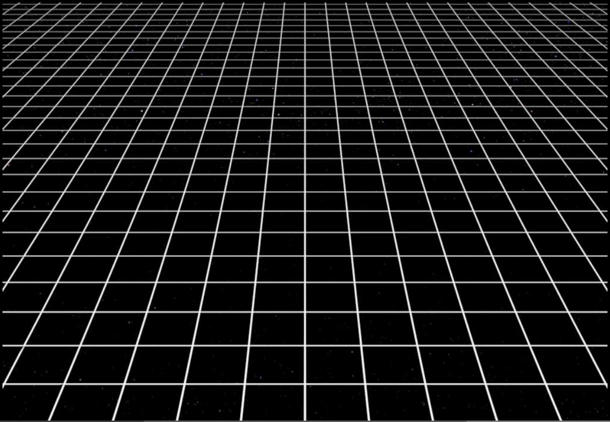
Flat Space–Time

- metric of Euclidean space

$$ds^2 = dx^2 + dy^2 + dz^2$$

- metric of Minkowski space-time

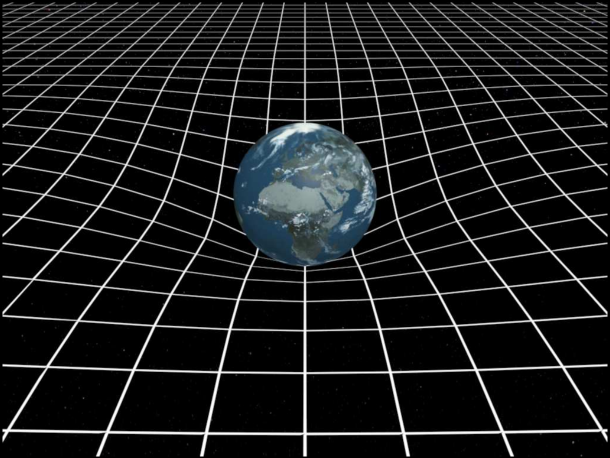
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$



Curved Space–Time

- metric of curved space-time

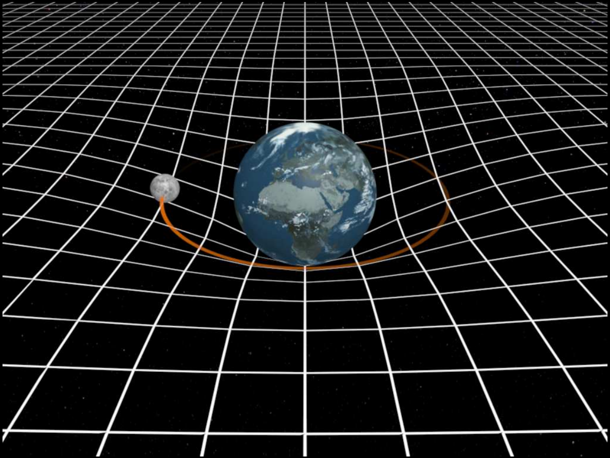
$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$



Motion in Curved Space–Time

- motion in curved space–time

$$0 = \frac{d^2 x^\mu}{ds^2} + \left\{ {\mu \atop \rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}$$



Strongly Curved Space–Time

- metric of curved space–time

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

Einstein Equations

- metric

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

- Einstein equations

  matter tells space how to curve

$$\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

  $G_{\mu\nu}$: Einstein tensor

  $T_{\mu\nu}$: energy-momentum tensor

- equations of motion for matter/radiation

  metric $g_{\mu\nu}$ tells matter how to move

Einstein Equations

Outline

Schwarzschild Metric

Schwarzschild 1916

- space-time outside a star: $T_{\mu\nu} = 0$

$$ds^2 = -N(r)\, c^2 dt^2 + \frac{1}{N(r)}\, dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$N(r) = 1 - \frac{2GM}{c^2 r}$$

static spherically symmetric metric

remark: Minkowski space-time has $N(r) = 1$

- space–time inside a star: $T_{\mu\nu} \neq 0$

Karl Schwarzschild 1873 — 1916

Schwarzschild Singularity

- Schwarzschild space-time

$$ds^2 = -N(r)\, c^2 dt^2 + \frac{1}{N(r)}\, dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

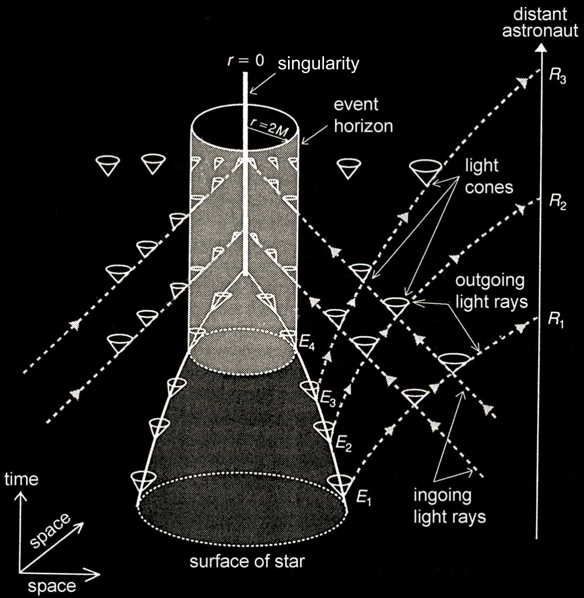
$$N(r) = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_{\rm H}}{r}$$

- black holes: M
- Schwarzschild radius $r_{\rm H}$

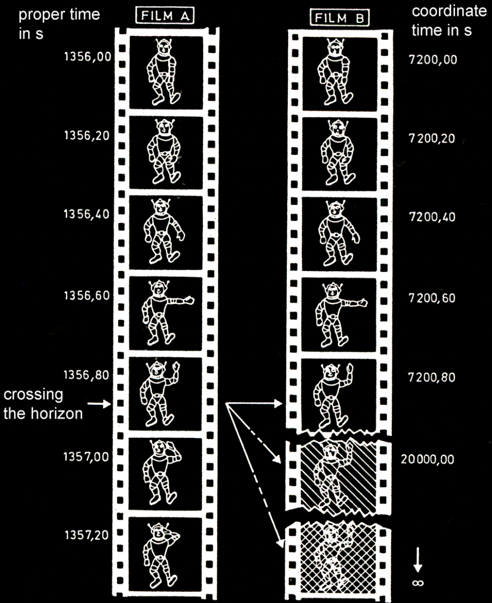
$$N(r_{\rm H}) = 0: \; r_{\rm H} = \frac{2GM}{c^2}$$

- event horizon
- coordinate singularity
- true singularity $r = 0$

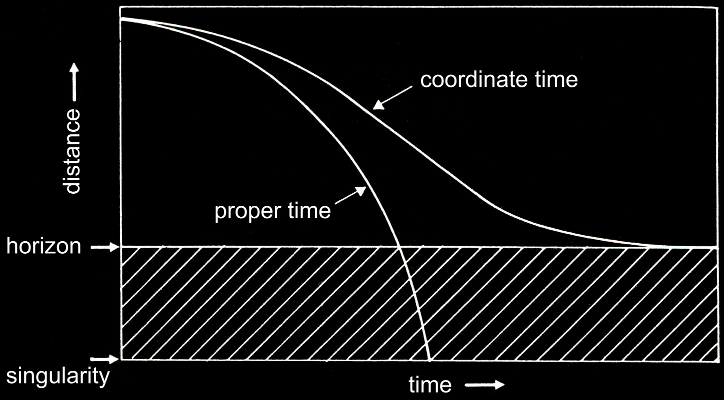
Formation of a Black Hole



Event Horizon



Event Horizon

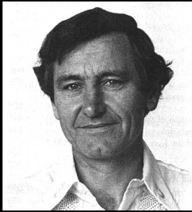


Outline

Rotating Black Holes

rotating generalization of the Schwarzschild black holes: Kerr (1963)

Kerr metric in Boyer–Lindquist coordinates



Roy Kerr *1934

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\rho^2}\left(adt - \rho_0^2 d\phi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta\ , \quad \rho_0^2 = r^2 + a^2\ , \quad \Delta = r^2 - 2Mr + a^2$$

a is the specific angular momentum: $a = \frac{J}{M}$

$a = 0$: Schwarzschild

Kerr Black Holes in the Equatorial Plane

metric in Boyer–Lindquist coordinates:
equatorial plane: $\theta = \pi/2$
through center of black hole, perpendicular to the spin axis

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 - \frac{4Ma}{r} dt d\phi + \frac{dr^2}{1 - \frac{2M}{r} + \frac{a^2}{r^2}} + \left(1 + \frac{a^2}{r^2} + \frac{2Ma^2}{r^3}\right) r^2 d\phi^2$$

comparison with Schwarzschild ($a \neq 0$)

- dt^2 Term: static limit
- $dt\, d\phi$ Term: frame dragging and Lense–Thirring
- dr^2 Term: event horizon

Event Horizon of Kerr Black Holes

First new feature

coordinate singularity:

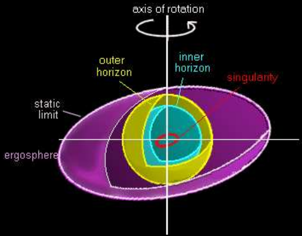
$$1 - \frac{2M}{r} + \frac{a^2}{r^2} = 0$$

radial coordinate of the horizon r_{H}

$$r_{\mathrm{H}} = M \pm \sqrt{M^2 - a^2}$$

- $a < M$
  - $+$: event horizon of the black hole
  - $-$: inner horizon

  maximal angular momentum $a = M$: extremal black hole
- $a > M$: naked singularity (Cosmic Censorship)



black hole with horizons

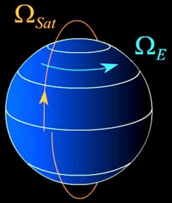
Sir
Roger Penrose
*1931

Gravitomagnetism

Second new feature

- The product $dt\,d\phi$ implies that the coordinates t and ϕ are intimately linked.
- The Kerr metric predicts Lense–Thirring effect and frame dragging.

What does Lense–Thirring mean?



- Schwarzschild: radial rocket thrust is required to keep a stationary observer at a fixed radius
- Kerr: additional tangential rocket thrust is required to keep an observer at a stationary position, i.e., a position from which the fixed stars do not appear to move

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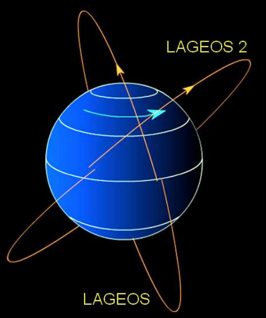

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Gravitomagnetism

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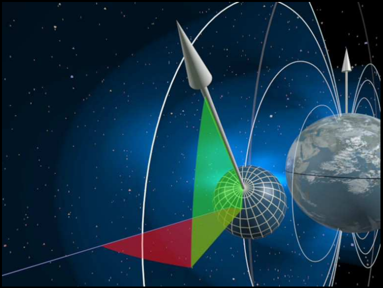
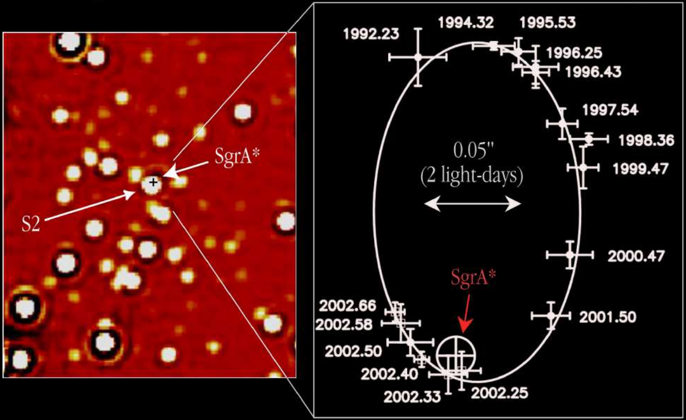
What does frame dragging mean?



- Schwarzschild: gyroscopes always point in the same direction
- Kerr: gyroscpes start to precess, i.e., the direction with respect to distant stars changes

Black Hole at the Center of the Milky Way



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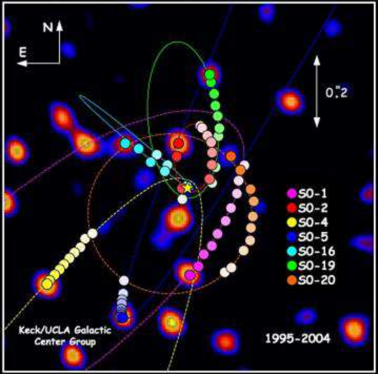

Black Hole: mass $3.7 \cdot 10^6\ M_\odot$ (Yusuf–Zasdeh *et al.* *Astrophys. J.* **644**, 198 (2006))
angular velocity $\sim 1/17\ \mathrm{min}$

R. Genzel (1995 – 2006)

Outline

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Maxwell Theory

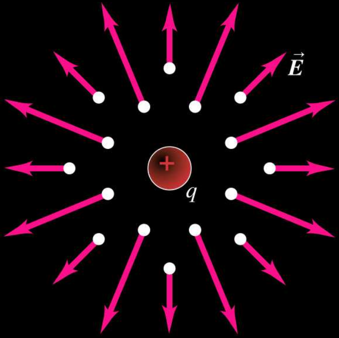
electric field $\vec{E}$, magnetic field $\vec{B}$
electromagnetic potential A^μ: $(\Phi, \vec{A})$

$$\vec{E} = -\nabla\Phi - \partial_t \vec{A} \,, \quad \vec{B} = \nabla \times \vec{A}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

the field strength tensor $F_{\mu\nu}$ is gauge invariant, while $A_\mu \to A_\mu + \partial_\mu \chi$

- the electromagnetic field can carry energy
- the electromagnetic field can carry momentum
- the electromagnetic field can carry angular momentum



Coulomb field

Outline

Einstein–Maxwell Equations

Einstein–Maxwell theory

- Einstein equations

Einstein tensor $\longrightarrow$ $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ $\longleftarrow$ stress–energy tensor

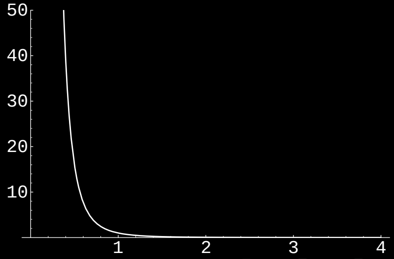
- Maxwell field equations

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = 0$$

For spherical symmetry:

$T_{00} \sim E^2 \sim \dfrac{1}{r^4}$

what are the properties of charged black holes?



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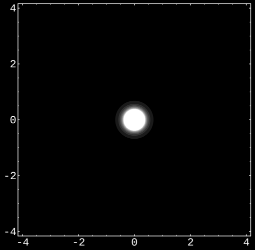

Reissner–Nordström Black Holes

1916: H. Reissner, 1918: G. Nordström
charged static spherically symmetric black holes:

$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

- electrically charged black hole: M, Q
  - horizons: $r_\pm = M \pm \sqrt{M^2 - Q^2}$
  - event horizon: $r_\mathrm{H} = M + \sqrt{M^2 - Q^2}$
- magnetically charged black hole: M, P

energy density outside the horizon due to the Coulomb field of the charge Q

$$M = M_\mathrm{H} + M_\mathrm{outside} = M_\mathrm{H} + 2\Phi_\mathrm{H} Q$$

Hans J. Reissner
1874 – 1967

Gunnar Nordström
1881 – 1923

Reissner–Nordström Black Holes

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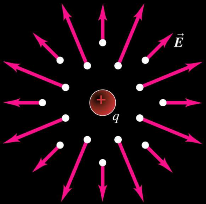
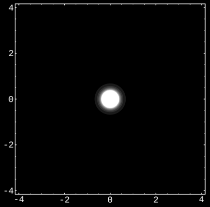
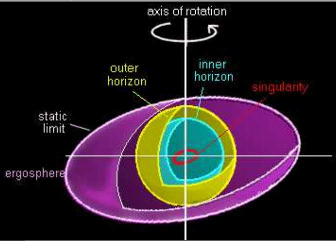




Kerr–Newman Black Holes

charged rotating black holes: adding charge to the Kerr solution

global charges:	M, $J = aM$, Q, P	
dipole moments:	$\mu_{\rm mag} = g_{\rm mag}\dfrac{Q}{2M}J, \quad \mu_{\rm el} = g_{\rm el}\dfrac{P}{2M}J$	$(g_{\rm Dirac} = 2)$
horizons:	$r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2 - P^2}$	$(\Delta = 0)$
static limit:	$r_0 = M + \sqrt{M^2 - a^2\cos^2\theta - Q^2 - P^2}$	$(g_{tt} = 0)$
horizon velocity:	$\Omega = \dfrac{a}{r_+^2 + a^2}$	



Summary: Einstein–Maxwell Black Holes

	static	rotating
spherically symmetric	Schwarzschild (M) Reissner-Nordström (M, Q, P)	–
axially symmetric	–	Kerr (M, J) Kerr–Newman (M, Q, P, J)

- Uniqueness theorem

  black holes are uniquely determined by their mass M, angular momentum J, charges Q and P
- Israel's theorem

  static black holes are spherically symmetric

Werner Israel

*1931

Outline

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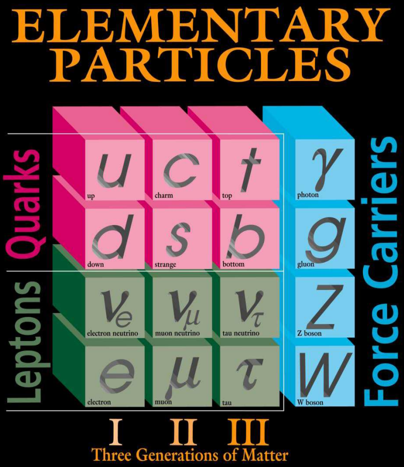
Standard Model of Particle Physics

- What types of black holes do current particle physics theories predict?
- Standard Model is modelled after Maxwell's Theory of Electromagnetism
- Standard Model: gauge field theory



Non–Abelian Fields

- standard model
  - QCD: gluons $a = 1, \ldots, 8$
  - WS: $W^{\pm}$, Z^0

    non-Abelian gauge fields

    non-linearity in field strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

What are the consequences of the presence of non–Abelian fields?

Non–Abelian Fields

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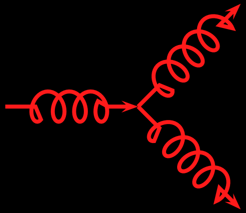
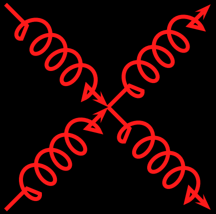




Einstein-Yang-Mills Theory

Einstein-Yang-Mills action

$$\mathcal{S} = \int \Big\{ \underbrace{\frac{R}{16\pi G}}_{\text{gravity}} - \underbrace{\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})}_{\text{Yang-Mills (YM)}} \Big\} \sqrt{-g}\, d^4x$$

- YM gauge potential $A_\mu = A_\mu^a \frac{\tau^a}{2}$
- YM field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu]$

Einstein equations

Einstein tensor $\longrightarrow$ $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ $\longleftarrow$ stress–energy tensor

Yang-Mills field equations

$$\frac{1}{\sqrt{-g}} D_\mu(\sqrt{-g} F^{\mu\nu}) = 0$$

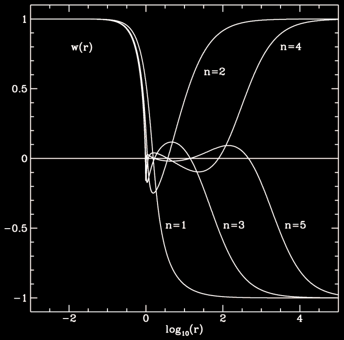
Static Spherically Symmetric EYM Solutions

globally regular solutions: Bartnik, McKinnon 1988

metric: $$ds^2 = -A^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 d\Omega^2$$

gauge potential: $$A_\mu dx^\mu = \frac{1 - w_k(r)}{2} [\tau_\varphi d\theta - \sin\theta \tau_\theta d\varphi]$$

- regular at $r = 0$
- asymptotically flat
- node number k
  $k = 1, ..., \infty$
- dimensionless mass M_k
  $M_1 = 0.83, ..., M_\infty = 1$
- no charge



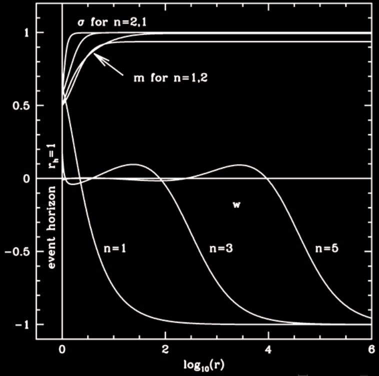
Static Spherically Symmetric EYM Solutions

black hole solutions: Volkov, Gal'tsov 1989, et al.

metric: $$ds^2 = -A^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 d\Omega^2$$

gauge potential: $$A_\mu dx^\mu = \frac{1 - w_k(r)}{2}\left[\tau_\varphi d\theta - \sin\theta \tau_\theta d\varphi\right]$$

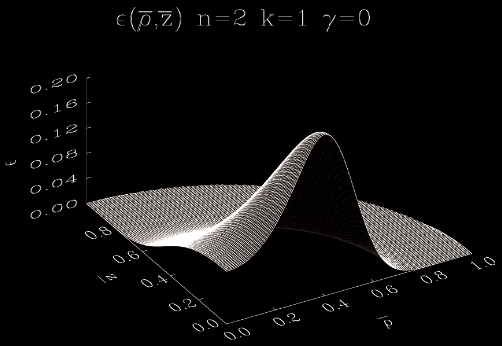
- regular at $r = r_{\rm H}$
- asymptotically flat
- node number k
  $k = 1, ..., \infty$
- limiting solution
  $k \to \infty$: RN
- no charge
- no uniqueness



Static Axially Symmetric EYM Solutions

globally regular solutions: Kleihaus, Kunz 1997

- regular
- asymptotically flat
- node number k
- winding number n
- no charge

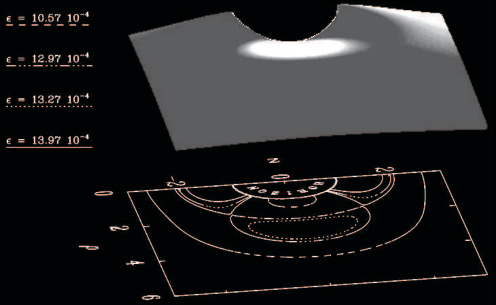


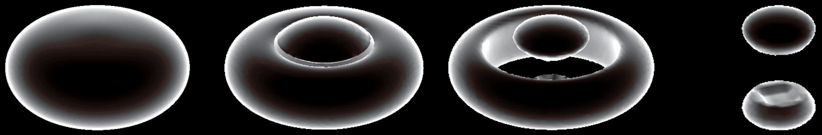
$$\epsilon = -T_0^0$$

Static Axially Symmetric Black Holes

black hole solutions: Kleihaus, Kunz 1997

- regular horizon
  $f(r_{\mathrm{H}}) = 0$
- asymptotically flat
- node number k
- winding number n
- no charge
- no uniqueness





$\epsilon = 10.57 \cdot 10^{-4}$ $\epsilon = 12.97 \cdot 10^{-4}$ $\epsilon = 13.27 \cdot 10^{-4}$ $\epsilon = 13.97 \cdot 10^{-4}$

Static Axially Symmetric EYM Black Holes

black hole solutions: Kleihaus, Kunz 1997

circumferences of horizon:

$$L_e = \int_0^{2\pi} \sqrt{\frac{l}{f}}\, x\ \sin\theta\, d\varphi\ , \quad L_p = 2\int_0^{\pi} \sqrt{\frac{m}{f}}\, x\, d\theta$$

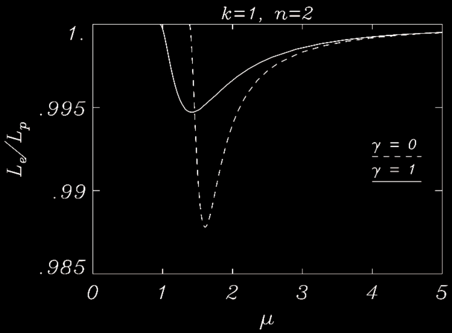
spherical symmetry:

$L_e = L_p$

prolate black holes:

$L_e < L_p$

Israel's theorem does not hold



New regular solutions

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New regular solutions with axial symmetry in Einstein–Yang–Mills theory

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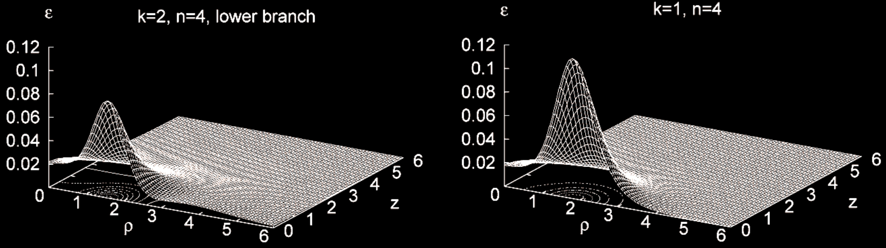
Abstract

We construct new regular solutions in Einstein–Yang–Mills theory. They are static, axially symmetric and asymptotically flat. They are characterized by a pair of integers (k, n), where k is related to the polar angle and n to the azimuthal angle. The known spherically and axially symmetric EYM solutions have $k = 1$. For $k > 1$ new solutions arise, which form two branches. They exist above a minimal value of n, that increases with k. The solutions on the lower mass branch are related to certain solutions of Einstein–Yang–Mills–Higgs theory, where the nodes of the Higgs field form rings.

New regular solutions

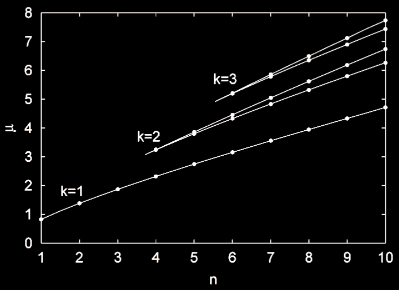
Ibadov, Kleihaus, Kunz, Shnir 2005



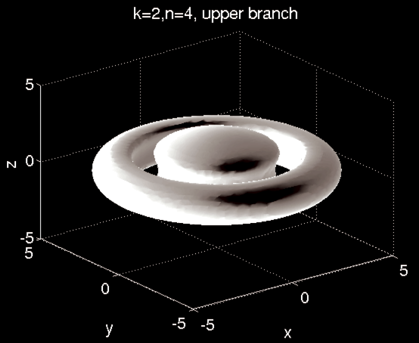
energy density of two of the new solutions

New regular solutions

Ibadov, Kleihaus, Kunz, Shnir 2005



mass of the new solutions



surface of constant energy density

New black hole solutions

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New black hole solutions with axial symmetry in Einstein–Yang–Mills theory

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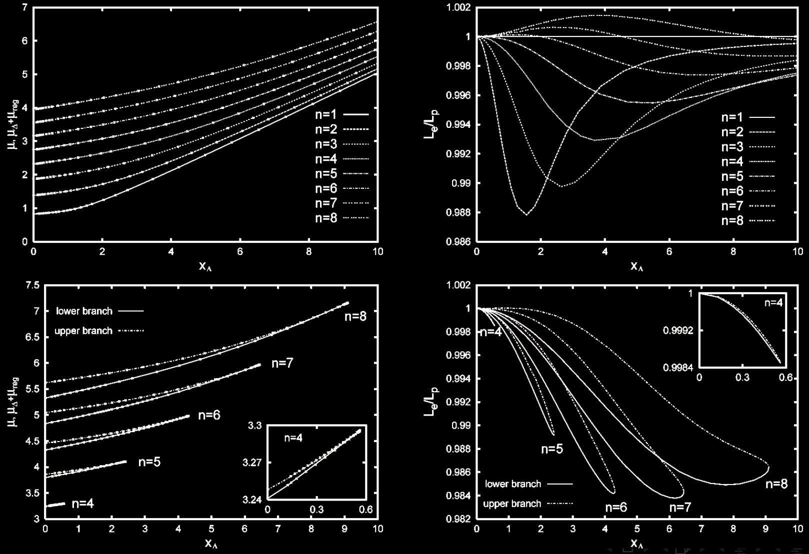
Abstract

We construct new black hole solutions in Einstein–Yang–Mills theory. They are static, axially symmetric and asymptotically flat. They are characterized by their horizon radius and a pair of integers (k, n), where k is related to the polar angle and n to the azimuthal angle. The known spherically and axially symmetric EYM black holes have $k = 1$. For $k > 1$, pairs of new black hole solutions appear above a minimal value of n, that increases with k. Emerging from globally regular solutions, they form

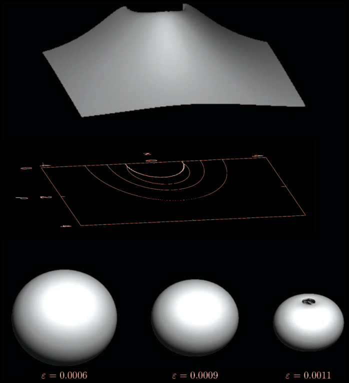
New black hole solutions

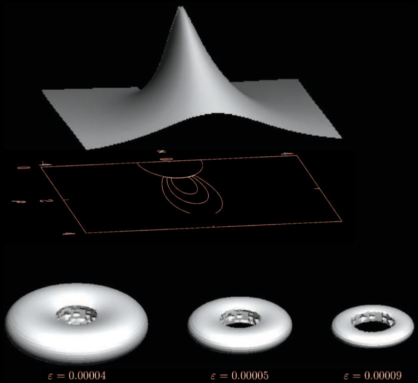
Ibadov, Kleihaus, Kunz, Wirschins 2005



Energy Density of Rotating EYM Black Holes

Kleihaus, Kunz 2001
Kleihaus, Kunz, Navarro-Lérida 2002





$$M = 2.4 \qquad J = 1.9$$

slow

$$M = 10.3 \qquad J = 103$$

fast

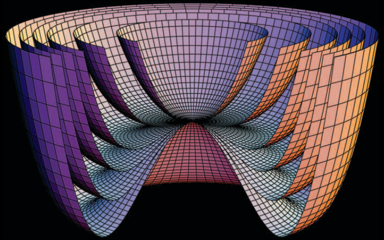
Outline

Einstein-Yang-Mills-Higgs Theory

Einstein-Yang-Mills-Higgs action

$$\mathcal{S} = \int \left\{ \underbrace{\frac{R}{16\pi G}}_{\text{gravity}} - \underbrace{\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})}_{\text{Yang-Mills (YM)}} - \underbrace{\frac{1}{4}\mathrm{Tr}\left\{D_\mu \Phi D^\mu \Phi\right\}}_{\text{Higgs}} - \underbrace{\frac{\lambda}{8}\mathrm{Tr}\left(\Phi^2 - \eta^2\right)^2}_{\text{Higgspotential}} \right\} \sqrt{-g}\, d^4x$$

Mexican hat potential:



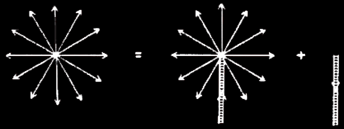
spontaneous symmetry breaking:
SU(2) $\longrightarrow$ U(1)

gauge bosons: $m_{W^\pm} = g\eta\,, m_\gamma = 0$
Higgs boson: $m_H = \sqrt{2\lambda}\eta$

Abelian and non-Abelian magnetic monopoles

Abelian monopole:

Dirac 1931/1948



Wu-Yang 1975

northern hemisphere

$$\vec{A}_{\rm D} = \frac{g_{\rm m}}{r} \frac{1 - \cos\theta}{\sin\theta} \vec{e}_\varphi$$

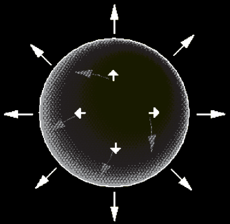
southern hemisphere

$$\vec{A}_{\rm D} = -\frac{g_{\rm m}}{r} \frac{1 + \cos\theta}{\sin\theta} \vec{e}_\varphi$$

non-Abelian monopole:

’t Hooft 1974, Polyakov 1974

- gauge group SU(2)
- globally regular static solutions
- magnetic charge
- finite energy

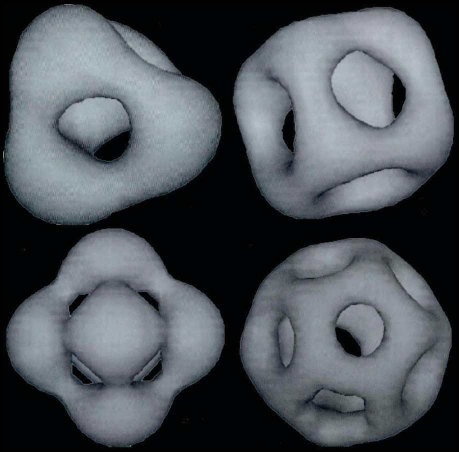


Monopoles and Multimonopoles

Houghton, Sutcliffe 1996 in flat space

$P = 3$
tetrahedral

$P = 4$
cubic

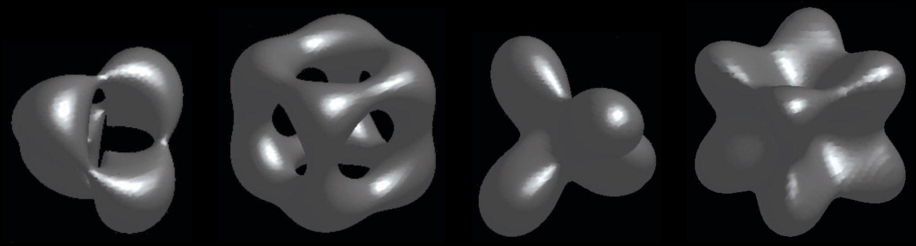


$P = 5$
octahedral

$P = 7$
dodecahedral

Monopoles and Multimonopoles

Ioannidou, Kleihaus, Kunz, Myklevoll 2006
in curved space (with approximations)

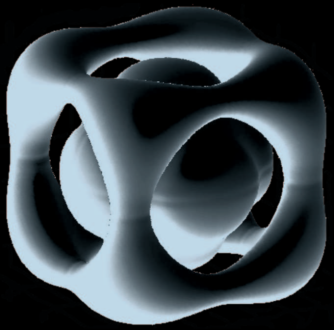


Platonic Black Holes

Challenge:
Are there black holes with only discrete symmetries?

Rustam Ibadov
Theodora Ioannidou,
Burkhard Kleihaus,
Jutta Kunz,
Kari Myklevoll

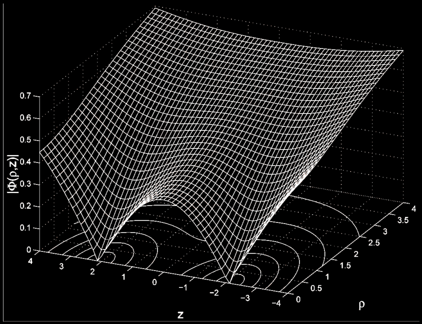
work in progress

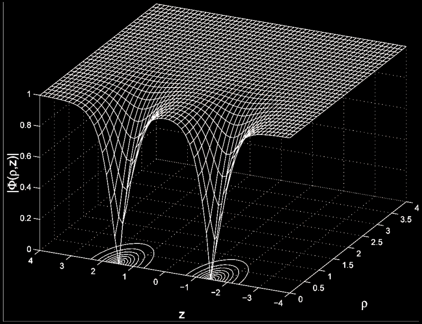


platonic black holes?

Monopol-Antimonopole Pairs: MAPs

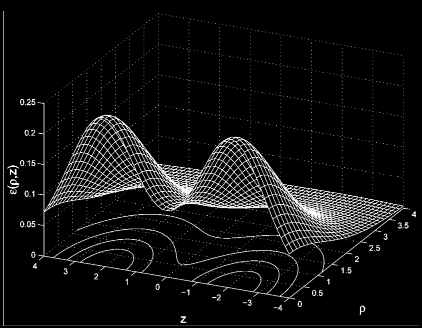
Kleihaus, Kunz 1999

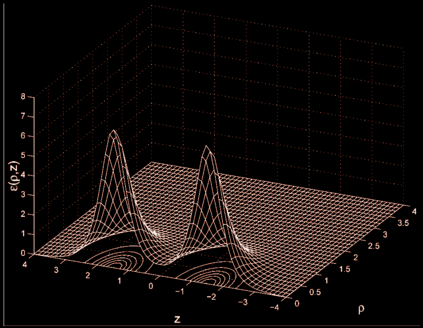




Monopol-Antimonopole Pairs: MAPs

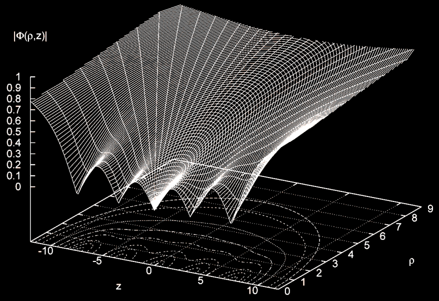
Kleihaus, Kunz 1999

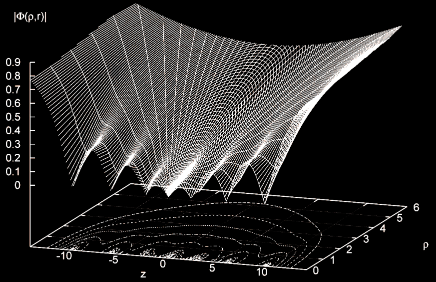




Monopol-Antimonopole Chains: MACs

Kleihaus, Kunz, Shnir 2003





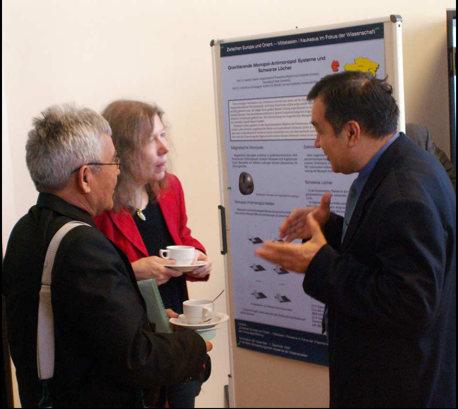
MACs and Big MACs

VW–Stiftung Project

Ibadov, Kunz 2005-2007

Proposal for International Co-operation Between Europe and the Orient – A Focus on Research and Higher Education in/on Central Asia and the Caucasus

“Gravitating dyons
dyonic monopole-antimonopole systems
and black holes”



VW–Stiftung Project: black holes

Gravitierende Monopol-Antimonopol Systeme und Schwarze Löcher

Prof. Dr. Rustam Ibadov
Department of Theoretical Physics and Computer Sciences, Samarkand State University
Prof. Dr. Jutta Kunz-Drolshagen
Institut für Physik, Carl von Ossietzky Universität Oldenburg

Unser heutiges Verständnis des Universums beruht zum einen auf der Einsteinschen Theorie der Gravitation und zum anderen auf dem Standardmodell der Teilchenphysik. Im frühen Universum sollten aufgrund der vorhandenen hohen Temperaturen auch Teilchen und Felder wichtig gewesen sein, die wegen ihrer großen Massen bislang experimentell an den großen Teilchenbeschleunigern noch nicht beobachtet werden konnten. Von besonderem Interesse in diesem Zusammenhang sind magnetische Monopole, deren Vorhersage zum Modell des inflationären Universums geführt hat. Die Untersuchung von magnetischen Monopolen und Systemen von Monopolen und Antimonopolen ist ein zentraler Aspekt dieses Projekts.
Schwarze Löcher gehören zu den faszinierendsten Objekten des Universums. Astrophysikalisch interessant sind Schwarze Löcher als Endstadien sehr schwerer ausgebrannter Sterne und superschwere Schwarze Löcher in den Zentren von Galaxien. Im frühen Universum können aber auch mikroskopische Schwarze Löcher entstanden sein, die überraschende neue Eigenschaften besitzen. Die mathematische Konstruktion und die Erforschung der Eigenschaften solcher kleiner Schwarzer Löcher ist der zweite zentrale Aspekt dieses Projekts.

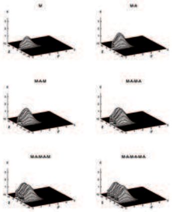
Magnetische Monopole

Magnetische Monopole entstehen in Großvereinheitlichten Feldtheorien der Teilchenphysik. Einfache Monopole sind kugelsymmetrisch. Monopole mit höheren Ladungen können platonischen Objekten gleichen.

Tetrahedron: Monopol mit magnetischer Ladung 3

Monopol-Antimonopol Ketten

Monopole und Antimonopole können lineare Ketten bilden, wobei sich immer Monopol (M) und Antimonopol (A) abwechseln müssen.



Energiedichten von Monopol-Antimonopol-Ketten

In diesem Projekt untersuchen wir den Einfluß der Gravitation auf solche Systeme.

Elektrische Ladung und Rotation

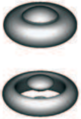
Diese magnetischen Monopole können nicht rotieren. Die Monopol-Antimonopol Paare entsprechen aber Dipolen, und diese können rotieren. Drehimpuls und Ladung sind einander dann proportional.

Wir untersuchen insbesondere auch die Effekte, die elektrische Ladung auf Monopol-Antimonopol Systeme hat.

Schwarze Löcher

In der Einsteinschen Theorie ist ein statisches Schwarzes Loch kugelsymmetrisch und ein rotierendes Schwarzes Loch axialsymmetrisch.

In Gegenwart weiterer Felder müssen statische Schwarze Löcher jedoch nicht mehr rund sein. Aufgrund der nicht kugelsymmetrischen Energiedichte dieser Felder können sie einen deformierten Horizont besitzen.



Energiedichten von Schwarzen Löchern

Wir beabsichtigen den Einfluß elektrischer Ladung auf die Eigenschaften solcher Schwarzen Löcher zu untersuchen.

Wichtige offene Fragen betreffen die Existenz von

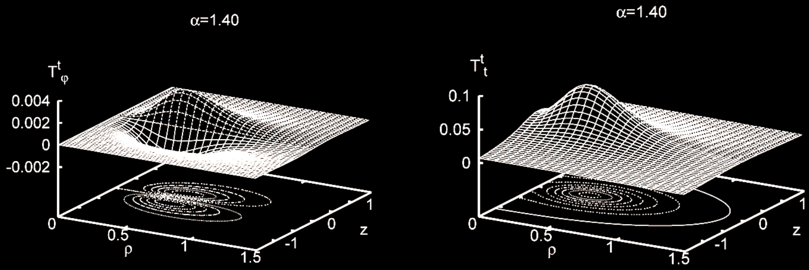
- platonischen Schwarzen Löchern
- Paaren von Schwarzen Löchern
- Ketten von Schwarzen Löchern
- anderen Systemen von Schwarzen Löchern.

Publikationen

1. R. Ibadov, B. Kleihaus, J. Kunz, and Y. Shnir, New Regular Solutions with Axial Symmetry in Einstein-Yang-Mills theory, Phys. Lett. **B609** (2005) 150.
2. R. Ibadov, B. Kleihaus, J. Kunz, M. Wirschins, New Black Hole Solutions with Axial Symmetry in Einstein-Yang-Mills Theory, Phys. Lett. **B627** (2005) 180.

Dyons

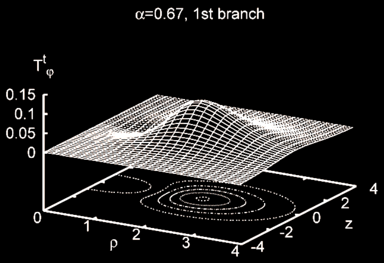
Kleihaus, Kunz, Neemann 2005

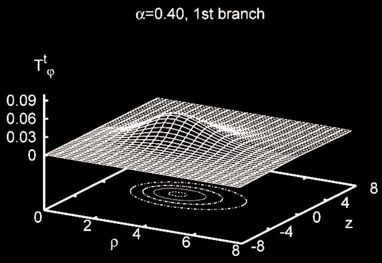


addition of electric charge Q: stationary not static solutions,
magnetic charge $P \neq 0$: $J = 0$

Rotating MAPs and vortex rings

Kleihaus, Kunz, Neemann 2005

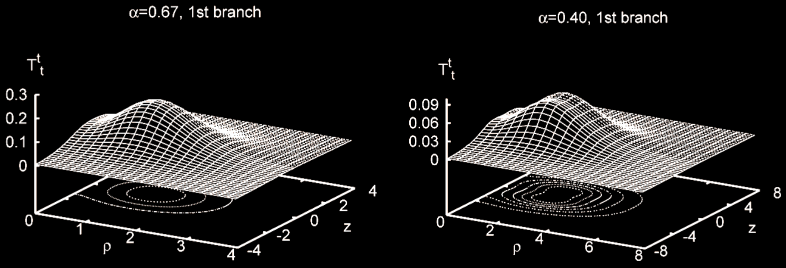




addition of electric charge Q: stationary rotating solutions,
magnetic charge $P = 0$: $J = Q$

Rotating MAPs and vortex rings

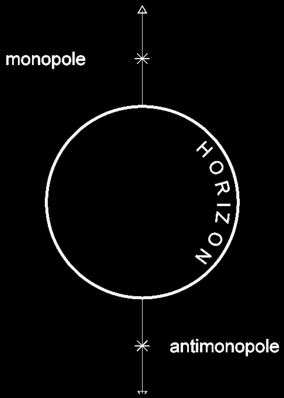
Kleihaus, Kunz, Neemann 2005



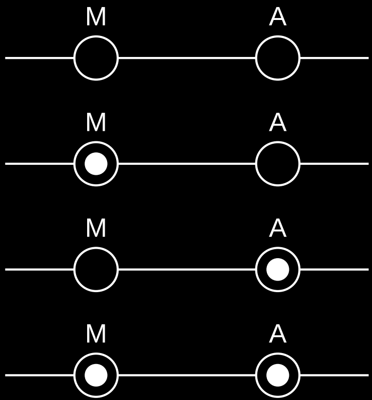
addition of electric charge Q: stationary rotating solutions,
magnetic charge $P = 0$: $J = Q$

VW–Stiftung Project: black holes

black hole between monopole and antimonopole

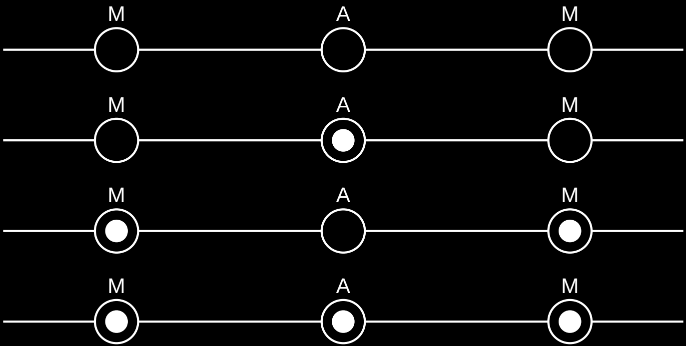


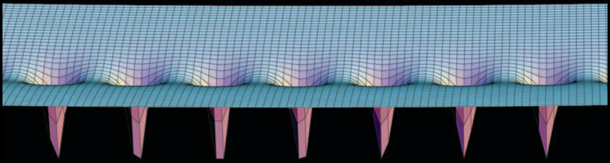
new possible types configurations ?



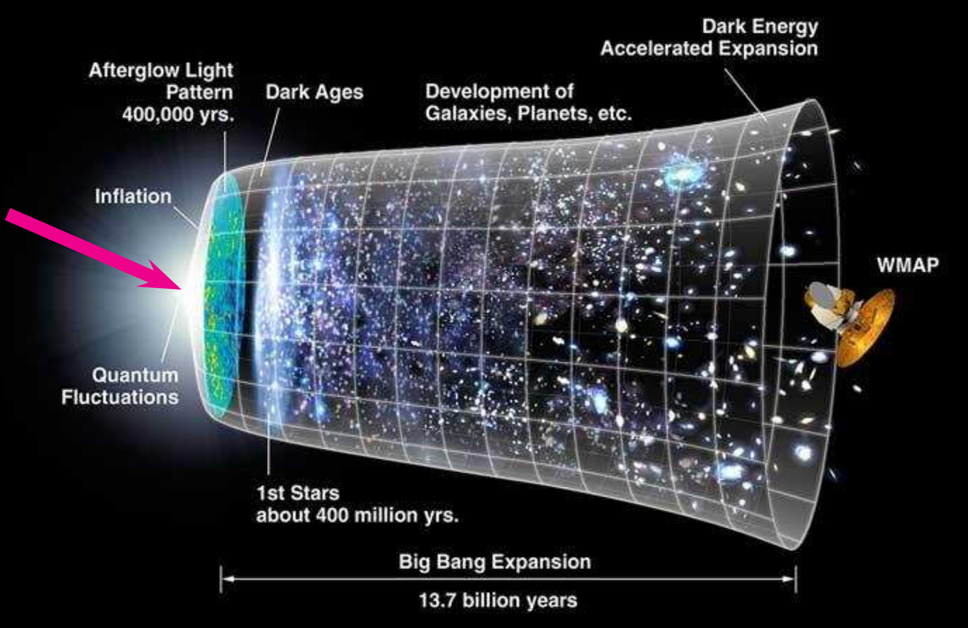
VW–Stiftung Project: black holes

further possible systems of black holes?





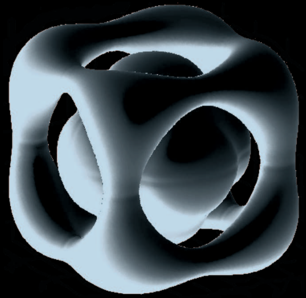
The evolution of our universe



Outline

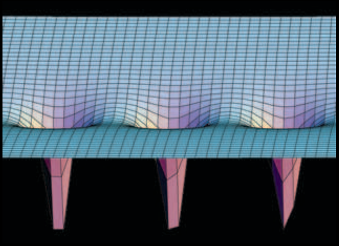
Outlook:

- platonic black holes?



- further surprises?

- systems of black holes?



- different horizon topology?
  black rings