Compact objects in gravity theories

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Outline

1. Introduction
2. Neutron Stars
   - GR
   - Beyond GR
3. Black Holes
4. Conclusions

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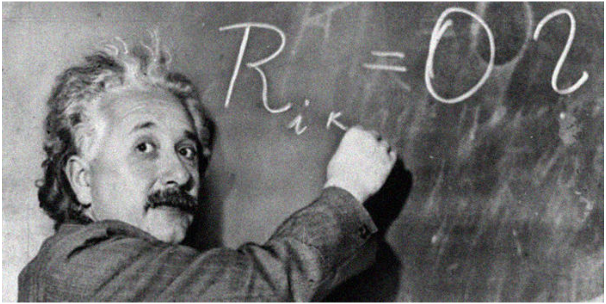
Introduction

General Relativity



- Incompatibility with Quantum Mechanics
- Singularities
- Dark Matter, Dark Energy
- ...

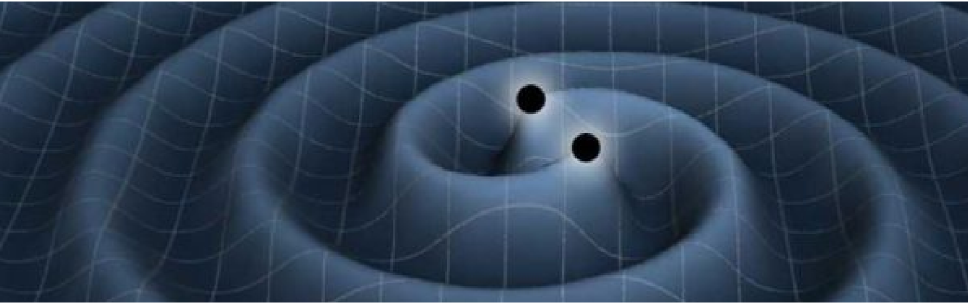
Introduction

GR or Alternative Theories of Gravity

- Scalar-tensor theories
- $f(R)$ theories
- Quadratic gravity (EsGB, CS, ...)
- ...

Introduction

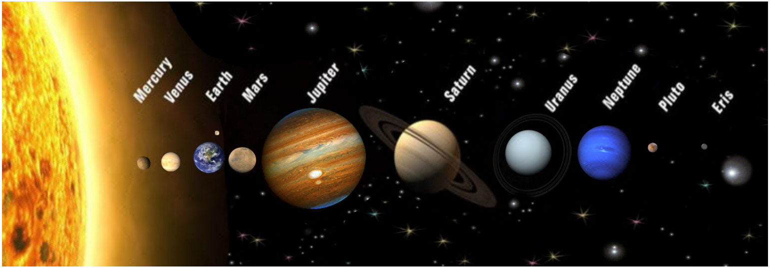
GR or Alternative Theories of Gravity



- Scalar-tensor theories
- $f(R)$ theories
- Quadratic gravity (EsGB, CS, ...)
- ...

Introduction

GR or Alternative Theories of Gravity



- Compatible with all solar system tests!
- Observational consequences: strong gravity?
  - Neutron stars
  - Black holes
  - Exotic objects

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4 Conclusions

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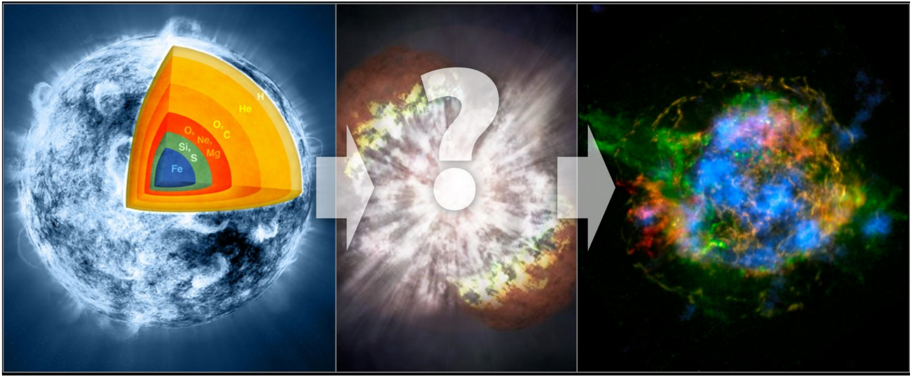
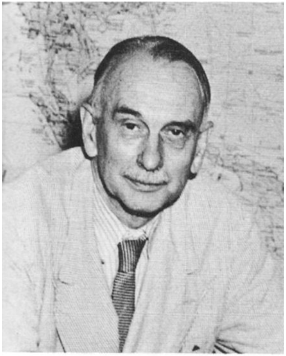
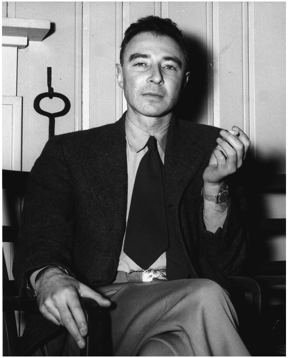
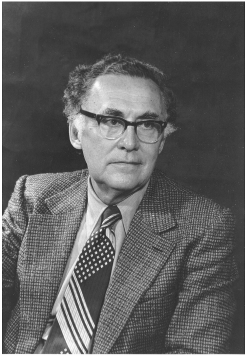
Neutron star basics



Supernova explosion

Neutron star basics

Richard C. Tolman (1881 – 1948)

J. Robert Oppenheimer (1904 – 1967)

George Michael Volkoff (1914 – 2000)

Tolman-Oppenheimer-Volkoff equations

Neutron star basics

Einstein equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

with stress-energy tensor of an isotropic perfect fluid

$$T_{\mu\nu} = (\varepsilon + P)\, u_\mu u_\nu + g_{\mu\nu} P$$

with four-velocity u^μ, energy density ε and pressure P

metric ansatz for static spherically symmetric neutron stars

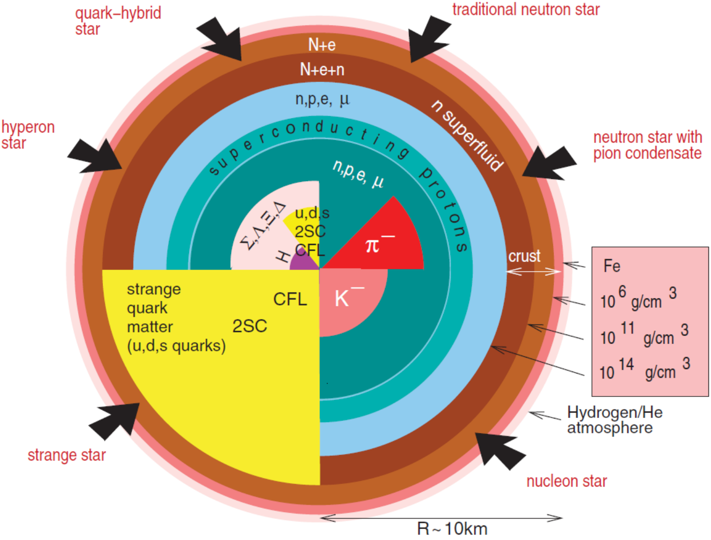
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^{2\Phi(r)} dt^2 + \frac{1}{1 - \frac{2m(r)}{r}} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

TOV equations

$$m' = \frac{\kappa}{2} \varepsilon r^2 \ , \qquad \Phi' = \frac{\frac{\kappa}{2} r^3 P + m}{\left(1 - \frac{2m}{r}\right) r^2}$$

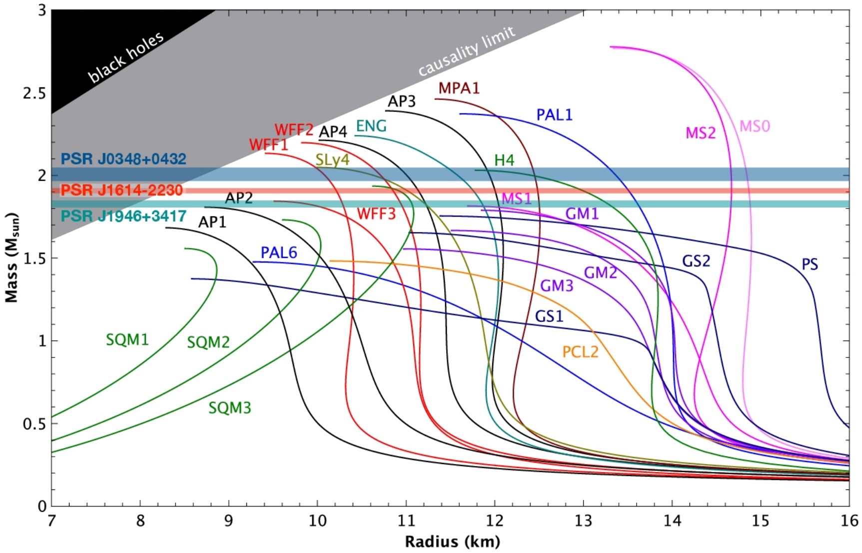
$$P' = -\frac{1}{r^2} \frac{(\varepsilon + P)\left(m + \frac{\kappa}{2} P r^3\right)}{1 - \frac{2m}{r}}$$

Neutron star basics



Weber et al. in Neutron Stars and Pulsars

Neutron star basics



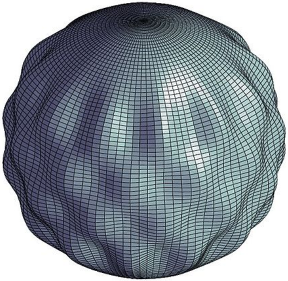
https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html

Neutron star basics

Universal relations

Relations between neutron star properties, that are to a large extent independent of the neutron star's internal structure (EOS).

- moment of interia I
- quadrupole moment Q
- Love number
- ...



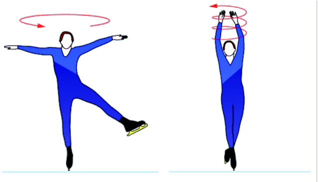
- quasi-normal modes: asteroseismology

no exact relations, but valid at the (few) percent level

Universal relations

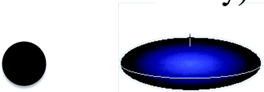
Yagi and Yunes I-Love-Q

moment of inertia



Q (spin-induced) quadrupole moment

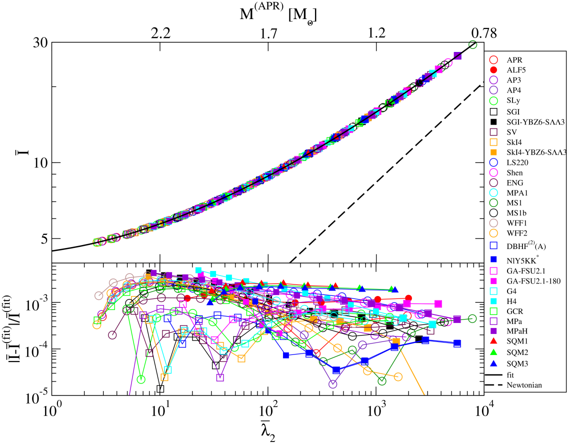
λ_2 tidal Love number (tidal deformability)



$$\lambda_2 = \frac{\text{(tidally induced) Q}}{\text{tidal field}}$$

Universal relations

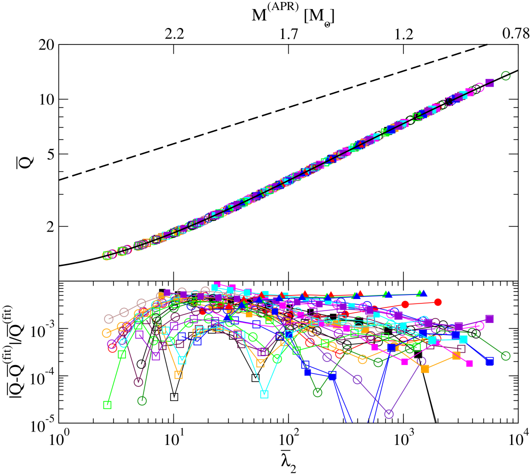
Yagi et al. 1302.4499, 1608.02582



I-Love relation: $\bar{I} = I/M^3$, $\bar{\lambda}_2 = \lambda_2/M^5$

Universal relations

Yagi et al. 1302.4499, 1608.02582

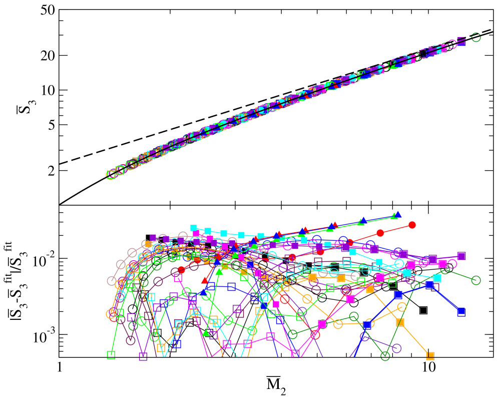


Q-Love relation: $\bar{Q} = -QM/J^2$, $\bar{\lambda}_2 = \lambda_2/M^5$

Universal relations

Yagi et al. 1608.02582, 1312.4532

three hair relations: relativistic results



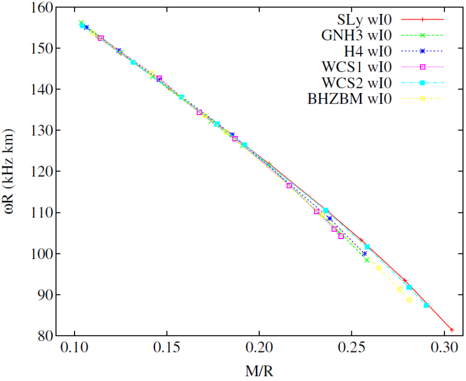
universal $\bar{S}_3$–$\bar{M}_2$ relation: $\sim 4\%$ fit (solid), Newton (dashed)

Universal relations

Blazquez-Salcedo et al. arXiv:1307.1063

quasi-normal modes: polar (parity even)

$\omega = \omega_R + i\omega_I$, frequency ω_R, damping time $\tau = 1/\omega_I$



universal $\omega_R R$–M/R relation

Universal relations

Blazquez-Salcedo et al. arXiv:1307.1063

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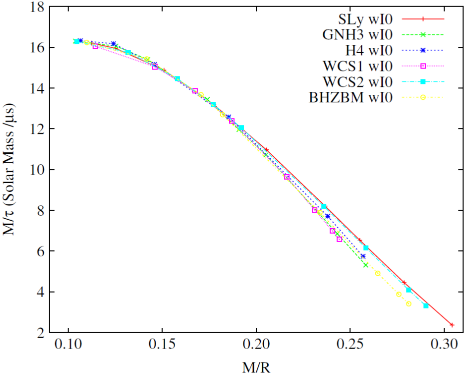

universal $M\omega_I$–M/R relation

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Scalar-Tensor Theories

action: Jordan frame

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-\tilde{g}} \left[F(\Phi)\tilde{\mathcal{R}} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi) \right] + S_m \left[\Psi_m; \tilde{g}_{\mu\nu}\right]$$

G_*: gravitational constant

$\tilde{\mathcal{R}}$: Ricci scalar with respect to $\tilde{g}_{\mu\nu}$

Φ: gravitational scalar field

S_m: matter action

Ψ_m: matter fields

Φ does not couple directly to Ψ_m: weak equivalence principle is satisfied

Scalar-Tensor Theories

transformation to Einstein frame

$$\left(\frac{d\varphi}{d\Phi}\right)^2 = \frac{3}{4}\left(\frac{d\ln(F(\Phi))}{d\Phi}\right)^2 + \frac{Z(\Phi)}{2F(\Phi)}$$

action: Einstein frame

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left[\mathcal{R} - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right] + S_m[\Psi_m; \mathrm{A}^2(\varphi)g_{\mu\nu}]$$

relations between the Jordan frame functions $F(\Phi)$ and $U(\Phi)$ and the Einstein frame functions $A(\varphi)$ and $V(\varphi)$

$$A(\varphi) = F^{-1/2}(\Phi)\ , \qquad 2V(\varphi) = U(\Phi)F^{-2}(\Phi)$$

Scalar-Tensor Theories

Brans–Dicke Theory

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[\Phi\tilde{\mathcal{R}} - \frac{\omega(\Phi)}{\Phi}\tilde{g}^{\mu\nu}\left(\partial_\mu\Phi\right)\left(\partial_\nu\Phi\right) - 2U(\Phi)\right] + S_m[\Psi_m, \tilde{g}_{\mu\nu}]$$

relation between Jordan-frame and Einstein-frame quantites

$$\Phi = A^{-2}(\varphi)\ , \quad 3 + 2\omega(\Phi) = \alpha(\varphi)^{-2}$$

$$\alpha(\varphi) \equiv d(\ln A(\varphi))/d\varphi$$

$\alpha(\varphi) = \alpha_0$ =constant, i.e., $\omega(\Phi)$ =constant
observational bound: $\omega > 40000$ (Cassini-Huygens)
limit: $\omega \to \infty$ GR

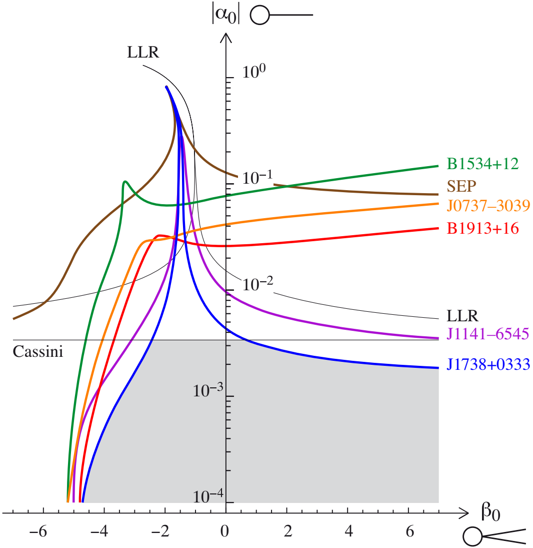
Scalar-Tensor Theories

Freire et al. 1205.1450

observational constraints on STT parameters

$$\ln A(\varphi) = \ln A(\varphi_0)$$
$$+\alpha_0(\varphi - \varphi_0)$$
$$+\frac{1}{2}\beta_0(\varphi - \varphi_0)^2$$

$$m_\varphi = 0$$



Scalar-Tensor Theories

VOLUME 70, NUMBER 15 PHYSICAL REVIEW LETTERS 12 APRIL 1993

Nonperturbative Strong-Field Effects in Tensor-Scalar Theories of Gravitation

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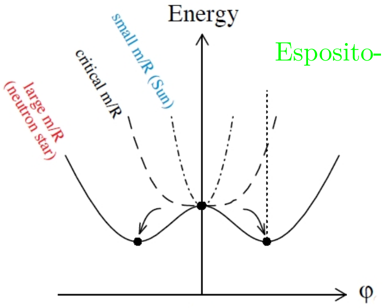
13288 Marseille CEDEX 9, France



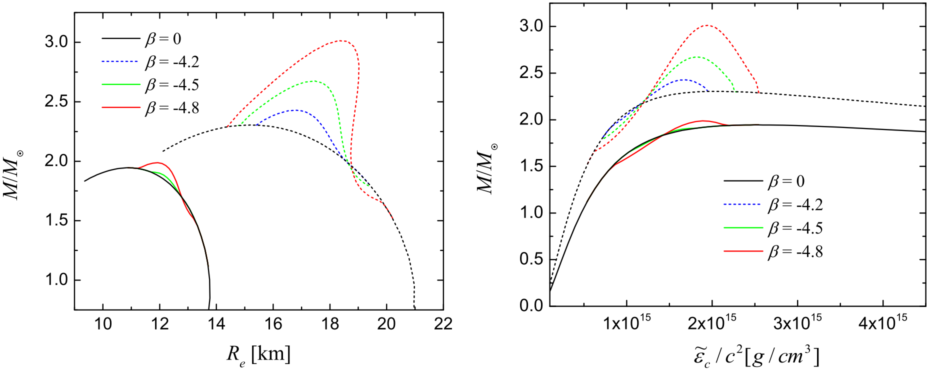
Esposito-Farese (lecture)

"spontaneous scalarization"

Scalar-Tensor Theories

Doneva et al. 1309.0605

$\ln A(\varphi) = \frac{1}{2}\beta_0\varphi^2$, $\beta_0 > -4.5$



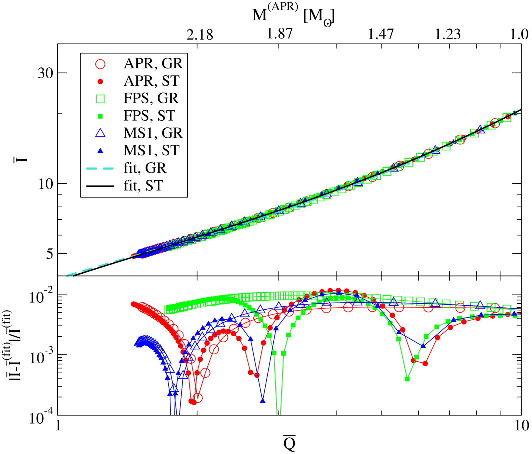
mass–radius

mass–energy density

spontaneous scalarization: static and Kepler limit

Scalar-Tensor Theories

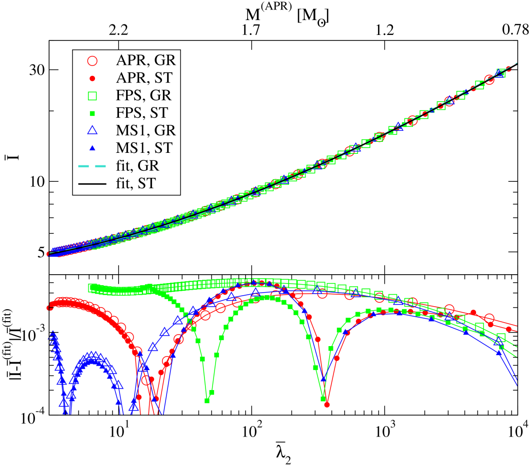
Pani et al. 1405.4547, Yagi et al. 1608.02582



I–Q: slow rotation (2nd order, $\beta_0 = -4.5$)

Scalar-Tensor Theories

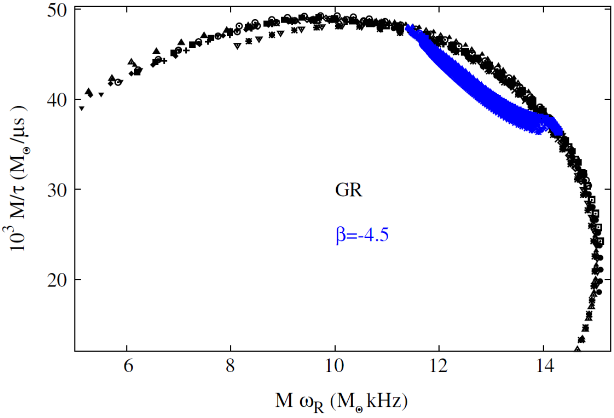
Pani et al. 1405.4547, Yagi et al. 1608.02582



I–Love: slow rotation (2nd order, $\beta_0 = -4.5$)

Scalar-Tensor Theories

Motahar et al. arXiv:1807.02598

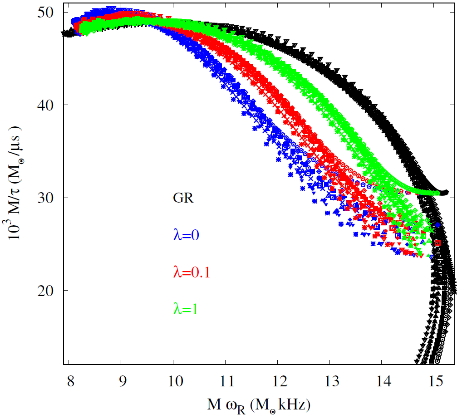


axial (odd parity) quasi-normal modes: $M\omega_I$–$M\omega_R$ ($\beta_0 = -4.5$)

Scalar-Tensor Theories

Motahar et al. arXiv:1902.01277

$V(\varphi) = 2m_\varphi^2 + \lambda\varphi^4 \qquad m_\varphi \sim 10^{-12}$ eV



axial (odd parity) quasi-normal modes: $M\omega_I$–$M\omega_R$ ($\beta_0 = -6$)

Scalar-Tensor Theories

Horndeski gravity

second-order field equations and one scalar field

$$
\begin{aligned}
S = \int d^4x\sqrt{-g}\Big\{ & K(\phi, X) - G_3(\phi, X)\Box\phi \\
& + G_4(\phi, X)R + G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] \\
& + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}(\phi, X)}{6}\big[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) \\
& + 2(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla_\sigma\phi)(\nabla^\nu\nabla^\sigma\phi)\big]\Big\},
\end{aligned}
$$

K and G_i's ($i = 1 \ldots 5$):
functions of the scalar field ϕ and of its kinetic term $X = -1/2\partial^\mu\phi\partial_\mu\phi$
$G_{i,X}$:
derivatives of G_i with respect to X

Scalar-Tensor Theories

Charmousis et al. 1106.2000, 1112.4866

subsector: Fab Four

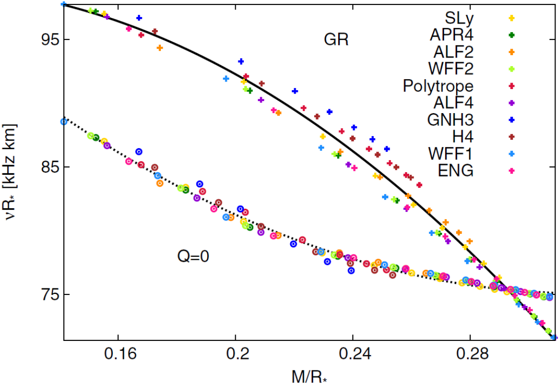
special cases of Horndeski gravity

- general relativity ("George")
- Einstein-dilaton-Gauss-Bonnet gravity ("Ringo")
- theories with a nonminimal coupling with the Einstein tensor ("John")
- theories involving the double-dual of the Riemann tensor ("Paul")

Scalar-Tensor Theories

Blazquez-Salcedo et al. arXiv:1803.01655

"George" + "John" $\phi(r,t) = Qt + F(r)$



axial (odd parity) quasi-normal modes: $\omega_R R$–M/R

Scalar-Tensor Theories

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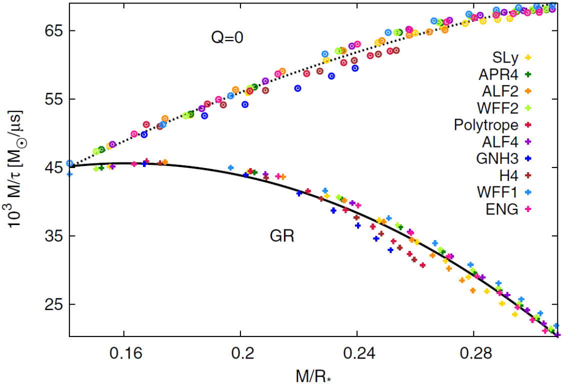
"George" + "John" $\phi(r,t) = Qt + F(r)$



axial (odd parity) quasi-normal modes: $\omega_I M$–M/R

Quadratic Gravity

Curvature invariants

$$R^2 \,, \quad R^2_{\mu\nu} \,, \quad R^2_{\mu\nu\rho\sigma} \,, \quad {}^*RR$$

$$R^2_{\mu\nu} \equiv R_{\mu\nu} R^{\mu\nu}$$

Kretschmann scalar

$$R^2_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

Pontryagin/Chern-Simons scalar

$${}^*RR \equiv \frac{1}{2} R_{\mu\nu\rho\sigma} \epsilon^{\nu\mu\lambda\kappa} R^{\rho\sigma}{}_{\lambda\kappa}$$

Levi-Civita tensor $\epsilon^{\mu\nu\rho\sigma}$

Gauss-Bonnet scalar

$$R^2_{GB} \equiv R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}$$

Einstein-Gauss-Bonnet-Dilaton Theory

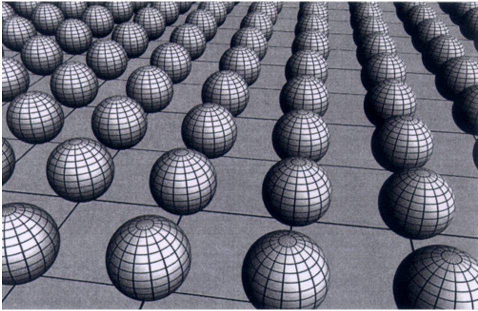
String Theory

unification of all fundamental interactions

dimensional reduction to 4 spacetime dimensions:

low energy effective theories

- additional fields
  - dilaton
  - axion
  - Maxwell fields
  - Yang-Mills fields
  - ...
- higher order curvature corrections
  - Gauss-Bonnet term
  - ...
- ...



Einstein-Gauss-Bonnet-Dilaton Theory

Action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} (\partial_\mu \phi)^2 + \frac{\alpha}{4} e^{-\gamma\phi} R^2_{\rm GB} \right]$$

Gauss-Bonnet term: quadratic in the curvature

$$R^2_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

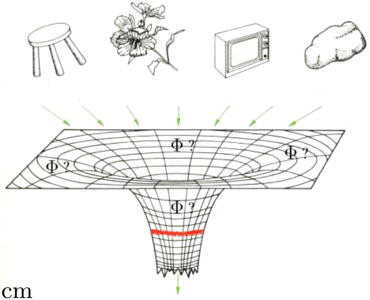
- α Gauss-Bonnet coupling constant
- γ dilaton coupling constant ($\gamma = 1$)

In 4 spacetime dimensions the coupling to the dilaton is needed.
The resulting set of equations of motion are of second order.

Einstein-Gauss-Bonnet-Dilaton Theory

consequences

- scalar "hair": dilaton "hair"
- negative energy density



bounds on α ($\gamma = 1$)

- observational
  - Shapiro time delay

$$\sqrt{\alpha} \lesssim 10^{13}\text{cm}$$

  - BH low-mass X-ray binaries

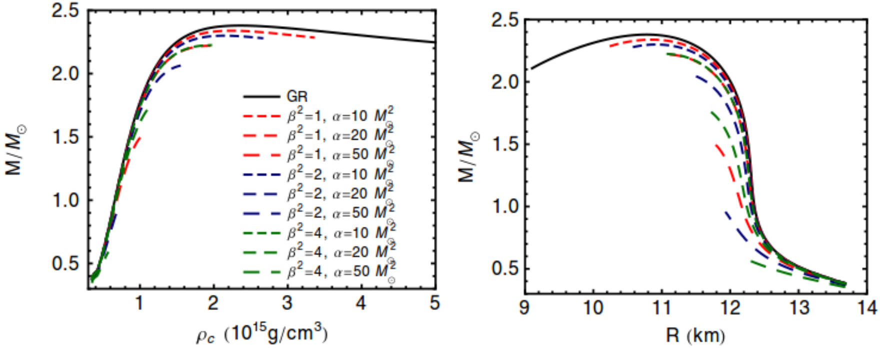
$$\sqrt{\alpha} \lesssim 3.8 \times 10^{5}\text{cm}$$

- theoretical/observational
  - lower bound on BH mass

$$\frac{\alpha}{M^2} \lesssim 0.691$$

Einstein-Gauss-Bonnet-Dilaton Theory

Pani et al. 1109.0928

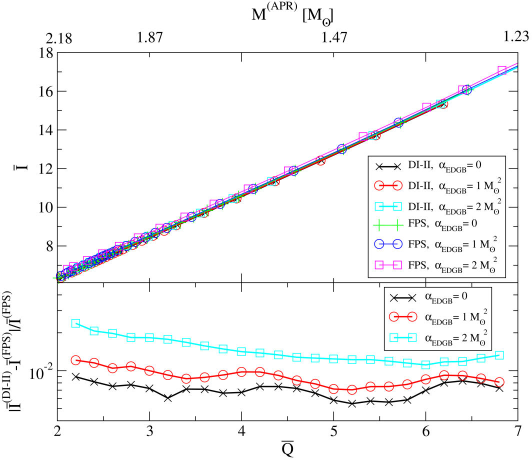


static neutron stars with APR EoS: dependence on α and β

branches end
expansion around origin: square roots
reality condition: condition on $\alpha\beta$, maximum central density

Einstein-Gauss-Bonnet-Dilaton Theory

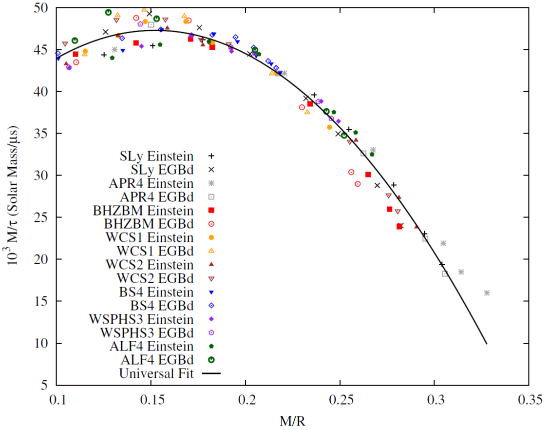
Kleihaus et al. 1601.05583 , Yagi et al. 1608.02582



$\bar{I}$–$\bar{Q}$: rapid rotation

Einstein-Gauss-Bonnet-Dilaton Theory

Blazquez-Salcedo et al. arXiv:1511.03960



axial (odd parity) quasi-normal modes: $M\omega_I$–M/R

Chern-Simons Gravity

action

$$S = \frac{1}{16\pi} \int \sqrt{-g} d^4x \left[R - 2\nabla_\mu \phi \nabla^\mu \phi - V(\phi) + \alpha_{\rm CS}\, \phi\, {}^*RR\right]$$

two cases

- dynamical:

  scalar true dynamical degree of freedom

  dCS gravity
- nondynamical:

  scalar kinetic term absent

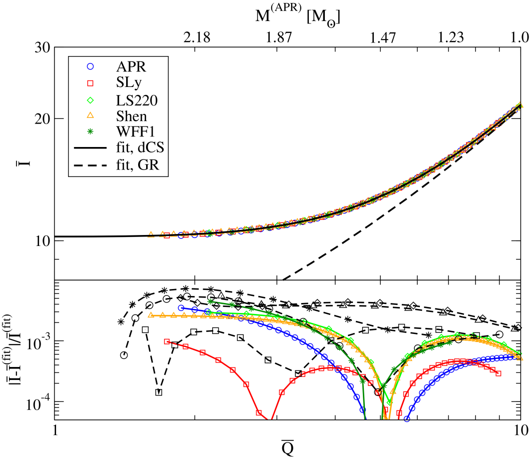
- a spherically symmetric solution of GR is also a solution of dCS gravity
- corrections in the presence of a parity-odd source such as rotation

bound: Gravity Probe B

$$\sqrt{|\alpha_{\rm CS}|} < \mathcal{O}(10^{13})\text{cm}$$

Chern-Simons Gravity

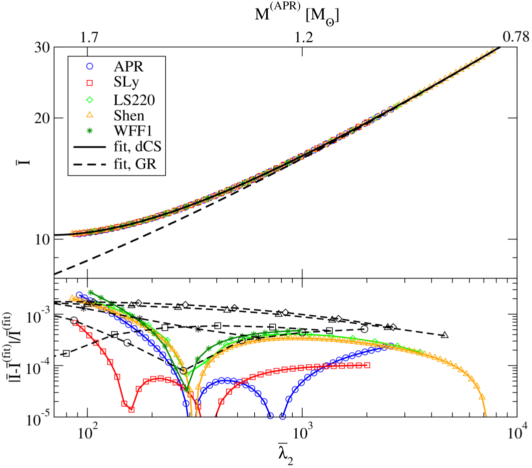
Yagi et al. 1608.02582



$\bar{I}$–$\bar{Q}$: slow rotation

Chern-Simons Gravity

Yagi et al. 1608.02582



$\bar{I}$–$\bar{\lambda}_2$: slow rotation

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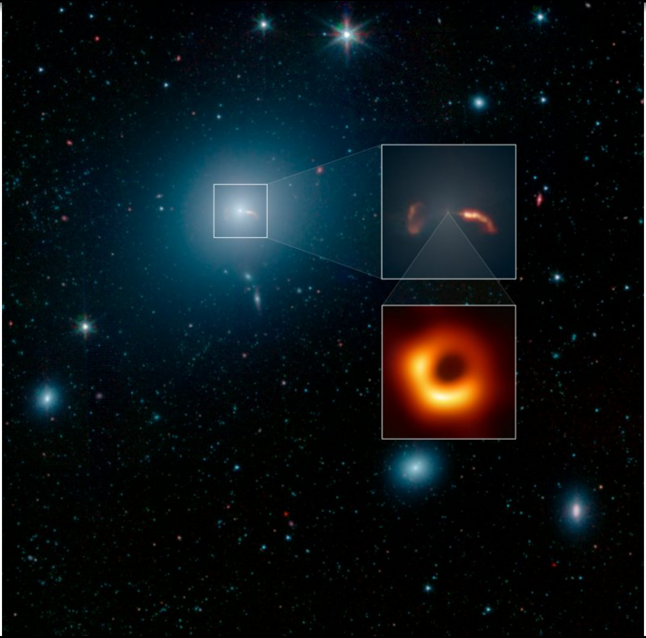
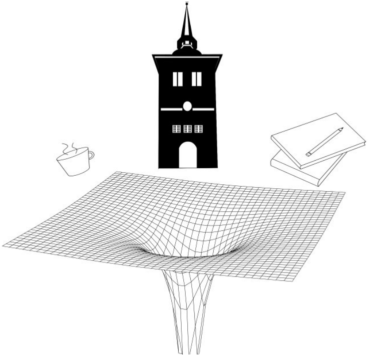
Black holes in GR



Black holes in GR



A Kerr black hole has no hair

A Kerr black hole is fully characterized in terms of only two global parameters: the mass M and the angular momentum J

Black holes in GR

Geroch, J. Math. Phys. (1970); Hansen, J. Math. Phys. (1974);
Thorne, Rev. Mod. Phys. (1980)

Multipole moments M_l (g_{00}) and S_l ($g_{0\phi}$)

All multiple moments can be expressed in terms of only two quantities

$$M_0 = M \qquad\qquad S_1 = J$$

$$M_l + iS_l = M\left(i\frac{J}{M}\right)^l$$

Quadrupole moment

$$M_2 = Q = -\frac{J^2}{M}$$

Black holes in GR

Grenzebach et al. arXiv:1403.5234

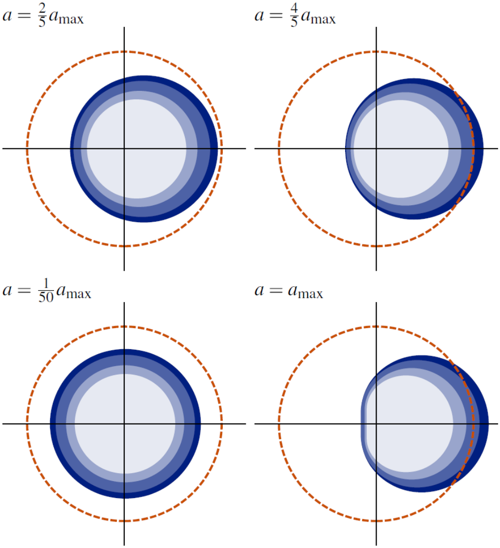
angular momentum bound

$$j = \frac{J}{M^2} \leq 1$$

- < 1 non-extremal black hole
- $= 1$ extremal black hole
- > 1 naked singularity

shadow

β	0	0	$\frac{5}{9}m^2$	0
ℓ	0	$\frac{3}{4}m$	$\frac{4}{3}m$	$\frac{4}{3}m$
Λ	0	0	$10^{-2}m^{-2}$	0
$a_{\max}$	m	$\frac{5}{4}m$	$1.51m$	$\frac{5}{3}m$
	Kerr	Kerr–NUT	KN–NUT with Λ	Kerr–NUT



EGBd black holes

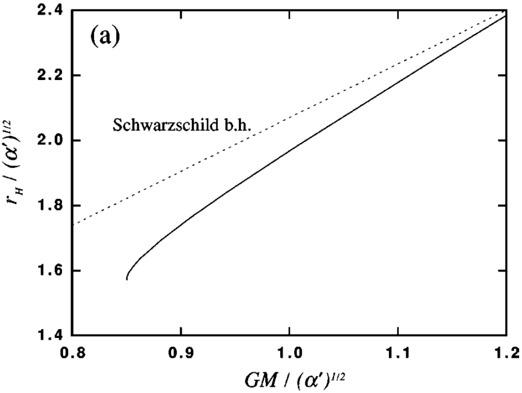
Kanti et al. arXiv:hep-th/9511071, Torii et al. arXiv:gr-qc/9606034

critical black holes:

horizon expansion

$$\sqrt{1 - 6\frac{\alpha'^2}{r_h^4}e^{2\phi_h}}$$

lower bound
on the horizon size
for fixed α'

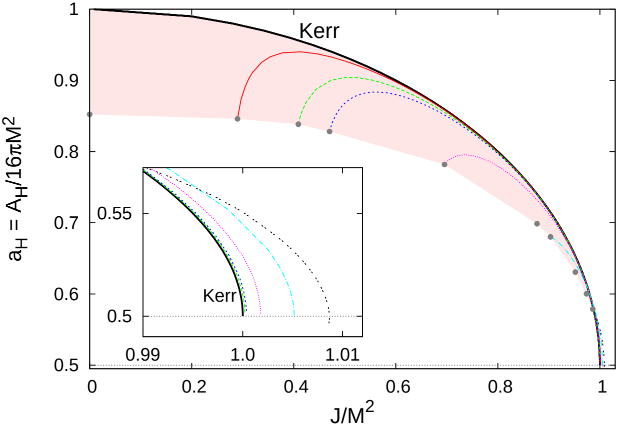


lower bound on the mass

EGBd black holes

Kleihaus et al. arXiv:1101.2868

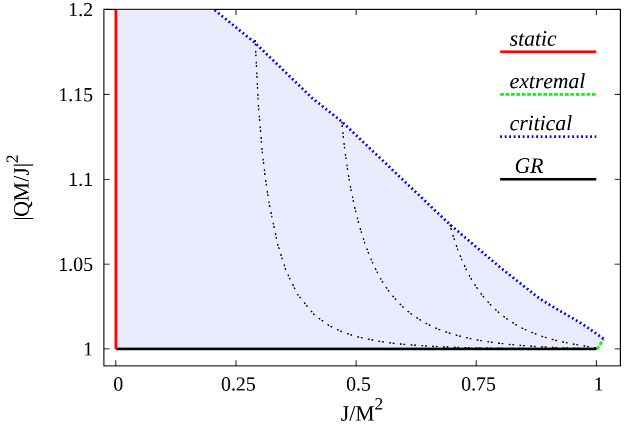
horizon area versus angular momentum



EGBd black holes

Kleihaus et al. arXiv:1101.2868

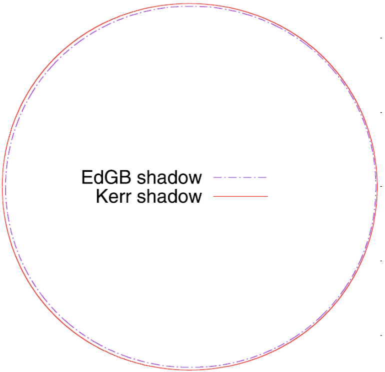
quadrupole moment versus angular momentum



EGBd black holes

Cunha et al. arXiv:1701.00079

shadow



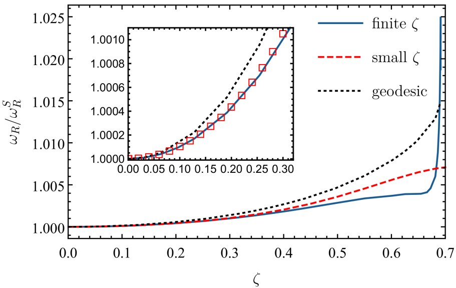
$$\alpha/M^2 = 0.172,\ J/M^2 = 0.41$$

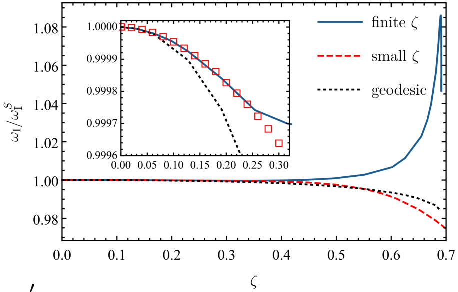
EGBd black holes

Blazquez-Salcedo et al. arXiv:1609.01286

quasi-normal mode (axial $l = 2$) versus coupling constant

normalized to the Schwarzschild values





real part

$$\zeta = \frac{\alpha'}{M^2}$$

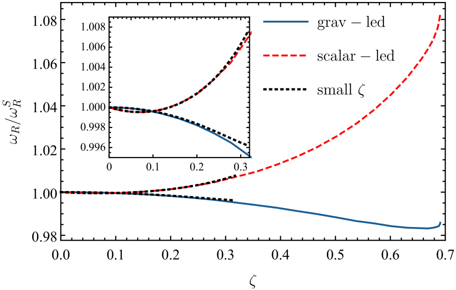
imaginary part

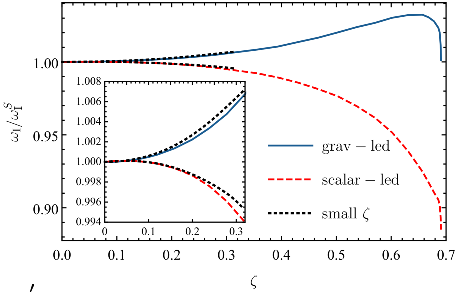
EGBd black holes

Blazquez-Salcedo et al. arXiv:1609.01286

quasi-normal mode (polar $l = 2$) versus coupling constant

normalized to the Schwarzschild values





real part $\quad \zeta = \dfrac{\alpha'}{M^2} \quad$ imaginary part

EsGB black holes

Doneva et al. arXiv:1711.01187, Silva et al. arXiv:1711.02080, Antoniou et al. arXiv:1711.03390

Curvature induced scalarized black holes

action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\Big[R - 2\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi) + \lambda^2 f(\varphi)\mathcal{R}^2_{GB}\Big]$$

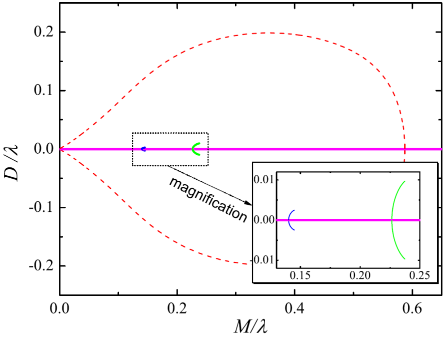
coupling function

$$f(\varphi) = \frac{1}{12}\left(1 - e^{-6\varphi^2}\right)$$

small φ

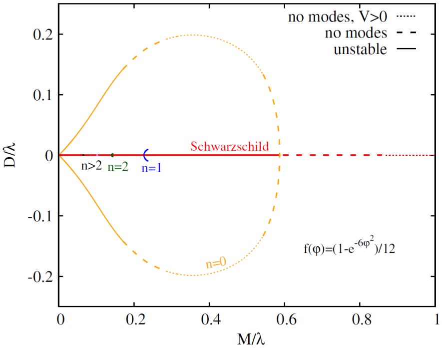
$$f(\varphi) = \frac{1}{2}\varphi^2$$

sequence of radial excitations



EsGB black holes

Blazquez-Salcedo et al. arXiv:1805.05755



solutions

Schwarzschild red

scalarized $n = 0$ orange

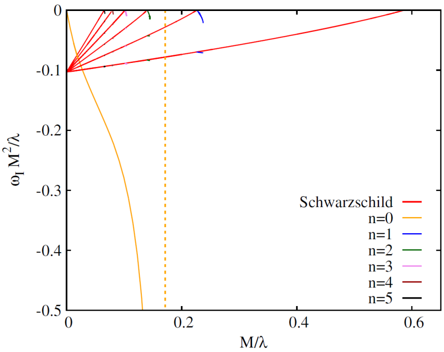
scalarized $n > 0$ ...

unstable modes

Schwarzschild red

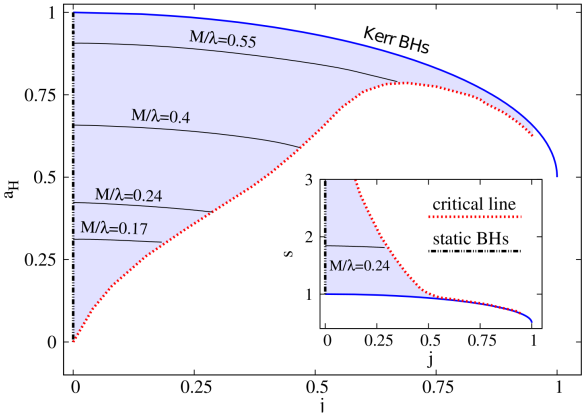
scalarized $n = 0$ orange

scalarized $n > 0$ ...



EsGB black holes

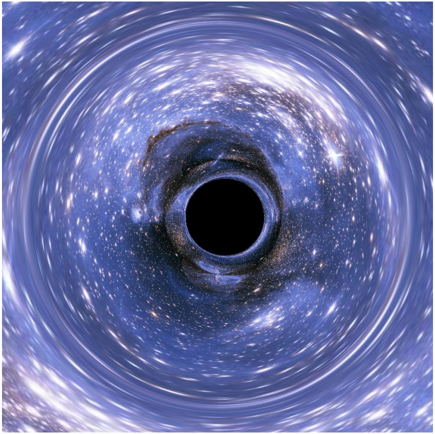
Cunha et al. arXiv:1904.09997



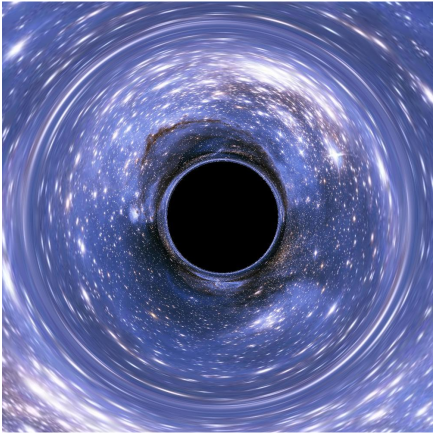
rotating EsGB black holes

EsGB black holes

Cunha et al. arXiv:1904.09997



EsGB



Kerr

$$M/\lambda = 0.237 (j = 0.24)$$

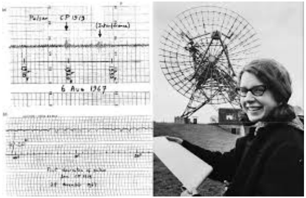
Outline

1. Introduction
2. Neutron Stars
   - GR
   - Beyond GR
3. Black Holes
4. Conclusions

Conclusions

GR versus generalized gravity theories

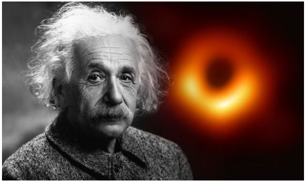
neutron stars



- properties
- quasi-normal modes
- universal relations
- ...

black holes

- properties
- shadow
- quasi-normal modes
- ...



Exotic Objects: EGBd Wormholes

Kanti et al. 1108.3003, 1111.4049

acceleration of a traveler at the throat?

- $g_{\oplus}$: acceleration of gravity at the surface of the earth
- acceleration on the order of $g_{\oplus}$: throat radius on the order of $(10 - 100)$ light-years

THANKS