Surprises with Rotating Black Holes

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Outline

1. Introduction to Black Holes
   - General Relativity
   - Schwarzschild Black Holes
   - Kerr Black Holes
2. Microscopic Black Holes
   - Maxwell Theory
   - Kerr–Newman Black Holes
   - $4D$ Einstein-Maxwell-Dilaton Black Holes
   - $5D$ Einstein-Maxwell-Chern-Simons Black Holes
   - Odd-D Einstein-Maxwell-Chern-Simons Black Holes
3. Conclusions

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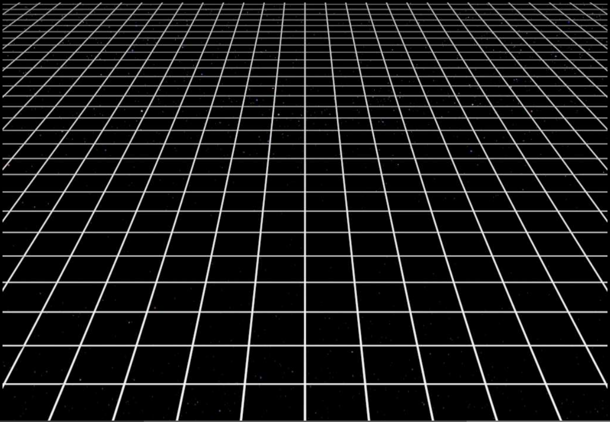
Flat Space–Time

- metric of Euclidean space

$$ds^2 = dx^2 + dy^2 + dz^2$$

- metric of Minkowski space-time

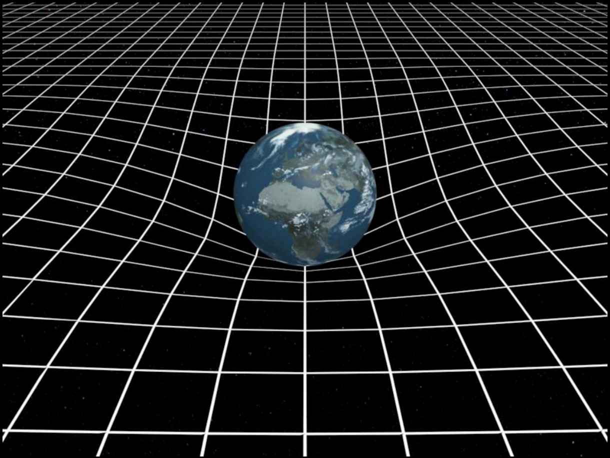
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$



Curved Space–Time

- metric of curved space-time

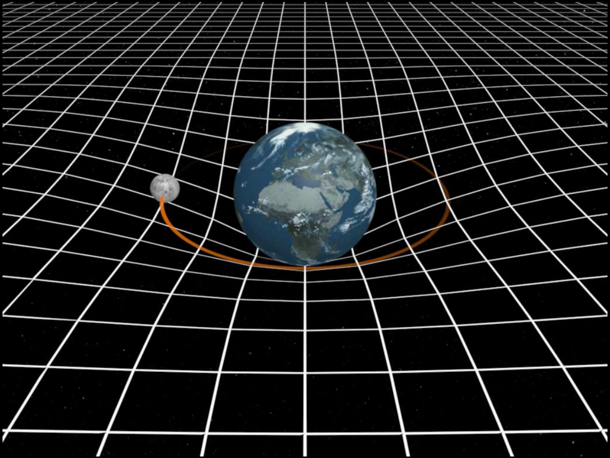
$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$



Motion in Curved Space–Time

- motion in curved space–time

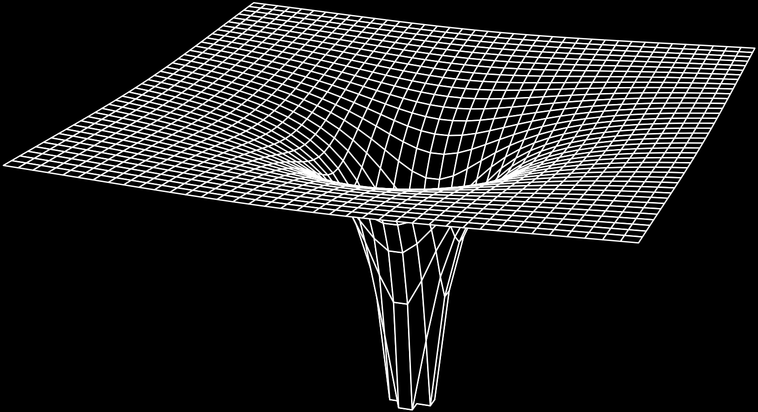
$$0 = \frac{d^2 x^\mu}{ds^2} + \left\{ {\mu \atop \rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}$$



Strongly Curved Space–Time

- metric of curved space–time

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$



Einstein Equations

- metric

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

- Einstein equations

  matter tells space how to curve

$$\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

  $G_{\mu\nu}$: Einstein tensor

  $T_{\mu\nu}$: energy-momentum tensor

- equations of motion for matter/radiation

  metric $g_{\mu\nu}$ tells matter how to move

Einstein Equations

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Schwarzschild Metric

Schwarzschild 1916

- space-time outside a star: $T_{\mu\nu} = 0$

$$ds^2 = -N(r)\, c^2 dt^2 + \frac{1}{N(r)}\, dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

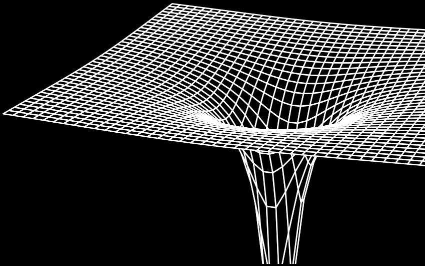
$$N(r) = 1 - \frac{2GM}{c^2 r}$$

static spherically symmetric metric

remark: Minkowski space-time has $N(r) = 1$

- space–time inside a star: $T_{\mu\nu} \neq 0$

Karl Schwarzschild 1873 — 1916



Schwarzschild Singularity

- Schwarzschild space-time

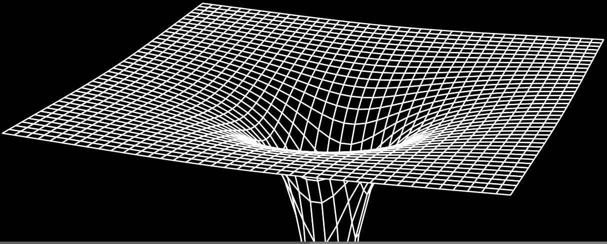
$$ds^2 = -N(r)\, c^2 dt^2 + \frac{1}{N(r)}\, dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$N(r) = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_{\mathrm{H}}}{r}$$

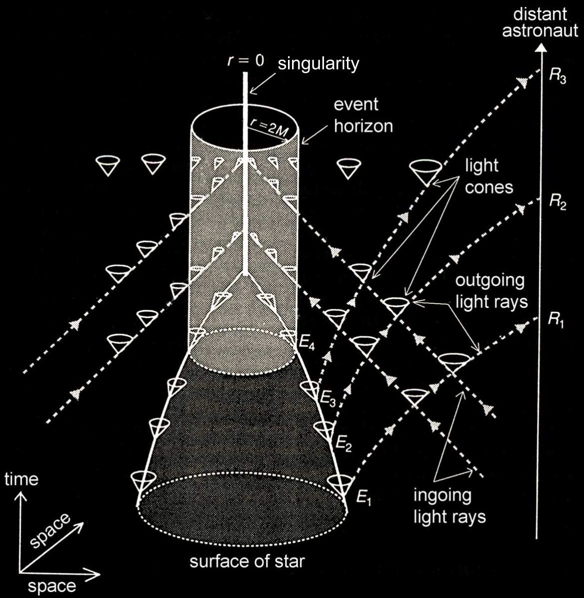
- black holes: M
- Schwarzschild radius r_{H}

$$N(r_{\mathrm{H}}) = 0: \;\; r_{\mathrm{H}} = \frac{2GM}{c^2}$$

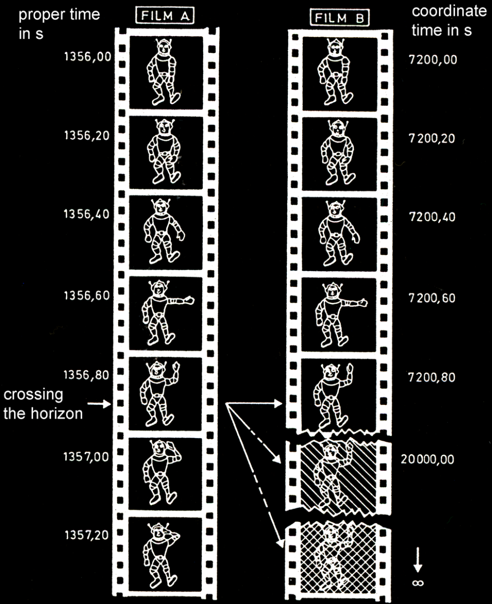
- event horizon
- coordinate singularity
- true singularity $r = 0$



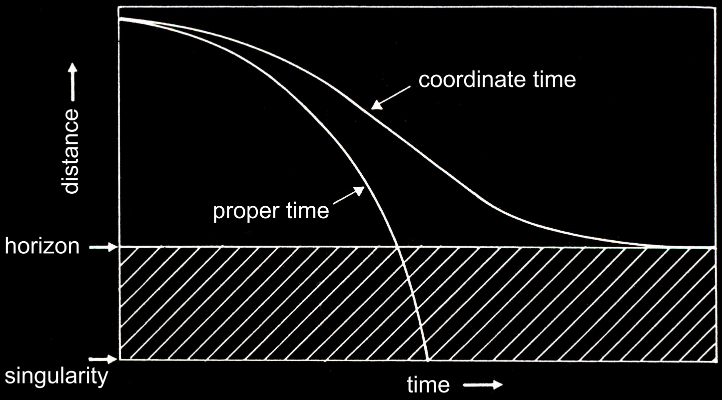
Formation of a Black Hole



Event Horizon



Event Horizon



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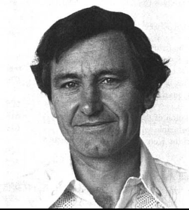
Rotating Black Holes

rotating generalization of the Schwarzschild black holes: Kerr (1965)



Roy Kerr *1934

Kerr metric in Boyer–Lindquist coordinates

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\rho^2}\left(adt - \rho_0^2 d\phi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta\ , \quad \rho_0^2 = r^2 + a^2\ , \quad \Delta = r^2 - 2Mr + a^2$$

a is the specific angular momentum: $a = \frac{J}{M}$

$a = 0$: Schwarzschild

Kerr Black Holes in the Equatorial Plane

metric in Boyer–Lindquist coordinates:
equatorial plane: $\theta = \pi/2$
through center of black hole, perpendicular to the spin axis

$$
\begin{aligned}
ds^2 &= -\left(1-\frac{2M}{r}\right)dt^2 - \frac{4Ma}{r}dt d\phi \\
&\quad + \frac{dr^2}{1-\frac{2M}{r}+\frac{a^2}{r^2}} + \left(1+\frac{a^2}{r^2}+\frac{2Ma^2}{r^3}\right)r^2 d\phi^2
\end{aligned}
$$

comparison with Schwarzschild ($a \neq 0$)

- dt^2 Term: static limit
- $dt\, d\phi$ Term: frame dragging and Lense–Thirring
- dr^2 Term: event horizon

Event Horizon of Kerr Black Holes

First new feature

coordinate singularity:

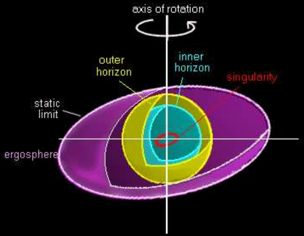
$$1 - \frac{2M}{r} + \frac{a^2}{r^2} = 0$$

radial coordinate of the horizon $r_{\rm H}$

$$r_{\rm H} = M \pm \sqrt{M^2 - a^2}$$

- $a < M$
  - $+$: event horizon of the black hole
  - $-$: inner horizon

  maximal angular momentum $a = M$: extremal black hole
- $a > M$: naked singularity (Cosmic Censorship)



black hole with horizons

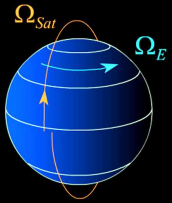
Sir
Roger Penrose
*1931

Gravitomagnetism

Second new feature

- The product $dt\,d\phi$ implies that the coordinates t and ϕ are intimately linked.
- The Kerr metric predicts Lense–Thirring effect and frame dragging.

What does Lense–Thirring mean?



- Schwarzschild: radial rocket thrust is required to keep a stationary observer at a fixed radius
- Kerr: additional tangential rocket thrust is required to keep an observer at a stationary position, i.e., a position from which the fixed stars do not appear to move

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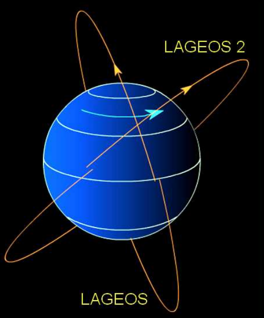

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Gravitomagnetism

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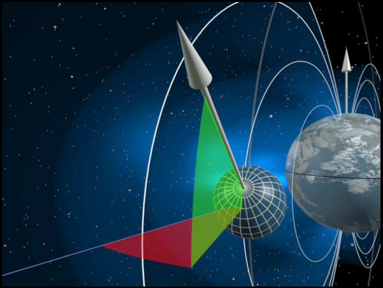
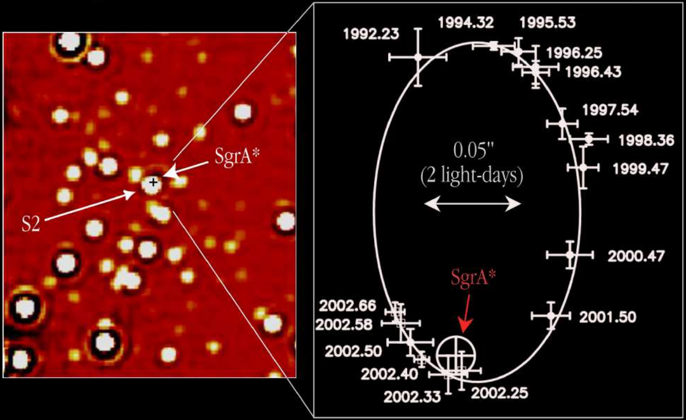
- The Kerr metric predicts Lense–Thirring effect and frame dragging.

What does frame dragging mean?



- Schwarzschild: gyroscopes always point in the same direction
- Kerr: gyroscpes start to precess, i.e., the direction with respect to distant stars changes

Black Hole at the Center of the Milky Way



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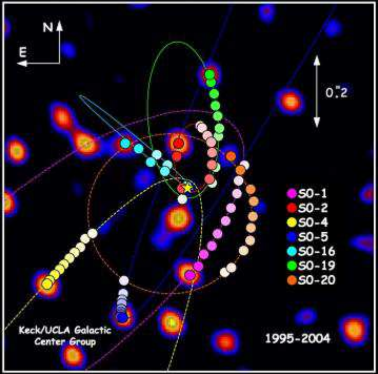

Black Hole: mass $3.7 \cdot 10^6\ M_\odot$ (Yusuf–Zasdeh *et al. Astrophys. J.* **644**, 198 (2006))
angular velocity $\sim 1/17\ \mathrm{min}$

R. Genzel (1995 – 2006)

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Maxwell Theory

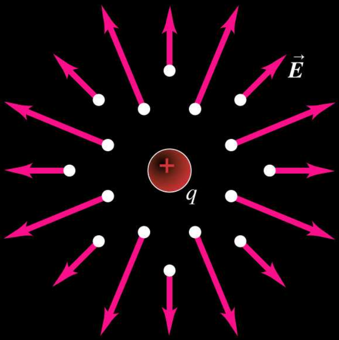
electric field $\vec{E}$, magnetic field $\vec{B}$
electromagnetic potential A^μ: $(\Phi, \vec{A})$

$$\vec{E} = -\nabla\Phi - \partial_t \vec{A} \,, \quad \vec{B} = \nabla \times \vec{A}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

the field strength tensor $F_{\mu\nu}$ is gauge invariant, while $A_\mu \rightarrow A_\mu + \partial_\mu \chi$

- the electromagnetic field can carry energy
- the electromagnetic field can carry momentum
- the electromagnetic field can carry angular momentum



Coulomb field

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Einstein–Maxwell Equations

Einstein–Maxwell theory

- Einstein equations

Einstein tensor $\longrightarrow$ $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ $\longleftarrow$ stress–energy tensor

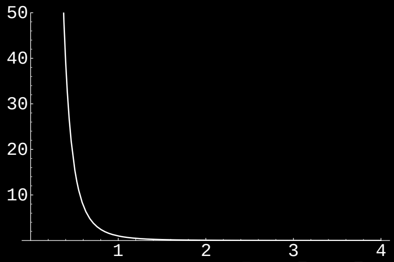
- Maxwell field equations

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = 0$$

For spherical symmetry:

$T_{00} \sim E^2 \sim \frac{1}{r^4}$

what are the properties of charged black holes?



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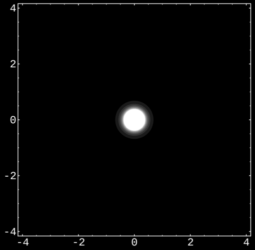

Reissner–Nordström Black Holes

1916: H. Reissner, 1918: G. Nordström
charged static spherically symmetric black holes:

$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

- electrically charged black hole: M, Q
  - horizons: $r_\pm = M \pm \sqrt{M^2 - Q^2}$
  - event horizon: $r_\mathrm{H} = M + \sqrt{M^2 - Q^2}$
- magnetically charged black hole: M, P

energy density outside the horizon due to the Coulomb field of the charge Q

$$M = M_\mathrm{H} + M_\mathrm{outside} = M_\mathrm{H} + 2\Phi_\mathrm{H} Q$$

Hans J. Reissner
1874 – 1967

Gunnar Nordström
1881 – 1923

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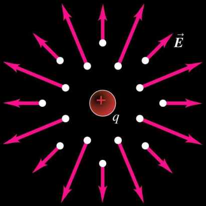
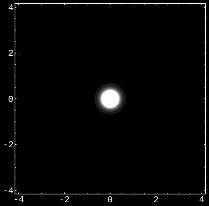
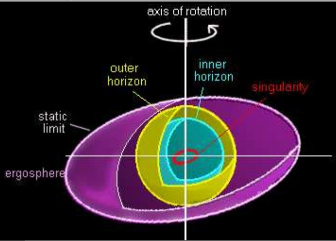




Kerr–Newman Black Holes

charged rotating black holes: adding charge to the Kerr solution

global charges:	M, $J = aM$, Q, P	
dipole moments:	$\mu_{\rm mag} = g_{\rm mag} \dfrac{Q}{2M} J$, $\quad \mu_{\rm el} = g_{\rm el} \dfrac{P}{2M} J$	$(g_{\rm Dirac} = 2)$
horizons:	$r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2 - P^2}$	$(\Delta = 0)$
static limit:	$r_0 = M + \sqrt{M^2 - a^2 \cos^2\theta - Q^2 - P^2}$	$(g_{tt} = 0)$
horizon velocity:	$\Omega = \dfrac{a}{r_+^2 + a^2}$	



Rotation and Deformation

- Kerr–Newman black holes co–rotate

$$\Omega > 0 \quad \Leftrightarrow \quad J > 0$$

they do not counter-rotate:

$$\Omega > 0 \ , \ J < 0$$

- a static horizon implies vanishing angular momentum

$$\Omega = 0 \quad \Leftrightarrow \quad J = 0$$

there are no black holes with

$$\Omega = 0 \ , \ J \neq 0 \qquad \text{or} \qquad \Omega \neq 0 \ , \ J = 0$$

- rotation implies an oblate deformation of the horizon

Summary: Einstein–Maxwell Black Holes

	static	rotating
spherically symmetric	Schwarzschild (M) Reissner-Nordström (M, Q, P)	–
axially symmetric	–	Kerr (M, J) Kerr–Newman (M, Q, P, J)

- Uniqueness theorem

  black holes are uniquely determined by their mass M, angular momentum J, charges Q and P
- Israel's theorem

  static black holes are spherically symmetric
- Staticity theorem

  stationary black holes with non-rotating horizon are static

Werner Israel

*1931

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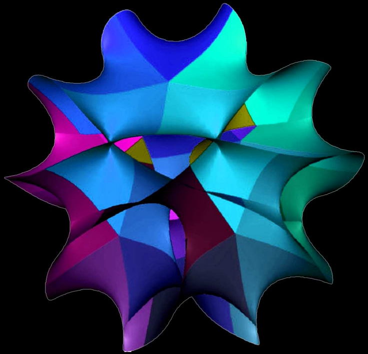
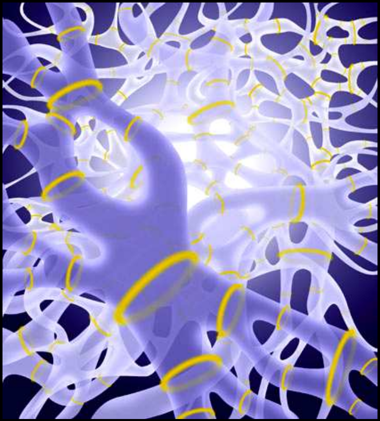
Motivation: String Theory

String theory:

- higher dimensions
- additional fields





influence on black hole physics ?

Einstein-Maxwell-Dilaton Theory

Einstein-Maxwell-dilaton action

$$S = \int \Big\{ \underbrace{\frac{R}{4}}_{\text{gravity}} \underbrace{- \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi}_{\text{dilaton}} \underbrace{- \frac{1}{4} e^{2\gamma\Phi} (\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu})}_{\text{Maxwell}} \Big\} \sqrt{-g} d^4x$$

dimensionless dilaton coupling constant γ

$\gamma = 0$: Einstein-Maxwell theory

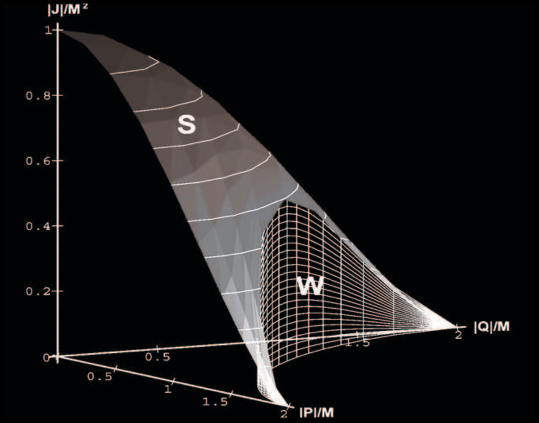
$\gamma = 1$: string theory

$\gamma = \sqrt{3}$: Kaluza-Klein theory

$\gamma > \sqrt{3}$

Kaluza-Klein Black Holes

Surfaces of extremal solutions in Kaluza-Klein theory: Rasheed 1995



vertical wall W:

stationary $\Omega = 0$ solutions

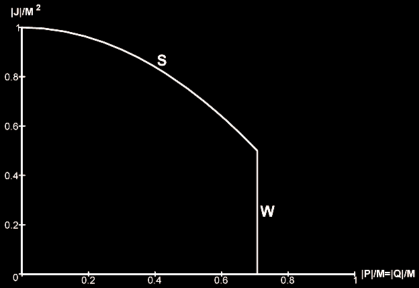
$$\left(\frac{P}{M}\right)^{\frac{2}{3}} + \left(\frac{Q}{M}\right)^{\frac{2}{3}} = 2^{\frac{2}{3}}$$

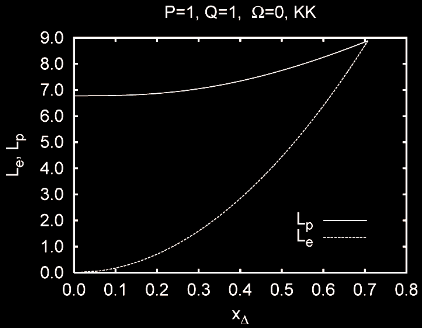
$$J \leq PQ$$

J increases, $M = \text{const}$

Kaluza-Klein Black Holes

extremal $|P| = |Q|$ solutions





vertical wall W:

stationary $\Omega = 0$ solutions

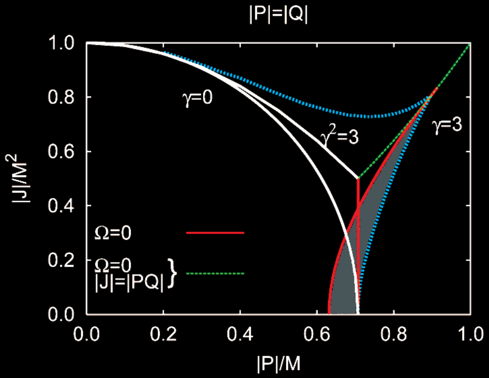
horizon circumferences:

$L_{\rm e}$ and $L_{\rm p}$

prolate deformation

Rotating EMD Black Holes

Kleihaus, Kunz, Navarro-Lérida 2004



extremal: $|P| = |Q|$

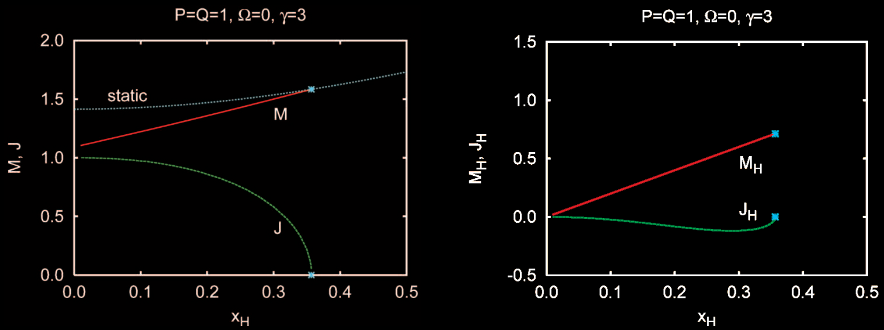
stationary: $\Omega = 0$

stationary: $\Omega = 0$, $J = PQ$

what is in the shaded region?

Non-Rotating Stationary EMD Black Holes

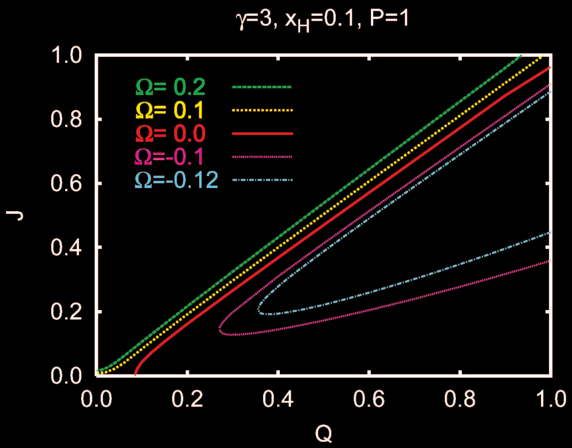
non-extremal stationary $\Omega = 0$ black holes



mass and angular momentum | horizon mass and angular momentum

- as J increases, M decreases
- a negative fraction of J resides behind the horizon: $J_{\rm H} < 0$
- effect of the rotation: prolate deformation of the horizon

Counter-Rotating EMD Black Holes



angular momentum versus charge

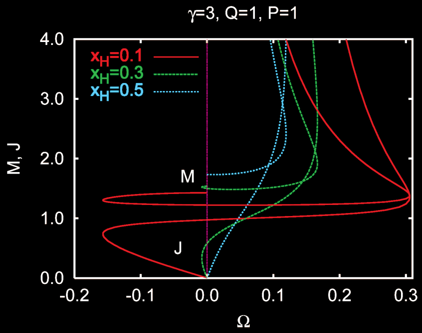
- co-rotation

  $J > 0$: $\Omega > 0$
- non-rotating horizon

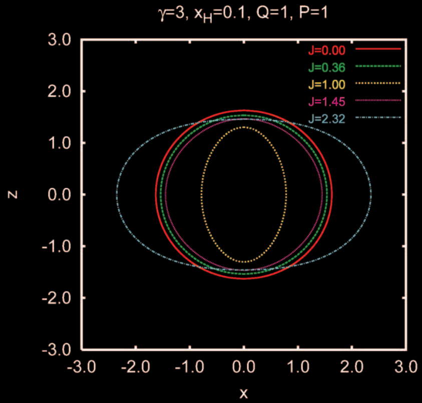
  $J > 0$: $\Omega = 0$
- counter-rotation

  $J > 0$: $\Omega < 0$

Shape of Counter-Rotating EMD Black Holes



angular momentum J versus Ω



embedding of the horizon shape

Outline

1. Introduction to Black Holes
   - General Relativity
   - Schwarzschild Black Holes
   - Kerr Black Holes
2. Microscopic Black Holes
   - Maxwell Theory
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   - $4D$ Einstein-Maxwell-Dilaton Black Holes
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   - Odd-D Einstein-Maxwell-Chern-Simons Black Holes
3. Conclusions

$D = 5$ Einstein-Maxwell-Chern-Simons Theory

In odd dimensions $D = 2n + 1$ the Einstein-Maxwell action may be supplemented by a 'AF^n' Chern-Simons term.

$D = 5$ Einstein-Maxwell-Chern-Simons action

$$S = \int \frac{1}{16\pi G_5} \left\{ \sqrt{-g}\,(R - \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}) - \underbrace{\frac{2\lambda}{3\sqrt{3}} \varepsilon^{mnpqr} A_m F_{np} F_{qr}}_{\text{Chern}-\text{Simons}} \right\} d^5x$$

Chern-Simons coupling constant λ

$\lambda = 0$: Einstein-Maxwell theory

$\lambda = 1$: bosonic section of minimal $D = 5$ supergravity

$\lambda > 1$

$\lambda = 0$: $D = 5$ Einstein-Maxwell Black Holes

- rotating vacuum black holes

  Myers, Perry 1986

  two angular momenta J_1, J_2

  rotation in two orthogonal planes

- rotating EM black holes

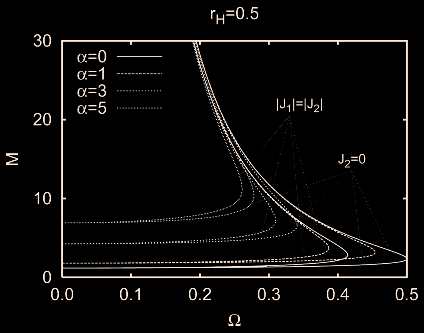
  surprise: no analytic solutions

  Kunz, Navarro-Lérida, Petersen 2005

  $g \neq 3$

$J_1 \neq 0$, $J_2 = 0$ black holes

$J_1 = J_2$ black holes



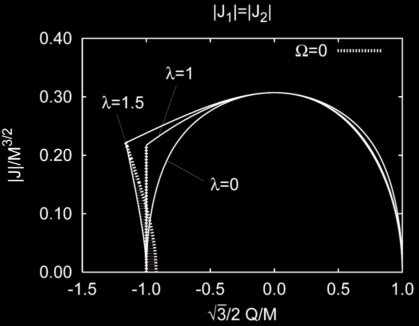
$\lambda = 1$: Supersymmetric Black Holes

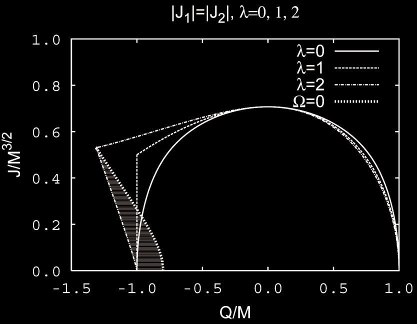
extremal $\lambda = 1$ EMCS black holes: Breckenridge, Myers, Peet, Vafa 1996

- mass saturates the bound: $M \geq \frac{\sqrt{3}}{2}|Q|$
- finite angular momenta: $|J| = |J_1| = |J_2|$
- angular momenta satisfy the bound: $|J| \leq \frac{1}{2}\left(\frac{\sqrt{3}}{2}|Q|\right)^{3/2}$
- horizon angular velocites vanish: $\Omega_i = 0,\ |J| \neq 0$
- angular momentum is stored in the Maxwell field
- negative fraction of the angular momentum is stored behind the horizon
- the effect of rotation is to deform the horizon into a squashed 3-sphere

$\lambda > 1$: Rotating $D = 5$ Black Holes

Kunz, Navarro-Lérida 2006





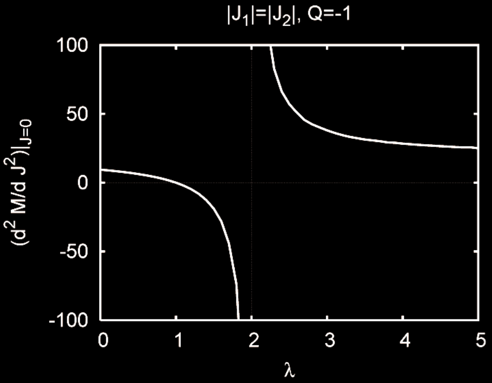
- black holes with $\Omega = 0,\ J \neq 0$
- black holes with $\Omega < 0,\ J > 0$

non-extremal

counter-rotating

Instability of $5D$ EMCS Black Holes

$\left.\frac{d^2M}{dJ^2}\right|_{J=0}$ for extremal black holes

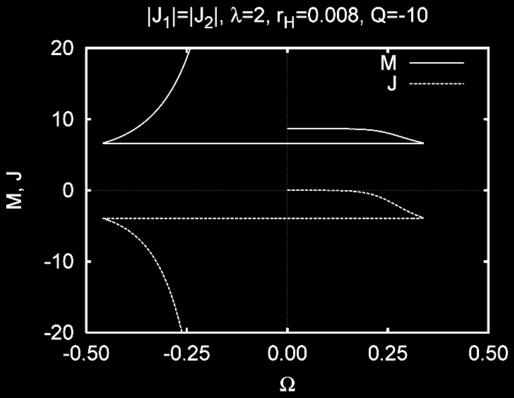


- instability beyond $\lambda = 1$

  supersymmetry marks the borderline between stability and instability
- $\lambda = 2$ is special

Non-Uniqueness of $5D$ EMCS Black Holes?

$\lambda = 2$ EMCS black holes

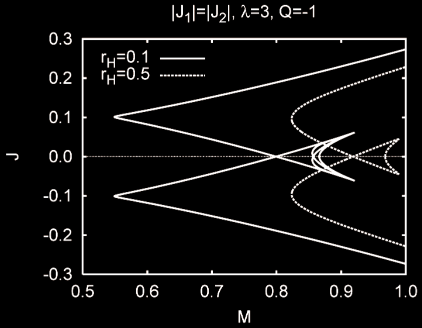


angular momentum and mass versus Ω

- $\lambda = 2$: set of extremal rotating $J = 0$ solutions appears to be present
- $\lambda = 2$: infinite set of extremal black holes with the same charges

Non-Uniqueness of $5D$ EMCS Black Holes

$\lambda > 2$ EMCS black holes

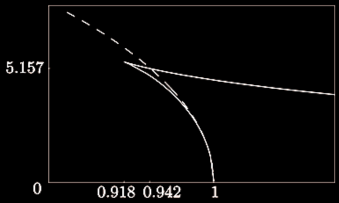


angular momentum versus mass

- black holes are not uniquely determined by M, J_i, Q

- non-uniqueness of $5D$ black holes with horizon topology of a sphere S^3
- non-uniqueness of $5D$ black holes and black rings ($S^1 \times S^2$)

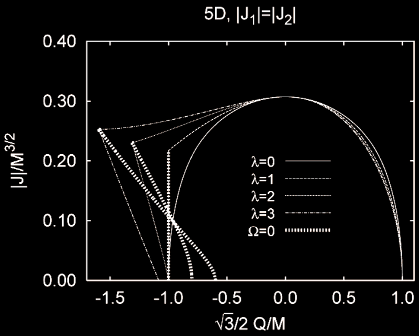
Emparan, Reall 2002



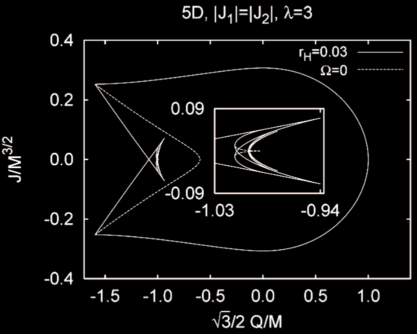
$$\frac{\mathcal{A}}{(GM)^{3/2}} \quad \text{versus} \quad \sqrt{\frac{27\pi}{32G}}\frac{J}{M^{3/2}}$$

Domain of Existence of $5D$ EMCS Black Holes

$\lambda > 2$ EMCS black holes



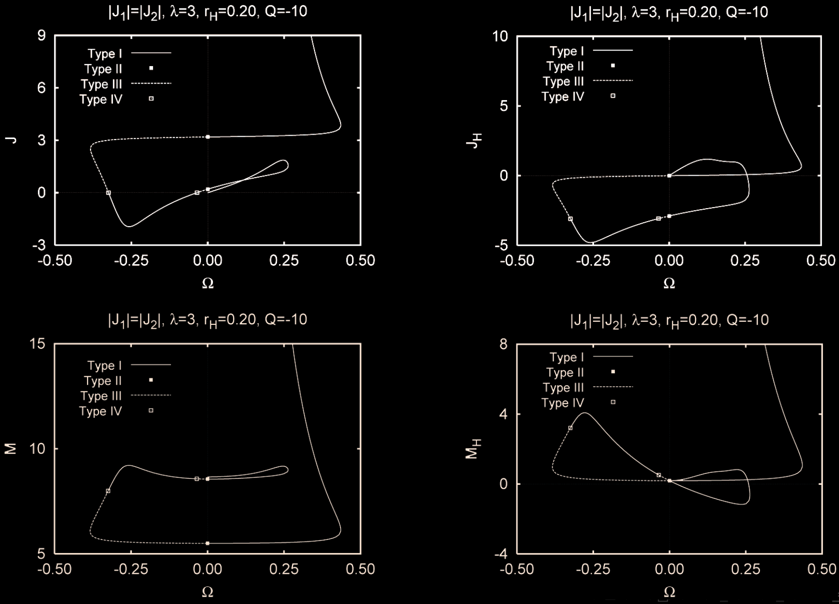
domain of existence



(almost) extremal black holes

- static extremal black hole is no longer on boundary
- $J = 0, \Omega \neq 0$ (type 3)
  continuous set of black holes

Negative Horizon Mass of $5D$ EMCS Black Holes



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3 Conclusions

Odd-D Einstein-Maxwell-Chern-Simons Theory

odd-D Einstein-Maxwell-Chern-Simons Lagrangian

$$L = \frac{1}{16\pi G_D}\Big\{R - \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \underbrace{\frac{8\tilde{\lambda}}{D+1}\epsilon^{\mu_1\mu_2\dots\mu_{D-2}\mu_{D-1}\mu_D} F_{\mu_1\mu_2}\dots F_{\mu_{D-2}\mu_{D-1}} A_{\mu_D}}_{\mathrm{Chern-Simons}}\Big\}$$

Chern-Simons coupling constant $\tilde{\lambda}$

$\tilde{\lambda} = 0$: Einstein-Maxwell theory

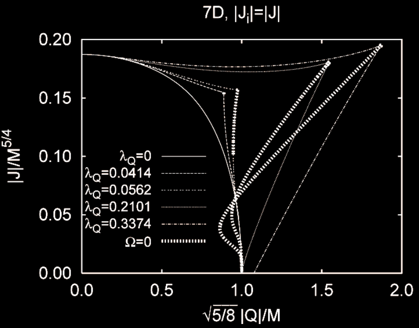
$\tilde{\lambda} \neq 0$: $\tilde{\lambda}$ dimensionful except for $D = 5$

scaling transformation: $D = 2N + 1$

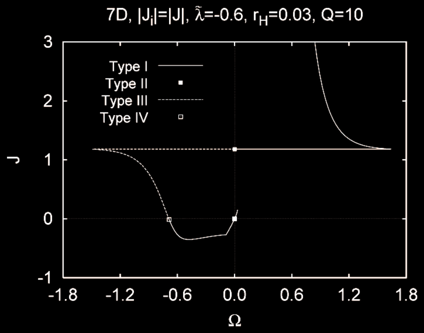
$$r_{\rm H} \to \gamma r_{\rm H}\ ,\ \ \Omega \to \Omega/\gamma\ ,\ \ \tilde{\lambda} \to \gamma^{N-2}\tilde{\lambda}\ ,\ \ Q \to \gamma^{D-3} Q\ , \dots$$

Rotating $D = 7$ EMCS Black Holes

Kunz, Navarro-Lérida 2006

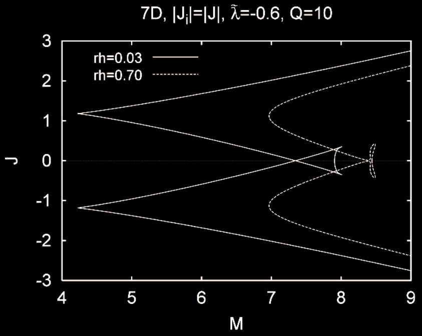


domain of existence

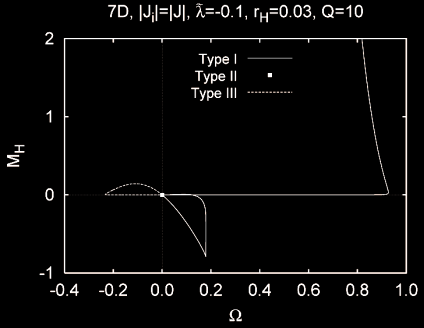


types of black holes

$\lambda > 1$: Rotating $D = 7$ Black Holes



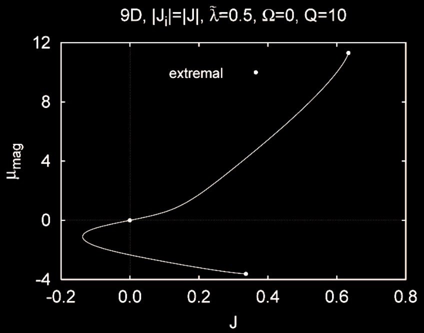
non-uniqueness



negative horizon mass

$\lambda > 1$: Rotating $D = 9$ EMCS Black Holes

Kunz, Navarro-Lérida 2006



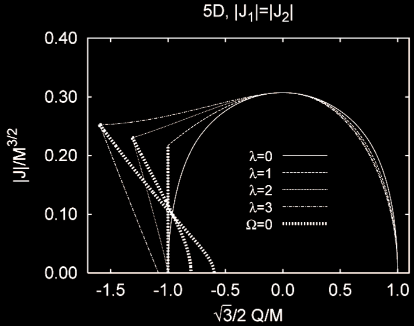
magnetic moment versus angular momentum

- non-static black holes with $J = 0$, $\Omega = 0$

Conclusions: Surprises with Rotating Black Holes

Einstein-Maxwell-Dilaton Black Holes

- $\Omega = 0$, $J > 0$ black holes
  stationary with static horizon
- $\Omega < 0$, $J > 0$ black holes
  counter-rotating black holes
- prolate horizon



$D = 5$ EM-Chern-Simons Black Holes

in addition: $\lambda \geq 2$

- $\Omega \neq 0$, $J = 0$ black holes
  rotating horizon, but vanishing J
- non-uniqueness of black holes
  with horizon topology S^3
- negative horizon mass

$D = 9$ EM-Chern-Simons Black Holes

in addition:

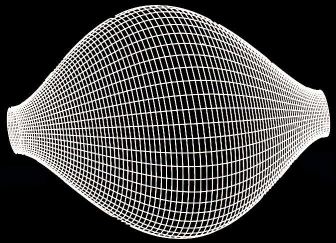
- $\Omega = 0$, $J = 0$ black holes
  stationary and non-static
- further surprises?

Outlook: Further Surprises?

higher dimensions:

- black holes

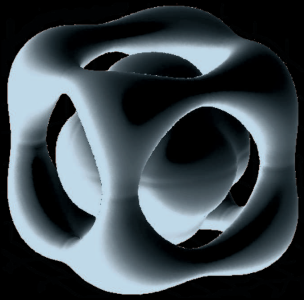
  different horizon topology?
- black strings



rotating non-uniform black strings

4 dimensions:

- platonic black holes?



- further surprises?