

GRATING DIFFRACTION-INTERFERENCE EFFECTS AND THEIR USE TO INTERFEROMETRY

Doctoral Thesis
by
LIREN LIU

Fachbereich Physik
Universität Oldenburg
Angewandte Optik

November 1991

Table of Contents

	page
Abstract	4
Acknowledgements	5
<u>1. INTRODUCTION</u>	6
1.1 Basic Idea	6
1.2 Historical Background	7
1.3 Preview	8
<u>2. PARTIALLY COHERENT DIFFRACTION EFFECTS</u>	9
2.1 Overview	9
2.2 Partially Coherent Diffraction Effect between Lau and Talbot Effects	11
2.3 Partially Coherent Diffraction Effect between Lau effect and Optical Fourier Transformation	11
2.4 Lau Effect in a Multi-Grating System	12
<u>3. JOINT TALBOT EFFECT</u>	14
3.1 Overview	14
3.2 Analysis of Joint Self-Imaging Effect	14
3.3 Logic-Operated Moiré	15
<u>4. TALBOT AND LAU EFFECTS ON INCIDENT BEAMS OF ARBITRARY WAVEFRONT</u>	16
4.1 Overview	16
4.2 Analysis and Experiment	17
<u>5. QUASI-INTERFEROMETRY</u>	18
5.1 Overview	18
5.2 Optical-Correlation Quasi-Interferometry	19

5.3	Quasi-Interferometry with Coded Correlation Filtering	20
5.4	Optical Biasing on Coded Correlation Filtering Quasi-Interferometry	21
5.5	Mask Functions in Quasi-Interferometry	22
5.6	Moiré Quasi-Interferometry	22
5.7	Color Coding of Fringes	23
5.8	Lensmeter	24
6.	<u>FOURIER-TRANSFORM DIFFRACTION INTERFEROMETRY</u>	25
6.1	Principle	25
6.2	Character Recognition of Phase Objects	26
7.	<u>INTERFEROMETRY BASED ON PARTIALLY COHERENT EFFECT BETWEEN TALBOT AND LAU EFFECTS</u>	27
8.	<u>TALBOT-INTERFEROMETRY USING LOGIC OPERATED MOIRÉ PHENOMENON</u>	28
9.	<u>CONCLUSIONS</u>	28
9.1	Future Prospects	29
9.2	Additional References	30
	Bibliography	32
	Statement of Independence	36
	Vita and Publications	37

Appendix A--Partially coherent diffraction effect between Lau and Talbot effects (J.Opt.Soc.Am.A,Vol.5,1709-1716,1988).

Appendix B--A new grating diffraction interference effect (Science in China, Series A,Vol.32,570-584,1989).

Appendix C--Lau effect in a multi-grating system (Optics Commun.Vol.70,267-271,1987).

- Appendix D--Joint Talbot effect and logic-operated moiré patterns
(J.Opt.Soc.Am.A,Vol.7,970-976,1990).
- Appendix E--Talbot and Lau effects on incident beams of arbitrary wavefront, and
their use (Appl.Opt.Vol.28,4668-4678,1989).
- Appendix F--Optical-correlation quasi-interferometry: a new viewpoint on spatial
frequency filtering (Appl.Opt.Vol.22,3024-3030,1983).
- Appendix G--Quasi-interferometry with coded correlation filtering
(Appl.Opt.Vol.21,2817-2816,1982).
- Appendix H--Optical biasing on quasi-interferometry with coded correlation filtering
(Appl.Opt.Vol.21,3839-3845,1982).
- Appendix I--Mask functions in quasi-interferometry (Optics Commun.Vol.44,301-
305,1983).
- Appendix J--Moiré quasi-interferometry (Optics Commun.Vol.45,215-220,1983).
- Appendix K--Color coding in quasi-interferometry (Appl.Opt.Vol.22, 3016-3023,1983).
- Appendix L--Specification of imaging-lensmeter.
- Appendix M--Observable imaging and character recognition of phase objects by coded-
spectrum diffraction interferometry (Kexue Tongbao(Chinese Science
Bulletin),Vol.33,685-689,1988).
- Appendix N--Interferometry based on the partially coherent effect lying between the
Talbot and Lau effects (J. of Modern Optics,Vol.35,1605-1618,1988).
- Appendix O--Talbot interferometry using logic-operated moiré (Optics
Commun.Vol.77,279-284,1990).

Abstract

Three related diffraction-interference phenomena involving two gratings, the partially coherent diffraction effect, the joint Talbot self-imaging effect, and the Talbot-Lau effect on incident beams of arbitrary wavefront, are proposed and investigated in the first part of this thesis. Some new and interesting phenomena are thus observed. On this basis, four kinds of optical contour-mapping methods are suggested in the second part, which are the quasi-interferometry, the Fourier-transform diffraction interferometry, the partially coherent diffraction interferometry, and the logic-operated Talbot interferometry. These interferometers have a large range of measurement, especially suitable to strongly distorted phase objects. The applications are demonstrated.

Acknowledgements

The work presented here in this thesis was drawn both from 2 years of research project involving nonconventional interferometry with incoherent illumination at Oldenburg and from the following years of relative work at Shanghai. I am deeply indebted to several people for helping me with experiments and for helping me begin the doctoral promotion procedure.

I sincerely thank Prof. Dr. Klaus Hinsch for welcoming me here in Oldenburg twice as a guest scholar, for laying the groundwork for my initial stipendium of the AvH Foundation, and for arranging the promotion procedure. He has created a relaxed research atmosphere in Oldenburg which encourages new ideas, experimentation and inventing. I have learned much about optics and finished quite a lot of research work in this environment. His unbelievable patience and implicit confidence in my work has finally resulted in completion of this thesis.

I also wish to thank Dr. Heinz Helmers for active help in experiments and computer operations. I thank Mr. Rolf Harms for patient help in mechanics. My thanks extend to the students and personnel there for frequent helps. Specifically, I am very grateful to Prof. Dr. Joachim Luther for his generous support to my promotion.

Lastly, my deepest gratitude goes to the Alexander von Humboldt-Stiftung for their financial support during my stays in Oldenburg. My gratitude also goes to the Chinese Natural Science Foundation for the support of my researches in Shanghai. The AvH stipendium awarded to me in 1981 enabled me to come to Oldenburg and work with the Coherent Optics group under Prof. Hinsch. Ich bedanke mich recht herzlich dafür!

1. INTRODUCTION

1.1 Basic Idea

The interference phenomena of two diffractive gratings can be observed in a general configuration as depicted in Fig. 1.1. Light from a source passes through the two gratings and finally projects on the observation screen. The black boxes represent in principle free spaces and/or optical elements. In this report, the interference phenomena with different light sources (coherent, incoherent, and partially coherent), different gratings (Ronchi gratings and the others), and different arrangements of the system are investigated. The results are then used to constitute optical interferometers.

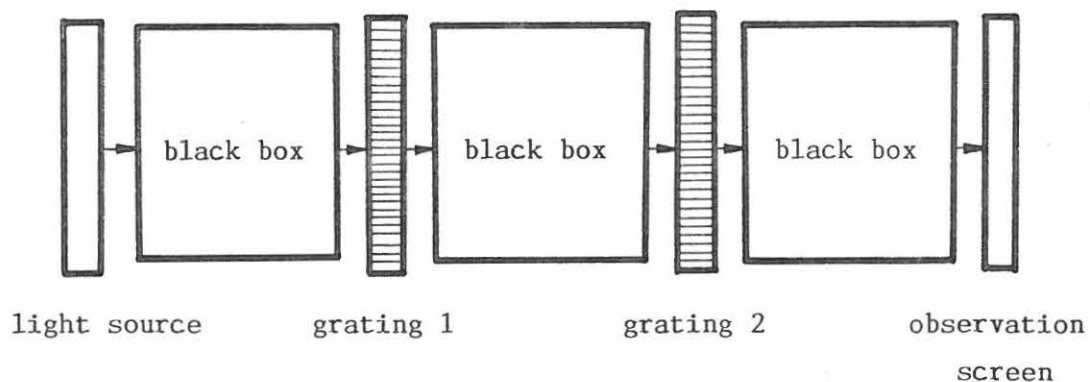


Fig. 1.1

1.2 Historical Background

Two kinds of imaging effect involving grating diffraction-interference are now well known: the Talbot effect and the Lau effect. When a grating is illuminated with a monochromatic and spatially coherent light source, exact and many more images can be found at finite distances from the grating. This self-imaging phenomenon is named the Talbot effect[1], it was first observed by Talbot more than 150 years ago. Only recently, considerable theoretical and experimental interest in the subject has been aroused[2-7]. In particular, Winthrop and Worthington[3] and Montgomery[4] gave detailed solutions on the basis of Fresnel diffraction theory. The Lau effect is an interference phenomenon observed at infinity when light from an extended source passes through two diffraction gratings of equal period in tandem. Grating-like fringes will be generated. The effect was discovered by Lau[8] in 1948. But only in recent years have a variety of theories and experiments been developed to explain this interesting effect[9-16]. For instance, Jahns and Lohmann[9] described the effect in terms of geometrical optics and scalar diffraction theory, Gori[10] and Sudol and Thompson[11] gave their analyses based on the theory of partial coherence, and Swanson and Leith[12] used a model of a generalized two-grating interferometer. However, all the theoretical analyses and experimental results were essentially based on the illumination of spatially incoherent light.

Correspondingly, there are two types of interferometry involving the self-imaging of diffraction grating: the Talbot interferometry and the Lau interferometry. Both these interferometers are shearing interferometers, but they differ in their modes of illumination, which are coherent and incoherent, respectively. The principle of the Talbot interferometer is essentially based on the Talbot self-imaging effect. In the basic as well as the simplest version[17], a second identical grating is placed in the observation plane coinciding with the self-imaging plane of the first grating illuminated with a coherent plane wavefront. A phase object located between the two gratings will change the moiré patterns resulted initially from the multiplicative superposition of the

two gratings. The amount of fringe distortion thus represents the lateral derivative of the phase variation. Since the Talbot interferometer consists only of two gratings, it is easy to align and to work with. In order to extend the applicability, modifications were then suggested, for instance, with circular gratings[18], spatial filters[18], rotated gratings[19,20], second gratings of higher spatial frequency[21], and anisotropic gratings[22]. In the earliest Lau interferometer[23] the two gratings used are imaged to each other by a lens combination giving an effective self-imaging distance of zero. Then a fringe pattern of infinite period is observed. The absolute value of the intensity depends on the lateral shift between the two gratings. Therefore, an interference pattern can be seen as a phase object is inserted in the plane between the gratings. Furthermore, by displacing one of the gratings axially, a Lau-fringes-biased interferogram results[15]. A great number of applications of these two kinds of interferometers have been demonstrated.

1.3 Preview

This thesis deals with the new diffraction-interference effects relating two gratings and their applications to optical interferometry. When a partially coherent source is used to illuminate the gratings, the partially coherent diffraction effects will be observed. These effects are discussed in Chapter 2, and the interferometry based on the partially coherent effect in Chapter 7. The joint Talbot self-imaging effect is seen as the observation is made apart from the two gratings illuminated by a coherent parallel wavefront. A mechanism of the moiré pattern, which is related not only to the independent self-images of the gratings but also to their mutual coupling, is thus found and interpreted in terms of the multiple-valued logic operations. The text is presented in Chapter 3, and the related logic-operated Talbot interferometer is given in Chapter 8. The Talbot and Lau effects of the two gratings illuminated by beams of arbitrary

wavefront are reported in Chapter 4. The two gratings in an optical correlator yield the general Lau effect. On this basis and in the limit of geometric optics, quasi-interferometry is suggested and described in Chapter 5, in which the concept of a grating is extended to the mask of coded pattern. In the limit of diffraction optics, the same configuration as the quasi-interferometer becomes the Fourier-transform diffraction interferometer, which is analyzed in Chapter 6.

A collection of my publications closely related to this thesis is attached as Appendices. In the following context, I shall give only the brief discussions on the principles.

2. PARTIALLY COHERENT DIFFRACTION EFFECTS

2.1 Overview

In fact, there exists an inherent connection between the Lau effect and the Talbot effect. It is apparent that the same specific distance is needed for the both effects, but the two effects differ in their modes of illumination, spatially incoherent or coherent. Thus some methods using the parageometrical approach[24], and operations of the Wigner distribution function(WF)[25] were proposed to treat both the Lau and the Talbot effects in a uniform fashion. In this context, naturally, it is of great interest to look for a new diffraction effect of gratings between the Lau and the Talbot effects with partially coherent illumination. This idea was first proposed by Lohmann et al[26,27] and developed by Indebetouw[28]. Some relations between the axial and the lateral periodicities of the mutual intensity of partially coherent illumination were thus found. In addition, Gori[10] and Sudol and Thompson[11] discovered the fact that an incoherent illuminated grating gives rise to a grating-like coherent function is the basis

of the Lau effect, and the second grating will sample this coherent function. Furthermore, Sudol[29] extensively analyzed the coherence of the light passing through the system. Recently, Hane et al[30] extended the study of the Lau effect by replacing the second transmission grating with a reflection grating and noticed that the fringe profile varies as one of the gratings is translated in a lateral direction.

These studies, however, seemed either not to concentrate on the direct determination of the possible fringes under partially coherent illumination or not to consider the partial coherence of light fields. So far, to our knowledge, no further proposal has been suggested to implement a full display of fringe distributions in the case of partially coherent illumination, which can be changed practically by continuously tuning the spatial coherence and in particular should become the well-established Lau fringes and the Talbot self-images of the used gratings at the two extremes of illumination coherence--zero coherence and pure coherence, respectively.

The aim of this Chapter has been to study the fringe characteristics under partially coherent illumination of variable coherence and to establish a straight link of the partially coherent fringes to the Lau fringes and the Talbot self-images or the Fourier transforms of the gratings.

In Section 2.2, the phenomenon in the Lau setup illuminated directly by a source slit is studied. According to the Van Cittert-Zernike theorem, the illumination on the Lau setup is a partially coherent field. The resulted partially coherent diffraction effect stands between the Lau and the Talbot effects. Then a collimating lens is introduced between the slit and the Lau setup, thus the illumination on the Lau setup is partially coherent quasihomogeneous. This leads to another partially coherent diffraction effect lying between the Lau effect and the optical Fourier transformation. Section 2.3 describes this situation. The single source slit may be replaced, as imagined, by a source grating. Moreover, a number of gratings may be used. This kind of multi-grating system is reported in Section 2.4.

The mutual intensity is most frequently used to analyse the partially coherent optical system in the quasichromatical approximation. But an advanced mathematical method,

the ambiguity function, is adopted in this Chapter. The ambiguity function(AF) was first introduced in radars in order to establish the ambiguity in range and velocity. The phase space representation like the AF can help simplify the treatment of partially coherent optical systems[31-33], especially the propagation can be described by a transport matrix as done in geometric optics. It is known that there is an inherent connection among the AF, the WF and the mutual intensity.

2.2 Partially Coherent Diffraction Effect between Lau and Talbot Effects

As shown in Appendix A, a source slit is used to illuminate the Lau setup. The width of the slit is changeable, so the spatially coherent state of the illumination can be varied continuously. If the slit width is adjusted to approach zero, the illumination is approximately coherent, and the Talbot effect can be seen. If the slit is opened wide enough, the illumination is incoherent, resulting in the Lau effect. Thus the partially coherent effect between the Lau effect and the Talbot effect can easily be realized. We employ the AF to evolve the suggested system. As a result, a general formula is achieved as a joint correlation among the three functions of the source slit and the two gratings for an exact description of the intensity distribution of fringes yielded by any state of illumination coherence. Here an interesting phenomenon between the Lau and the Talbot effects is predicted: that the fringe pattern changes in its profile not only as the degree of spatial coherence is varied with the adjustment of the slit width but also as one of the gratings is shifted in the direction perpendicular to the grating lines. In describing the experiments, photographs of the intensity distributions in the cases of coherent, partially coherent, and incoherent illumination are shown.

2.3 Partially Coherent Diffraction Effect between Lau Effect and Optical Fourier Transformation

Our aim of this Section is to study the fringe characteristics in the Lau setup illuminated by a partially coherent quasihomogenous source, i.e. a lens is introduced between the slit and the Lau setup to collimate the illumination. It can be seen that the Lau fringes and the optical Fourier transform of the gratings are the situations at the two extremes of coherence--zero coherence and pure coherence, respectively. The suggested system is analysed in terms of the AF, it leads to a general analytic expression for describing the intensity distribution on the observation screen. For a better understanding, numerical examples are shown. An experimental verification with an arrangement of one-dimensional configuration is carried out. Thus it allows us to say a new effect between the Lau effect and optical Fourier transformation is seen that the fringe pattern varies in its profile not only as the state of coherence is changed by the adjustment of the slit width but also as one of the gratings is translated laterally. All the details are given in Appendix B.

2.4 Lau Effect in a Multi-Grating System

We have studied the diffraction-interference phenomena of gratings under partially coherent illumination generated by a slit source. It was shown that the resultant phenomena characterized by the changeable fringe profile belongs to the partially coherent effect. When the distance between the slit source and the first grating is within the range as set in the near-field approximation, the partially coherent effect will fall somewhere between the Lau and the Talbot effects(Section 2.2). When the distance is elsewhere for a far-field diffraction to occur, the partially coherent fringes will look like something between the Lau fringes and the intensity diffraction pattern of the gratings(Section 2.3).

Hane et al[30] studied the Lau effect by replacing the second transmission grating with a reflection one. This setup is an equivalent 3-grating system. In their work, a

formula in the form of series was obtained through the use of diffraction theory to describe the fringe intensity distribution. Yet the formula implies no physical meaning for each term and is therefore only amenable to numerical simulation. Especially, the question of change, if any, in the intrinsic properties of coherence has not been touched on. From the above considerations, we can extend the configuration of the reflective Lau effect to include a system with three independent gratings. Otherwise, we can develop the single source slit further to the case of a grating-like illumination. Thus our first aim of this Section is to study the interference phenomenon in a 3-grating system, where the distance between any two gratings is changeable and the period of the first grating may differ from the other two. As seen in Ref. 3 in Section 9.2, we employ the AF to evolve the intensity distribution of the interference fringes. As a result of calculations, a transparent and elegant expression is obtained, which describes mathematically the fringe intensity distribution as a joint convolution-correlation integral of three equivalent functions of the gratings. By changing the slit width of the illuminating grating, the Talbot effect, the partially coherent effect, and the Lau effect can be observed in sequence. It is also noticed that the profile of the partially coherent fringes varies with the relative shift of any one of the gratings. All these phenomena are predicted by utilizing numerical simulations.

The main aim of this Section is to study the diffraction-interference phenomenon on the Lau setup with any number of gratings of equal period in tandem, which is a natural extension of the 3-grating system. The details are given in Appendix C. Alternatively, the multi-grating system is analysed in terms of the off-axis Fourier transform model. A general and elegant formula for describing exactly the fringe intensity distribution is then resulted, which is a joint correlation among the functions of gratings. It is recognized that the fringe profile varies not only as one of the gratings is translated but also as one of the separations between two sequent gratings is changed, when more than two gratings are used. In the same way, fringe sharpening or down-conversion in spatial frequency can be achieved.

3. JOINT TALBOT EFFECT

3.1 Overview

Previous analyses and experiments on the Talbot effect involved only a single grating[1-7]. Our aim of this Chapter is to extend the concept of the normal Talbot effect into that with two separated and crossed gratings of substructures. The joint self-images are observed as the observation is made apart from the second grating, they are related to independent self-images of the two gratings and their mutual coupling.

In general, the joint self-images of two gratings are the moiré patterns. Because of the mutual coupling, these moiré patterns may differ from those resulting from the direct overlapping of the two gratings or of their independent self-images. This mechanism of the formation of the moiré patterns is different from that in the conventional moiré effect. A way is thus found to create new kinds of moiré patterns from two gratings by diffraction.

3.2 Analysis of Joint Self-Imaging Effect

A brief description to the details shown in Appendix D is as follows. Each period of the both gratings is divided into N stripes of identical width. The distance between the first grating and the second one, the distance between the second grating and the observation screen, and the distance between the first grating and the screen all satisfy the fractional self-imaging conditions. In terms of the Fresnel diffraction theory, we can obtain the field distribution equation for describing the joint self-images on the screen. From the mathematics, it is concluded that the joint self-images are related to both the

independent self-images of the gratings and the coupling between them. In order to discuss the effect in a way easy to understand, we define the original moiré pattern as the direct multiplicative superposition of the two gratings, the in-between moiré pattern as the field distribution behind the second grating, and the final moiré pattern as the field on the screen. Moreover, we decompose a moiré pattern into a structural element, which is rhombic and consists of $N \times N$ basic cells, and an array function. In this treatment, research on the structural element is essentially enough to explain the whole moiré pattern, and the structural element of the final moiré pattern can be easily evaluated.

3.3 Logic-Operated Moiré Phenomenon

The conventional moiré patterns are formed from the superposition of periodic structures in optics. The moiré fringe has been explained theoretically[34] and its essential characteristics have been analysed[35]. The moiré technique has found wide use in measuring purposes. In principle one usually needs to eliminate the grating carriers, to extract one of the beats, and to sharpen the fringes. An orthogonal-state-grating moiré in using colors or polarizations was proposed to solve these questions[36,37]. In addition, gratings with a narrow opening width[38] or with pseudo-random sequence[39] were used to sharpen the moiré fringes. We have studied the moiré fringes by use of logic operations between two gratings of substructures(Ref.9, Section 9.2). From the point of view of optical logic, normal moiré operation formed by two Ronchi gratings can be regarded as an AND logic operation. In fact each of the logic operations corresponds to a moiré pattern, so we can select a special pattern by performing its corresponding logical operation. Furthermore, we can have as many stripes as we wish in the period of the gratings, thus we can select more complicated moiré patterns by the help of the multiple-valued logic. This leads to flexibility and

variability in processing of the moiré patterns. Optical logical operation is a nonlinear processing of information, and it is the fundament of optical computing as well as some optical image processing. Several approaches for the optical implementation of logic operations using spatial encoding techniques[40-42] have been presented. But they are difficult to operate in real time. Thus our aim is in terms of the multiple-valued logic to study such joint self-imaging conditions, under which the required moiré fringes can be synthesized in real time. So-called logic-operated moiré provides a new principle to obtain sharpened, carrier-free, and/or beat-selected moiré fringes directly from two gratings.

As seen in Appendix D, the design principle can be summarized as follows. The values of basic cells in a structural element can be represented by a table. However, this kind of table is just like the truth table of multiple-valued logic[43]. We can thus design the table of the structural element of the final moiré pattern as a certain logic operation capable of carrier suppression, fringe sharpening, and/or beat selection. This can be done only by appropriate control of the self-imaging conditions of the gratings. The design examples for the moiré patterns with binary logic, ternary logic, and quarternary logic are given.

4. TALBOT AND LAU EFFECTS ON INCIDENT BEAMS OF ARBITRARY WAVEFRONT

4.1 Overview

As we have learned from the above mentioned studies, all the explanations are based on the fundamental assumption that the apertures of the incident light as well as the optical system are considered to be of infinite extent. Moreover, there are no

discussions of the experiment with a beam of arbitrary wavefront illuminating both the Lau and the Talbot setups. It is interesting to explore the Talbot and the Lau effects on beams of arbitrary wavefront and finite extent in using Ronchi gratings or binary phase gratings. This is the aim of this Chapter. We are particularly concerned with Gaussian beams. The Gaussian beam has two very useful properties in that its Fresnel diffraction and its Fourier transform are still Gaussian.

Recently, a binary optics technology is emerging[44]. Binary optics refers to the two-level nature of the phase relief pattern to control the phase, the amplitude, and the polarization of an optical wavefront. The analysis of the Talbot and the Lau effects on beams of arbitrary wavefront also leads to some interesting conclusions, such as beam modulation via the grating shift and phase compensation by the second phase grating. Therefore, we can apply these phenomena to beam modulation, beam splitting or coupling, beam spread measurement, aperture filling of phased laser arrays, and phase locking of laser arrays. But the detailed discussion is beyond the topic.

4.2 Analysis and Experiment

As shown in Appendix E, the analysis is performed in terms of the Fresnel diffraction theory. First, the Talbot effect of Ronchi gratings and binary phase gratings on beams of arbitrary wavefront is analysed, resulting in a general expression for describing the field distribution on the observation screen. Then numerical simulations on the self-imaging profiles of Ronchi gratings and binary phase gratings illuminated by Gaussian beams are given. Next, the Lau effect on beams of arbitrary wavefront is discussed. As a result, an analytic expression for describing the far-field distribution on the observation plane in the Lau setup with different combinations of Ronchi gratings and binary phase gratings is obtained. The expression can be reduced to describe separately the standard Lau fringes if the incident illumination is incoherent, and the diffraction pattern of the product of the two gratings if the incidence is equivalent to

that of a plane wave. In the following, numerical simulations on the zero-order diffraction profiles of the Gaussian beams are demonstrated. To examine the established theory, some fundamental experiments are carried out, which are in good qualitative agreement with the predictions.

5. QUASI-INTERFEROMETRY

5.1 Overview

Many methods exist to determine or visualize phase variations of a transparent object of interest, such as optical interferometry, holographic intrerferometry, phase contrast, schlieren methods, and others. With these conventional methods, however, the measurable relative phase variation should not be much larger than the order of magnitude of wavelengths. Much effort has been made to find noncoherent methods suited to measure especially strongly varying or distorted phase objects. In this Chapter, optical quasi-interferometry is proposed.

From the description of the Lau interferometers in Section 1.2, it is seen that the arrangement of these Lau interferometers is similar to that of an optical correlator. In Section 5.2, on the basis of optical correlation in the limit of geometric optics we suggest the optical correlation quasi-interferometer for measuring the strongly distorted phase objects. This kind of noninterferential contour-mapping method can be also explained by Fourier filtering. In other words, the same optical system can be described both in the space domain and in the spatial frequency domain relying on different principles. The interpretations with the Fourier filtering in the cases of in-focus and defocus are presented in Section 5.3 and Section 5.4, respectively. The contour-mapping system depends on the correlation pattern of the two used coded masks. But there are not too

many choices to generate a correlation pattern of high contrast from two patterns. In Section 5.5, therefore, we introduce a simplified quasi-interferometer where only one mask is needed, and then study the mask functions for the desired contour-mapping systems. Various contour-mapping systems can be easily achieved by using only a mask. Furthermore, we use the moiré technique associated with double exposures to quasi-interferometers, as shown in Section 5.6. As a result, the high-order phase derivatives are contour-mapped. In Section 5.7, pseudo-color coding of the fringes is discussed.

The main advantages of the quasi-interferometry are: the system operates under incoherent extended illumination; various contour-mapping systems with different sensitivities in mapping phase variations can be obtained by changing the patterns of coded masks; background fringes can be yielded, which resemble an interferogram with or without background fringes. It is evident that this kind of quasi-interferometry can be used extensively for qualitative or quantitative measurements of various kinds of phase objects. Thus in Section 5.8, practical applications such as lensmeters for testing spectacle lenses are illustrated.

5.2 Optical-Correlation Quasi-Interferometry

It is well known that a defocused system serves as an optical correlator(convolver)[45,46]. The 2-D intensity transmittances of two transparent masks are correlated(convolved) with each other in such a system, and the intensity distribution across the output screen is their 2-D correlation(convolution) function. Our proposal uses the following facts: The two masks are purposely designed to have a well-defined pattern of correlation. When the object under test is inserted somewhere between these masks and imaged onto the output screen, the phase variations of the object may cause a modulation in the correlation pattern due to the angles of refraction. Consequently, contour fringes will result on the object image, which resembles an interferogram with or without background fringes. This procedure is known as optical-

correlation quasi-interferometry, since the contour fringes are generated by the operation of correlation not by coherent interference.

Appendix F describes the details. Complete expressions for describing the performance of the system are derived by the backward impulse response method described in this paper. We also find it convenient to characterize generally the resultant correlation by three main parameters: K the scaling ratio of mask 2 to its projection on mask 1; R the center shift factor of the projection of mask 2; and C the equivalent length that relates the refraction by the object to the center shift. Then we discuss the condition of invariant correlation, under which the mapping of angles of refraction is linear. The relation between the two treatments in the space and spatial frequency domains are explored.

5.3 Quasi-Interferometry with Coded Correlation Filtering

By comparison, it can be understood that in the space domain the system is readily synthesized mathematically, but in the spatial frequency domain with Fourier filtering the process of contour-mapping allows clearer physical analysis. In this Section, the above mentioned quasi-interferometry will be explained by Fourier filtering[47], it is called quasi-interferometry with coded correlation filtering here.

The basic configuration is similar to a typical Fourier filtering system, but with a mask illuminated by an extended light source instead of a point source. The interpretation is as follows: A coded mask is illuminated by an extended white light source, and the resulting transmittance is Fourier-transformed by a lens. The transformation of the coded mask acts as a spatial frequency carrier for the object under study. With the use of another lens the object light distribution is also transformed. A convolved spatial frequency spectrum will be produced and then filtered by the second coded mask. It can be proved that the total filter function is the

correlation of the light transmittances of the two masks. It is proved as well that the spatial frequency spectrum of the object is essentially identical with its phase gradient. So the information about the phase gradient, which appears on the object image in the form of fringes, can be obtained by a specially designed filter function. Therefore, various information about the phase gradient can be obtained simply by changing the filtering function, for example, contours of equal gradient, contours of the direction of varying phase gradient, contours of the gradient only along a pregiven direction, as well as nonequal interval contours of the gradient.

Appendix G gives the details. The conclusion, that the total filter function is the correlation between the two mask functions, is achieved by the theory of linear shift-invariant system. The Fourier transform spectrum of a transparent object is treated by geometrical optics. Some experiments and possible applications are then demonstrated.

5.4 Optical Biasing on Quasi-Interferometry with Coded Correlation Filtering

Like conventional interferometry, additional and variable background fringes are usually desirable. In the quasi-interferometry there is the possibility of providing the background fringes, as discussed in Section 5.2. To do this by the explanation of Fourier filtering, we must use the idea of optical biasing: a quadratic phase factor can be added to the object under test by defocusing of masks in optical Fourier transformation[48], and a linear phase factor can be added by off-axis positioning of masks.

The details are given in Appendix H. First, the correlation filtering process of quasi-interferometry is explored for the case of longitudinal defocusing of one of the two coded masks from its Fourier transform focal plane. The basic equations show that the effective transforming focal length and the scaling factor between the two masks are changed. Therefore, the effect of defocusing can be fully characterized by an equivalent transforming focal length f_q , an equivalent distance related to the distance between

object and corresponding mask l_q , and a quadratic phase coefficient f_o . The condition of invariance, when the defocusing of the mask generates just the quadratic phase, is investigated. In this case, the quadratic phase factor is directly proportional to the amount of defocusing. Next, we treat the optical linear phase biasing by lateral off-axis shifting of the masks. Finally, experiments and some applications of optical biasing are demonstrated.

It is interesting to find that there is a close relation between the general parameters K , C and R in the space domain and the general parameters f_q , l_q and f_o in the spatial frequency domain.

5.5 Mask Functions in Quasi-Interferometry

In this Section, a general method is given of finding the filtering function required for a desired contour-mapping system for coding the phase variation. For the present purpose we carry out our work in a so-called simplified quasi-interferometer. In such a system one of the masks is a pinhole or a point light source, and thus only one mask alone determines the filtering function, without any correlation. In other words, this is a typical Fourier filtering system[47].

Appendix I shows the details. First, we establish a general mathematical connection between the desired contour system for mapping the phase gradient and the corresponding pattern function of the coded mask. Then, as example, we give some useful and interesting functions for the coded masks. Experiments and possible applications are demonstrated, too.

5.6 Moiré Quasi-Interferometry

The moiré technique is powerful and has been utilized in metrology. Especially for phase objects, the information about the relative phase variation can be extracted in the form of moiré fringes in so-called carrier frequency photography[49-51]. In this Section we shall develop the quasi-interferometry with the moiré technique associated with double exposures.

As shown in Appendix J, two methods are suggested to yield the moiré fringes in the quasi-interferometry. If, between the two exposures, there is only a little change of the phase distribution of the object, this can be visualized in the form of moiré fringes. The second-order phase derivatives can be measured by the moiré fringes when the object is shifted laterally by a small amount to the optical axis of the system between the two exposures. The necessary condition to obtain appropriate background fringes for a correct moiré superposition is considered. In particular, an experimental demonstration for a progressive spectacle lens is given. By the contour-mapping of the second-order phase derivatives, the different areas for vision in the progressive lens are clearly recognized.

5.7 Color Coding of Fringes

The use of color in optical processing has received considerable attention, and a variety of optical methods are known for performing color coding. Because human observation can perceive variations in colors better than in the grey level, adding color to the output of an optical processor will result in enhancement of detail or contrast and lead to an output that is qualitatively simpler to analyze and visually more pleasant.

Historically, the use of color filters in microscopy has been discussed[52]. On the other hand, schlieren visualization in color has been accomplished by using colored filters or a prism[53,54]. We shall demonstrate that the quasi-interferometry can be extended to color coding of fringes with available techniques from diffraction, dispersion, and color filtering by coded masks. In this Section we shall summarize our

proposals for color coding as shown in Appendix K.

In the Appendix, we first discuss color coding for 1-D filtering by means of a prism combined with a correlation filter of parallel slits. It renders a map of colored contour lines on the image of the object, which displays contours of equal magnitude of the phase gradient along the direction normal to the slits. Based on the same analysis we discuss the other methods for color coding in 1-D filtering by a diffractive grating and 2-D filtering by a cone prism or by a diffractive circle grating. It is seen that the coding colors for phase variations consist of pure spectral colors as well as of a mixing of them. Subsequently, we consider some methods of a fully different principle, where the color coding of the phase variations is accomplished by color correlation filtering with two colored masks. As usual, three colors (red, green, and blue) are used as the primary colors of the coded masks. Some interesting examples of the color patterns of masks for 2-D filtering are studied in the CIE chromaticity diagram. Furthermore, optimization of colored masks is investigated to yield as many colors, with some periodic repetition, as possible. Experiments are then given. It is thus seen that the color coding techniques provide an additional important advantage to the quasi-interferometry. Its simplicity makes it particularly suitable for adaption in practical systems.

5.8 Lensmeter

Two prototypes of lensmeters were built during the past years on the basis of the quasi-interferometer.

5.8.1. Imaging-Lensmeter

Besides the reading of diopter, the direct determination and recognition of optical centers and areas for vision of multi-focal lenses and progressive lenses are necessary. The existing lensmeters do not work like this. For an effective solution, the imaging-lensmeter was developed by us. Its principle is given in Section 5.4, and a specification is

shown in Appendix L.

5.8.2. Lens-Center-Meter

The above lensmeter is somewhat complex in configuration and expensive to fabricate. Thus a prototype of lens-center-meter was designed for measuring only the optical centers and the vision areas. Its principle is a direct copy of that described in Section 5.2, and only a conventional camera lens is needed in the system. The instrument can be also used to test strongly distorted transparent objects such as glass products.

6. FOURIER-TRANSFORM DIFFRACTION INTERFEROMETRY

6.1 Principle

In the above Chapter we discussed the quasi-interferometry, which is realizable since the diffraction of the used masks is negligible and the tested phase variation is quite strong. If the diffraction of the masks can not be neglected due to the high density of mask periodicity and the phase variation under test is weak to the order of magnitude of wavelengths, we should look for another appropriate theoretical analysis on the same system.

In terms of the Fresnel diffraction theory, an elegant formula for describing the performance of this diffraction interferometry is obtained. The observed field distribution is related to a correlation between the Fourier transforms of the two masks and the phase distribution of the object. Thus various forms of interference can be obtained easily by using different coded gratings. In Appendix M, there is a brief discussion on it.

6.2 Observable Imaging and Character Recognition of Phase Objects

Most of biological and medical specimens are phase objects and can not be observed directly under normal microscopes. Usually these specimens must be dyed, but it is harmful to living cells. Using the methods of phase contrast, schlieren, central dark ground etc., transparent objects can be changed into absorbed constructions visible to the naked eye. These methods are also applicable to physics, chemistry, and industry though without any further information display. In application, character recognition is always needed for phase objects. Therefore, it is very significant to realize observable imaging and character recognition at the same time. The famous optical method for character recognition is to employ the correlation principle with holographic complex spatial filters[55]. But this method can not be applied immediately to phase objects, phase objects must be first transformed into visible pictures. One way to do so is to prepare the specimen with dyeing technique[56]. The other way is to use phase-contrast transparencies[57], or it would be better to use the specimen directly as the input of a phase-contrast microscope[58]. Note that the complex spatial filtering does not lead to a true image of the object.

The realization of character recognition using the principle of correlation is demonstrated by a correlation peak approximate to the Delta function on the screen. From the above Section we have found that the field amplitude of an image is connected with a correlation between the phase variation and the product of the two Fourier transforms of the masks. This means that the character recognition of phase objects can be performed directly in such a diffraction interferometer. To do so, the product of the Fourier transforms is designed to consist of a character function and a reference function. The character function is the same as the phase character required to identify. And the reference function is added to modulate interferentially the intensity of the correlation peaks for distinguishing them from the background intensity

of the image. Note that a transparent object is simultaneously visualized by the joint action of the character function and the reference function in the process of character recognition. The details are shown in Appendix M.

7. INTERFEROMETRY BASED ON PARTIALLY COHERENT EFFECT BETWEEN TALBOT AND LAU EFFECTS

From Chapter 2, it has been known that the fringe profile can be changed by a lateral shift between the two gratings in the setup for observing the partially coherent diffraction effect. In particular, two well-defined fringe patterns of double spatial frequency can be yielded during the shift. This interesting phenomenon is now used to build an interferometer.

The tested object is inserted elsewhere between the two gratings in the Lau setup illuminated by a slit source, which is similar to the experimental setup as described in Section 2.2. The phase variations of the tested object cause equivalently lateral shifts between the two gratings, thus the resultant interferogram is dominantly characterized by two grating-like carriers of double spatial frequency. These two carriers are the major partially coherent fringe patterns with different relative shifts between the two gratings. This entirely new kind of interferogram is essentially different from either a normal interferogram or a moiré interferogram. By simply changing the width of the source slit to one of the two extremes of coherence or another, the same setup immediately becomes either a Talbot interferometer or a Lau interferometer.

Appendix N gives the details. The system is analysed in terms of the AF. As a result, an elegant expression is deduced for describing the intensity distribution on the observation screen. On this foundation, the performance of the partially coherent interferometer is discussed. The optimum choice of the system parameters is given. Finally, experimental results are shown.

8. TALBOT INTERFEROMETRY USING LOGIC-OPERATED MOIRÉ PHENOMENON

In the interferential measurement, it is usually wanted in practice to determine the fringe position with high accuracy. The measuring accuracy of the Talbot interferometer can be improved as the moiré patterns are processed by eliminating the grating-carriers to enhance the contrast, by sharpening the fringes to increase the visible resolution or/and by extracting the difference-beat to exclude the interference of the sum-beat. But, for the former suggested techniques in the Talbot interferometer such as spatial filtering[18] and complementary compensation[22], these requirements were partially met. A solution, which we found to be very simple and effective, is to use the logic-operated moiré phenomenon based on the joint self-imaging effect as discussed in Chapter 3. In this Chapter, our aim is to study such a new type of logic-operated Talbot interferometer.

As described in Appendix O, such an interferometer can be constituted simply by inserting the tested object elsewhere in the front of the first grating or between the two gratings in just the setup for observing the joint Talbot effect. In other words, only two slight modifications to the basic version of the Talbot interferometer are required: to move the observation screen apart from the second grating to a special distance, and to choose the opening ratios of the gratings suitable for a desired logic operation.

9. CONCLUSIONS

The main objective of this thesis has been to investigate the diffraction-interference

phenomena involving two gratings, and their use to interferometry. The thesis consists of two major parts. In the first part(Chapter 2--Chapter 4), three kinds of diffraction-interference phenomena are studied: the partially coherent diffraction effect, the joint Talbot self-imaging effect, and the Talbot and Lau effects on incident beams of arbitrary wavefront. Some interesting and useful effects are thus found. The fringe profile changes as one of the gratings is translated in the case of partially coherent illumination(Chapter 2). The joint self-images are related to both the independent self-images of the gratings and their mutual coupling(Chapter 3). The distortion of the self-image caused by a beam of arbitrary wavefront is closely linked to that of the far-field distribution(Chapter 4). Moreover, a logic-operated moiré is proposed on the basis of the joint self-imaging effect(Chapter 3). In the second part(Chapter 5--Chapter 8), four kinds of interferometer are developed. The quasi-interferometer is a noninterferential method specially suitable to strongly distorted phase objects(Chapter 5). Various forms of interference as well as the identification of phase characters can be realized in the Fourier-transform diffraction interferometer(Chapter 6). In the partially coherent diffraction interferometry, the resulted interferogram is characterized by two grating-like carriers of double spatial frequency(Chapter 7). Sharpened, carrier-free, and/or beat-selected moiré fringes are easily achieved in the logic-operated Talbot interferometry(Chapter 8). As the practical applications of the quasi-interferometer, two prototypes of lensmeters are outlined in Section 5.8.

9.1 Future Prospects

According to the general configuration as depicted in Fig. 1.1, the diffraction-interference phenomena of two gratings, such as the partially coherent diffraction effect, the joint Talbot effect, and the Talbot and Lau effects on incident beams of arbitrary wavefront, are studied independently. However, it seems that there are inherent relations among them. It is thus interesting to develop a uniform theory to

describe all the diffraction-interference phenomena of two gratings in a general arrangement of optical configuration.

In this report the experiments for applications of the suggested interferometers are demonstrated. However, more experimental tries, such as the mapping of density gradient fields in wind-tunnels, the measurements of temperature distributions in flames, the testing of optical components, the mapping of liquid surfaces, and the measurement of refractive indices are needed to show further the full potentials of the proposed interferometers.

9.2 Additional References

Much work on this topic has been done during the past years. But some of them were published in Chinese journals. Here I list some publications as additional references.

About the Lau and Talbot effects:

1. L.Liu, "Ambiguity function and general Talbot-Lau effects," *Acta Optica Sinica*, Vol.7, 501-510(1987).
2. L.Liu, "Theory for Lau effect of plane objects," *Acta Optica Sinica*, Vol.6, 807-814(1986).
3. L.Liu, "Interference phenomenon in a 3-grating system," *Acta Physica Sinica*, Vol.38, 15-24(1989).(in English)

About the quasi-interferometry:

4. L.Liu and Y.Yin, "Fringe sharpening in quasi-interferometry by using pseudo-random sequence gratings," *Acta Optica Sinica*, Vol.10, 88-91(1990).
5. L.Liu, "Application of crystal interference figures in conoscope to phase measurement," *Acta Optica Sinica*, Vol.4, 582-587(1984).

About the Fourier-transform diffraction interferometry:

6. L.Liu, "Coded grating Fourier-transformation diffraction interferometry with extended polychromatic illumination: theory," *Acta Optica Sinica*, Vol.4, 970-978(1984).
 7. L.Liu, "Diffraction correlation and Lau interferometry," *Acta Physica Sinica*, Vo.35, 1556-1566(1986).
 8. L.Liu, H.Huang, and Z.Wang, "Diffraction interferometry with computer-generated gratings in geometric approach of Fourier transformation," *Acta Optica Sinica*, Vol.6, 988-996(1986).
- About the moiré effect:
9. J.Zhang and L.Liu, "Optical logical operation and moiré pattern," *Optics Commun.* Vol.66, 179-182(1988).

Bibliography

1. F.Talbot, "Facts relating to optical scienc No.IV," *Phil.Mag.* 9, 401-407(1836).
2. J.M.Cowley and A.F.Moodie, "Fourier images: I-The point source," *Proc.Phys.Soc.* 70, 486-496(1957).
3. J.T.Winthrop and C.R.Worthington, "Theory of Fresnel images. I. Plane periodic objects in monochromatic light," *J.Opt.Soc.Am.* 55, 373-381(1965).
4. W.D.Montgomery, "Self-imaging objects of infinite aperture," *J.Opt.Soc.Am.* 57, 772-778(1967).
5. W.D.Montgomery, "Algebraic formulation of diffraction applied to self-imaging," *J.Opt.Soc.Am.* 58, 1112-1124(1968).
6. R.Jozwicki, "The Talbot effect as a sequence of quadratic phase correlation of the object Fourier transform," *Opt.Acta* 30, 73-84(1983).
7. A.W.Lohmann, "An array illuminator based on the Talbot effect," *Optik* 79, 41-45(1988).
8. E.Lau, "Beugungserscheinungen an Doppellrastern," *Ann.Phys.* 6, 417-423(1948).
9. J.Jahns and A.W.Lohmann, "The Lau effect (a diffraction experiment with incoherent illumination)," *Opt.Comm.* 28, 263-267(1979).
10. F.Gori, "Lau effect and coherence theory," *Opt.Comm.* 31, 4-8(1979).
11. R.Sudol and R.J.Thompson, "Lau effect," *Opt.Comm.* 31, 105-110(1979); "Lau effect, theory and experiment," *Appl.Opt.* 20, 1107-1116(1981).
12. G.J.Swanson and E.N.Leith, "Lau effect and grating imaging," *J.Opt.Soc.Am.* 72, 552-555(1982); "Analysis of the Lau effect and generalized grating imaging," *J.Opt.Soc.Am.* A2, 789-793(1979).
13. K.H.Brenner, A.W.Lohmann, and J.Ojeda-Castaneda, "Lau effect, OTF theory," *Opt.Comm.* 46, 14-17(1983).
14. K.Patorski, "Incoherent superposition of multiple self-imaging Lau effect and moiré fringe explanation," *Opt.Acta* 30, 745-750(1983).

15. S.Cartright and J.Lightman, "Coherence analysis of the Lau interferometer," Appl.Opt. 25, 3141-3148(1986).
16. K.Hane and C.P.Grover, "Imaging with rectangular transmission gratings," J.Opt.Soc.Am. A4, 706-711(1987).
17. A.W.Lohmann and D.E.Silva, "An interferometer based on the Talbot effect," Opt.Comm. 2, 413-415(1971).
18. D.E.Silva, "Talbot interferometer for radial and lateral derivatives," Appl.Opt. 11, 2613-2624(1972).
19. K.Patorski, "Talbot interferometry with increased shear," Appl.Opt. 24, 4448-4453(1985).
20. K.Patorski, "Talbot interferometry with increased shear: further considerations," Appl.Opt. 25, 1111-1116(1986).
21. K.Patorski, "Talbot interferometry with increased shear: part 3," Appl.Opt. 27, 3875-3878(1988).
22. H.J.Rabal, W.D.Furlan, and E.E.Sicre, "Talbot interferometry with anisotropic gratings," Opt.Comm. 57, 81-83(1986).
23. H.O.Bratelt and J.Jahns, "Interferometry based on the Lau effect," Opt.Comm. 30, 268-274(1979).
24. J.Jahns, A.W.Lohmann, and J.Ojeda-Castaneda, "Talbot and Lau effects, a parageometrical approach," Opt.Acta 31, 313-324(1984).
25. J.Ojeda-Castaneda and E.E.Silva, "Quasi ray-optical approach to longitudinal periodicities of free and bounded wavefields," Opt.Acta 32, 17-26(1985).
26. A.W.Lohmann and J.Ojeda-Castaneda, "Spatial periodicities in partially coherent fields," Opt.Acta 30, 475-479(1983).
27. A.W.Lohmann, J.Ojeda-Castaneda, and N.Streible, "Spatial periodicities in coherent and partially coherent fields," Opt.Acta 30, 1259-1266(1983).
28. G.Indebetouw, "Propagation of spatially periodic wavefields," Opt.Acta 31, 531-539(1984); "Spatially periodic wavefields: An experimental demonstration of the relation between the lateral and the longitudinal spatial frequencies,"

- Opt. Commun. 49, 86-90(1984).
29. R.Sudol, "Lau effect: An interference phenomenon in partially coherent light," PH. D. dissertation(University of Rochester, N.Y.), 1981.
 30. K.Hane, S.Hattori, and C.P.Grover, "Lau effect in reflection," J.Mod.Opt. 34, 1481-1490(1987).
 31. A.Papoulis, "Ambiguity function in Fourier optics," J.Opt.Soc.Am. 64, 779-788(1974).
 32. J.P.Gurigry, "The ambiguity function in diffraction and isoplanatic imaging by partially coherent beams," Opt.Comm. 26, 136-138(1978).
 33. K.H.Brenner and J.Ojeda-Castaneda, "Ambiguity function and Wigner distribution function applied to partially coherent imagery," Opt.Acta 31, 213-225(1984).
 34. G.Oster, M.Wasserman, and C.Zwerling, "Theoretical interpretation of moiré patterns," J.Opt.Soc.Am. 54, 169-175(1964).
 35. O.Bryngdahl, "Characteristics of superposed patterns in optics," J.Opt.Soc.Am. 66, 87-94(1976).
 36. O.Bryngdahl, "Beat pattern selection-multi-color-grating moiré," Opt.Comm. 39, 127-131(1981).
 37. O.Bryngdahl, "Orthogonal-states-grating moiré," Opt.Comm. 41, 249-254(1982).
 38. S.Yokozeki and K.Patorski, "Moiré fringe profile prediction method and its application to fringe sharpening," Appl.Opt. 17, 2541-2547(1978).
 39. R.H.Katyl, "Moiré screens coded with pseudo-random sequences," Appl.Opt. 11, 2278-2285(1972).
 40. J.Tanda and Y.Ichioka, "Optical logic array processor using shadowgrams," J.Opt.Soc.Am. 73, 800-809(1983).
 41. H.Bartelt, A.W.Lohmann, and E.E.Sicre, "Optical logical processing in parallel with theta modulation," J.Opt.Soc.Am. A1, 944-951(1984).
 42. J.Weigelt, "Binary logic by spatial filtering," Opt.Eng. 26, 28-33(1987).
 43. S.L.Hurst, "Multiple-valued threshold logic: its status and its realization," Opt.Eng. 25, 47-55(1986).

44. W.Veldkamp, "Binary optics: An emerging diffractive optics technology,"
Phys.Today S49-S50(Jan. 1987).
45. J.W.Goodman, Introduction to Fourier Optics (McGraw-Hill, San Francisco),
p.163,1986.
46. J.Knopp nad M.F.Becker, "Generalized model for noncoherent optical convolvers
and correlators," Appl.Opt. 17, 984-985(1978).
47. See Ref.45, Chapter 7.
48. J.D.Gaskill, Linear System, Fourier Transforms, and Optics (Wiley, New York),
397-442,1978.
49. C.P.Grover, S.Mallick, and M.L.Roblin, "Observation of a phase object using
carrier-frequency photography," Opt.Commu. 3, 181-183(1971).
50. D.E.Silva and Y.M.Hong, "Circular carrier-frequency photography for observing
phase objects," Opt.Communi. 7, 283-285(1973).
51. O.Kafri, "Noncoherent method for mapping phase objects," Opt.Lett. 5, 555-
557(1980).
52. J.Rheinberg, J.R.Microsc.Soc. 373(Aug. 1986).
53. G.J.North and R.F.Cash, Natl.Phys.Lab. U.K.Aero Note 383(Teddington, 1959).
54. G.J.North, Natl.Phys.Lab. U.K.Aero Note 397(Teddington 1959).
55. See Ref. 45, pp.171-184.
56. R.L.Bon, et al. "Fine structure of muscle of human testicular capsule: Basis of
testicular contractions," Science 149, 571-573(1973).
57. S.P.Almeida and J.K.T.Eu, "Water pollution monitoring using matched spatial
filters," Appl.Opt. 15,510-515(1976).
58. J.K.Partin, et al. Holography in Medicine and Biology (Ed. Bally,G.), Springer-
Verlag, Berlin, Heidelberg, New York, 73-76(1979).

Statement of Independence

I, Liren Liu, hereby certify that I have independently prepared this manuscript with complete and proper reference literature used.

Ich, Liren Liu, versichere hiermit, daß ich diese Arbeit selbständig angefertigt habe. Die verwendete Literatur habe ich vollständig angegeben.

Oldenburg in November 1991

(Liren Liu)

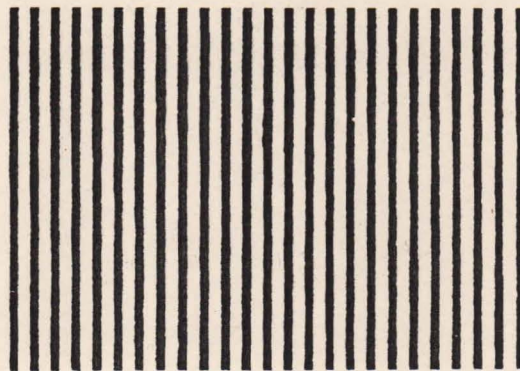
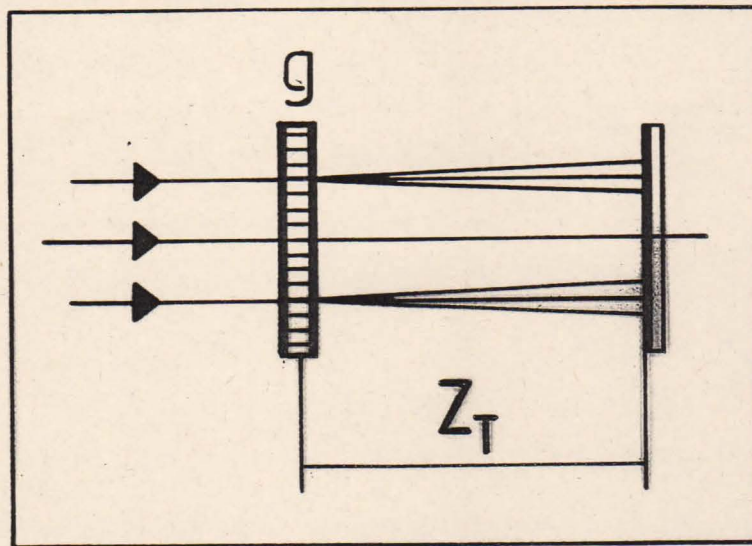
**GRATING DIFFRACTION-INTERFERENCE EFFECTS
AND THEIR USE TO INTERFEROMETRY**

Doctoral Thesis
by
Liren Liu

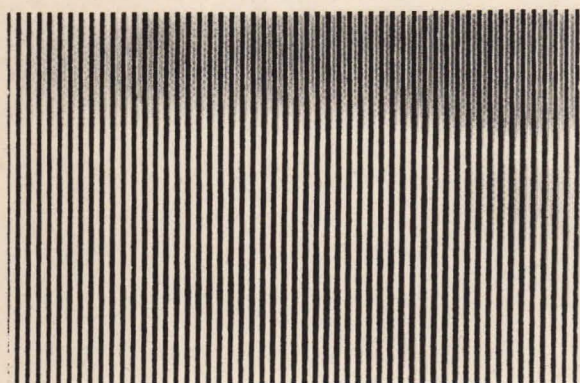
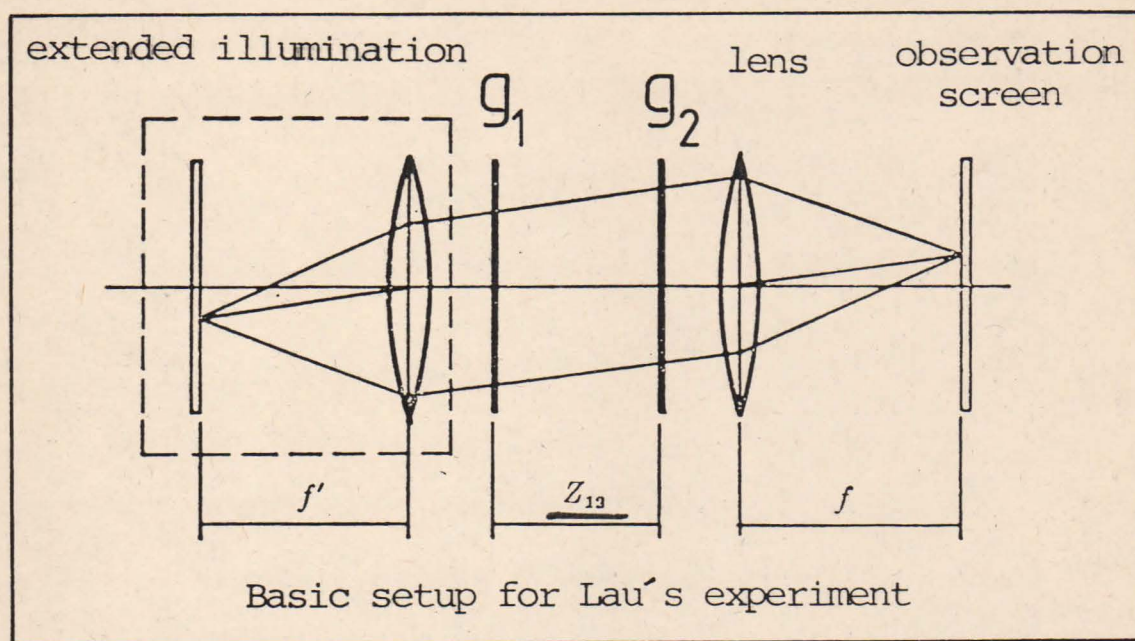
Fachbereich Physik
Universität Oldenburg
Angewandte Optik
November 1991

1.2 Historical Background

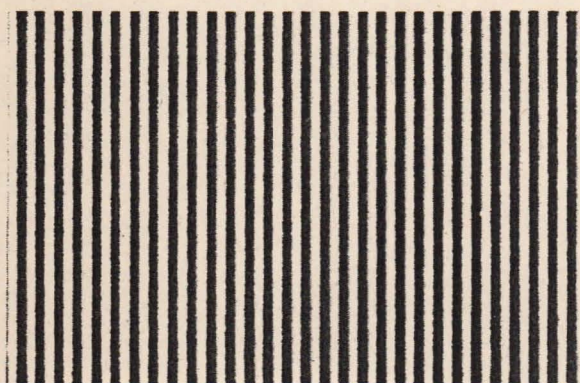
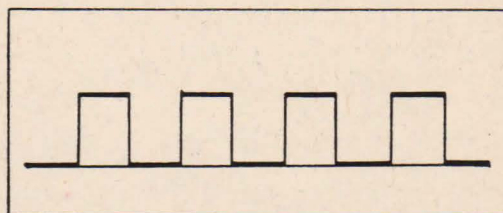
Talbot effect



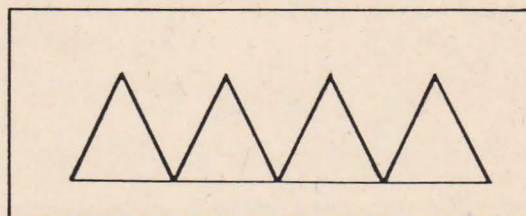
Lau effect



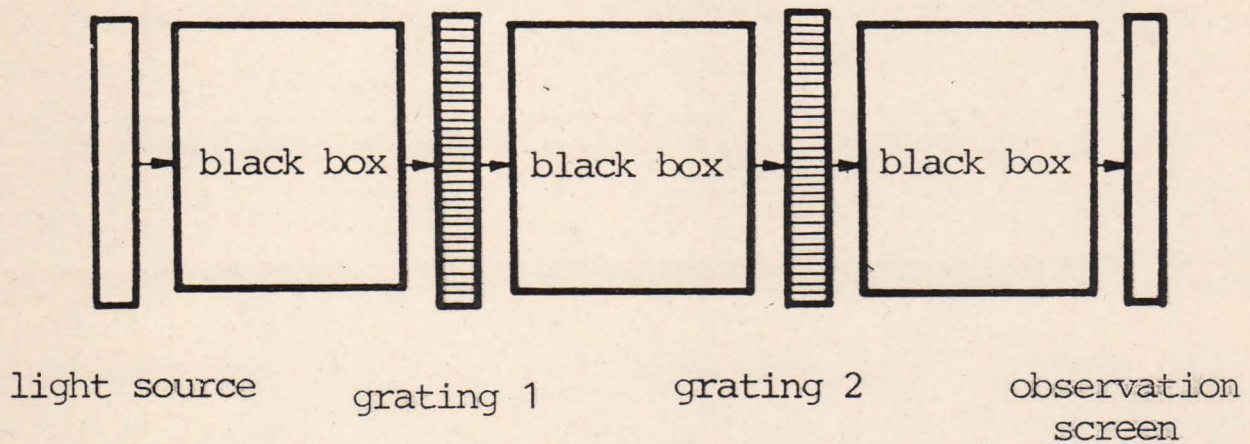
gratings



Lau fringes



1.1 Basic Idea



The interference phenomena of two diffractive gratings

- different light sources (coherent, incoherent, and partially coherent)
- different gratings (Ronchi gratings and the others)
- different arrangements of system

Spatial coherence

Monochromatic illumination

1.3 Preview

1. Partially coherent diffraction effects
2. Joint Talbot self-imaging effect and logic-operated moire patterns
3. Talbot-Lau effects for beams of arbitrary wavefront
4. General Lau effect and optical correlation interferometry

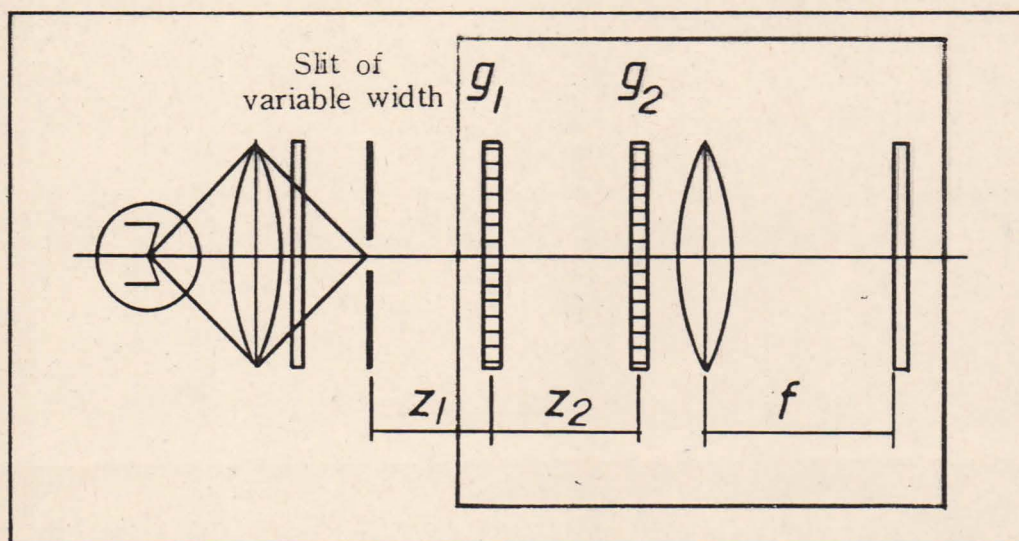
Mathematics

1. Fresnel diffraction theorem (linear approximation):
 - coherent systems
 - incoherent systems
2. Ambiguity function:
 - partially coherent systems

2. PARTIALLY COHERENT DIFFRACTION EFFECTS

Lau effect	incoherent
Talbot effect	coherent
<div style="border: 1px solid black; padding: 5px; display: inline-block;">?</div>	partially coherent

2.1 Partially Coherent Diffraction Effect between Lau and Talbot Effects



slit width	observed effect
small	Talbot effect
large	Lau effect
in between	partially diffraction effect

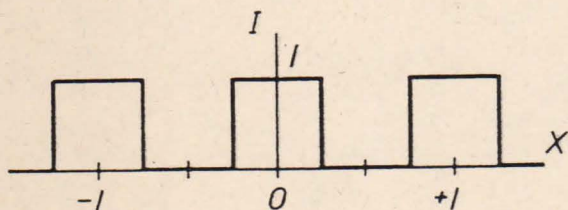
7

$$I(x) = \int s(\alpha) g_1(\alpha + k_1 x) g_2(\alpha + k_2 x) d\alpha$$

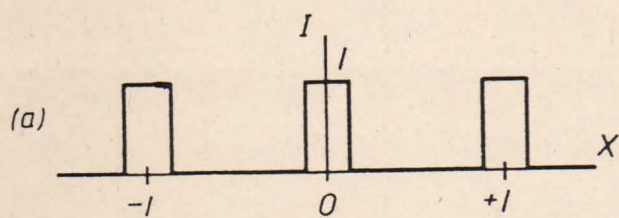
Characteristics:

Fringe profile changes
by shift of grating
change of coherence

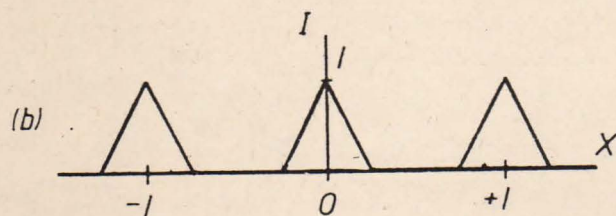
Coherence change by slit width:



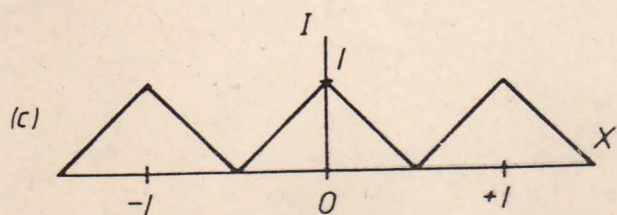
profiles of two gratings



self-image of gratings

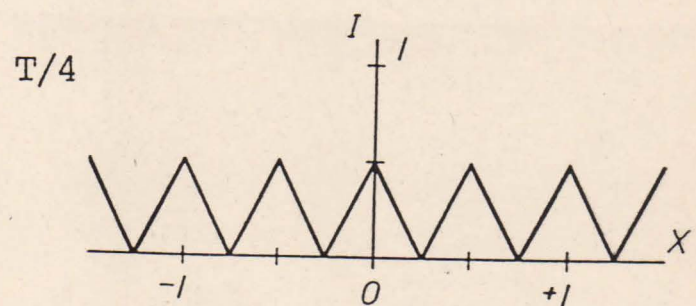
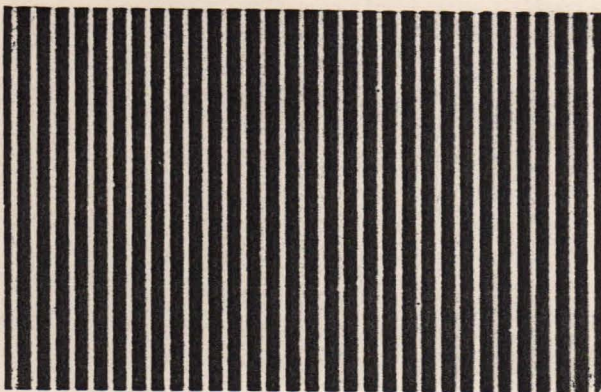
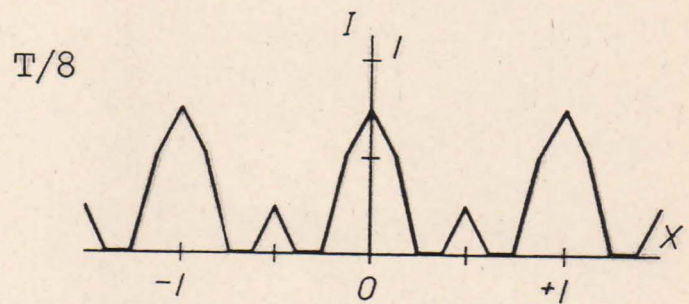
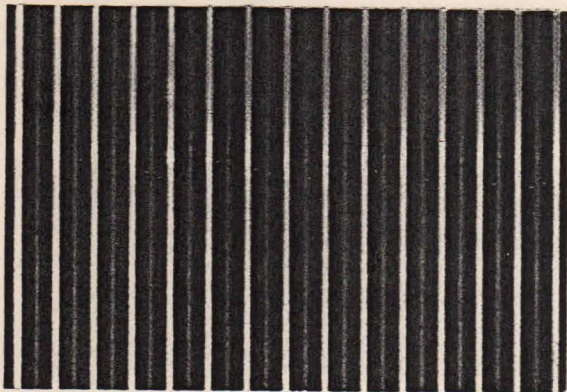
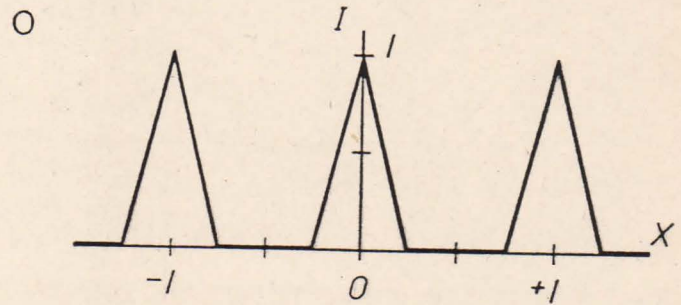
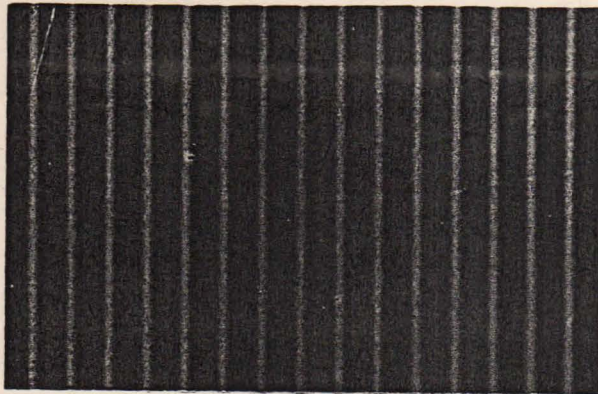
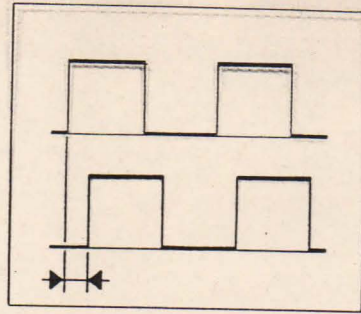


partially coherent fringes



Lau fringes

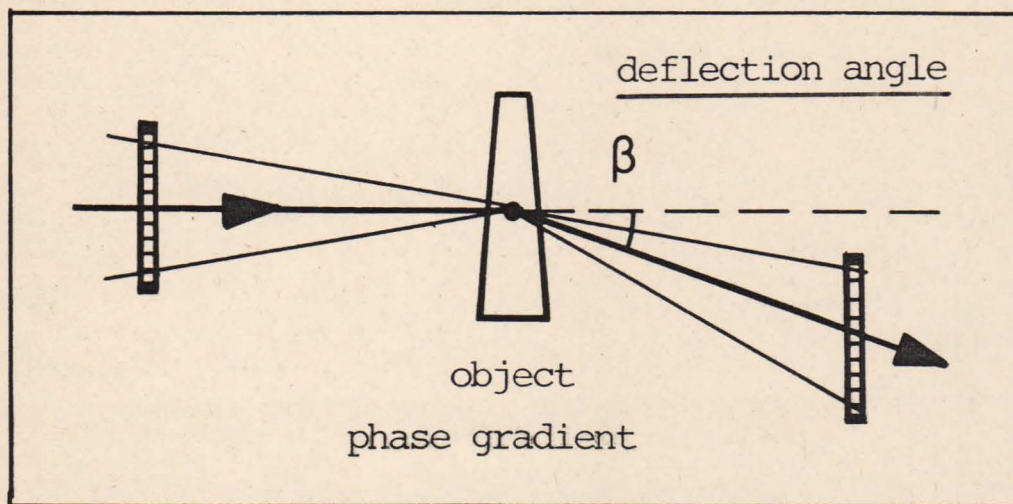
Shift of one grating:



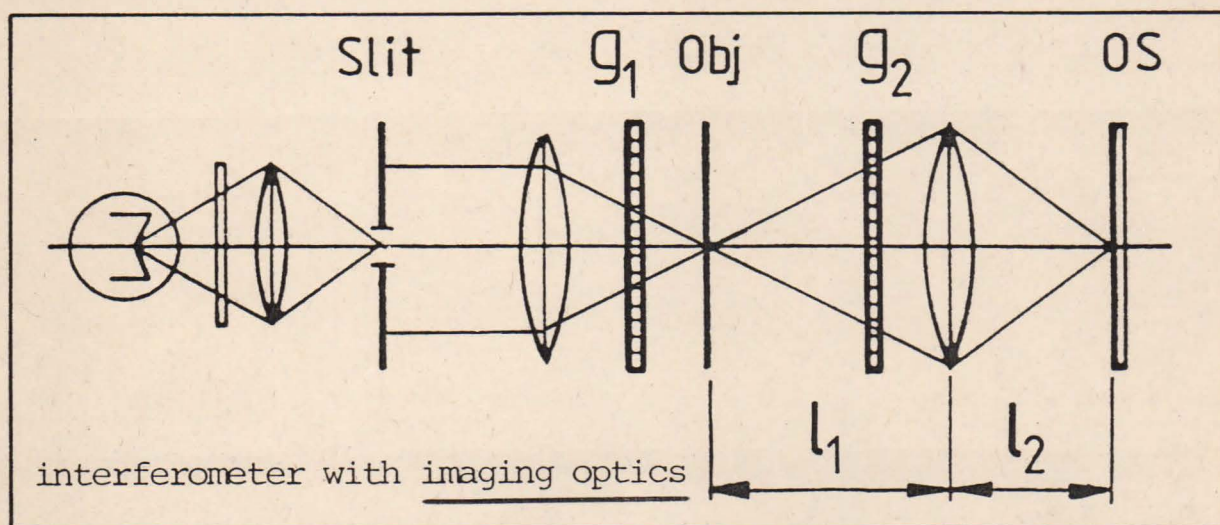
Space-Frequency Doubling Phenomenon

9

2.2 INTERFEROMETRY BASED ON PARTIALLY COHERENT EFFECT BETWEEN TALBOT AND LAU EFFECTS

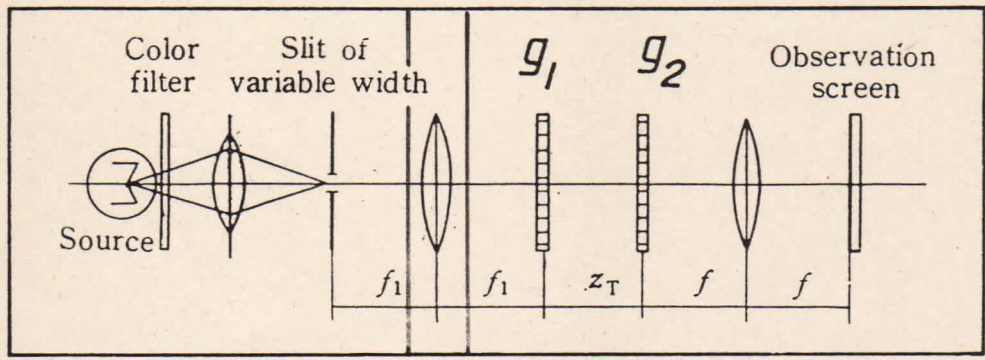


phase gradient \rightarrow beam deflection \rightarrow grating shift \rightarrow frequency doubling \rightarrow contour-mapping

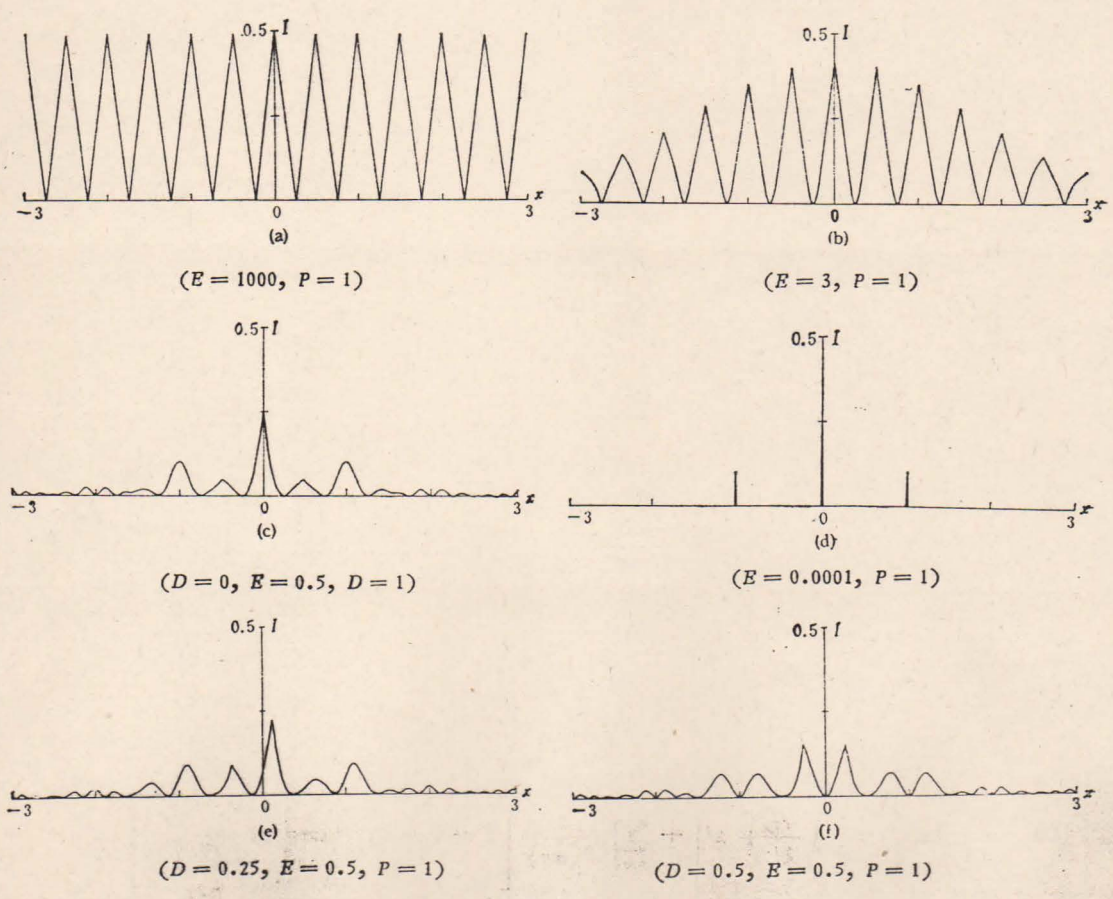


2.3 Partially Coherent Diffraction Effect between Lau effect and Optical Fourier Transformation

collimating lens ——— quasihomogeneous

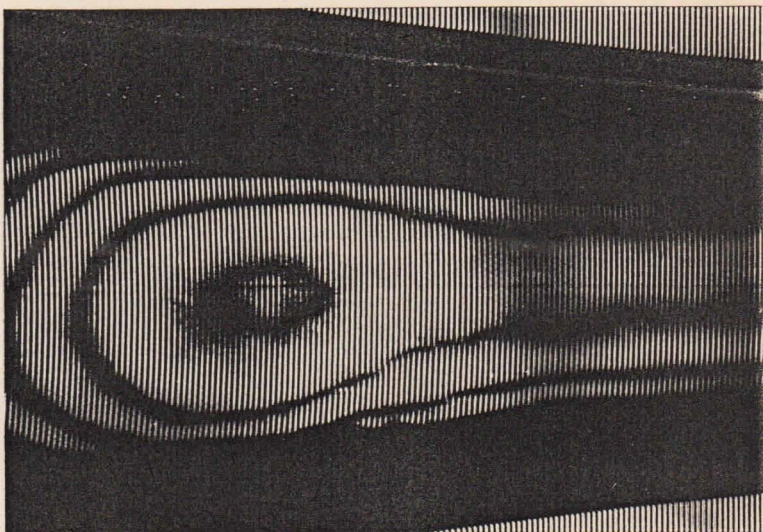
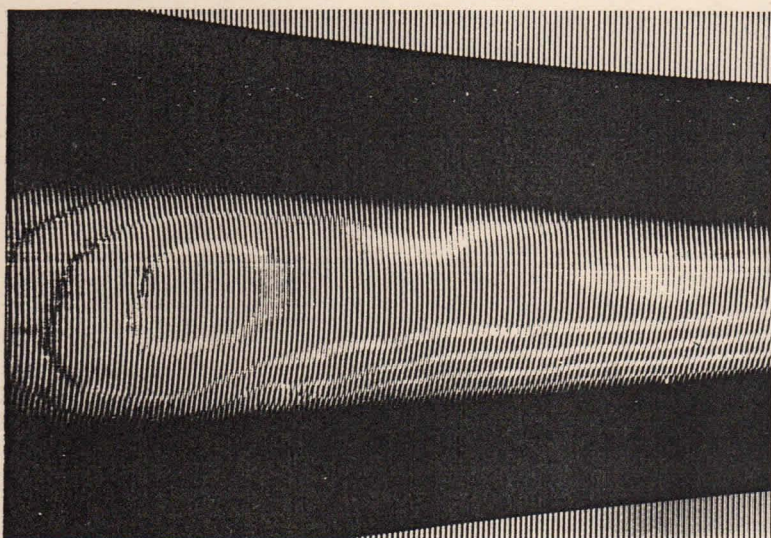


slit width	observed effect
small	optical Fourier transform
large	Lau effect
in between	partially coherent effect



Partially coherent interferogram

Moire interferogram

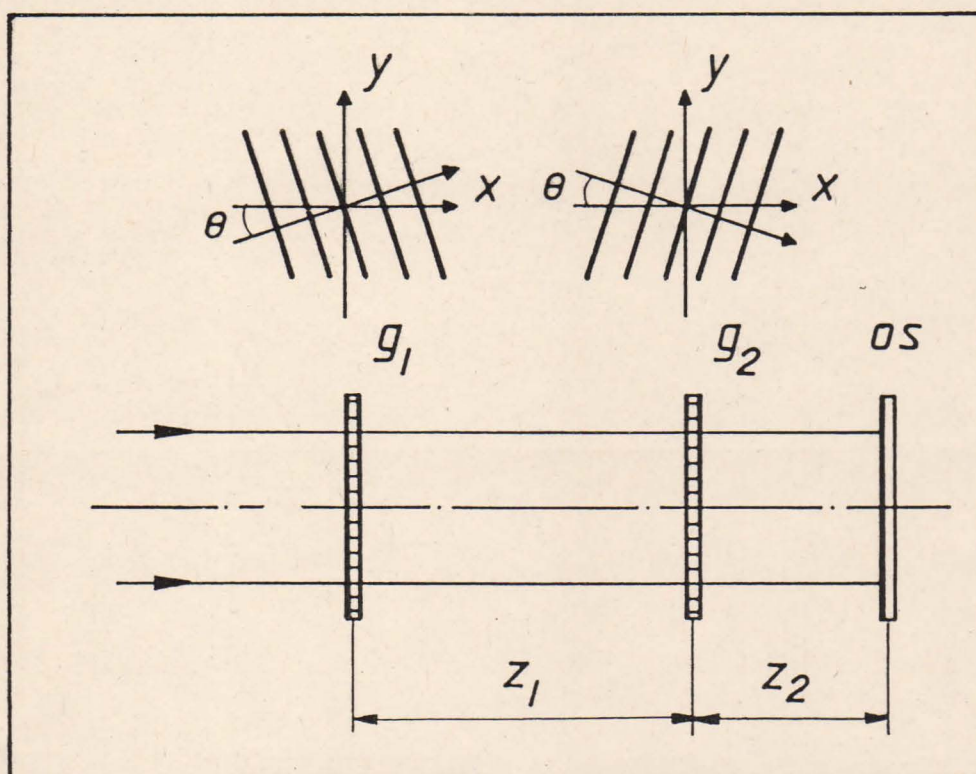


contour-mapping	carrier	fringes
partially coherent interferometry	low-frequency	high-frequency
moire technique	grating	difference-beat
interferometry	no	bright/dark
speckle metrology	speckle	decorrelation

3. JOINT TALBOT EFFECT

Talbot effect	one grating	self-images
	diffraction	
moire effect	two gratings	beat patterns
	overlapping	
joint Talbot effect	two gratings	moire patterns self-images ?
	diffraction	

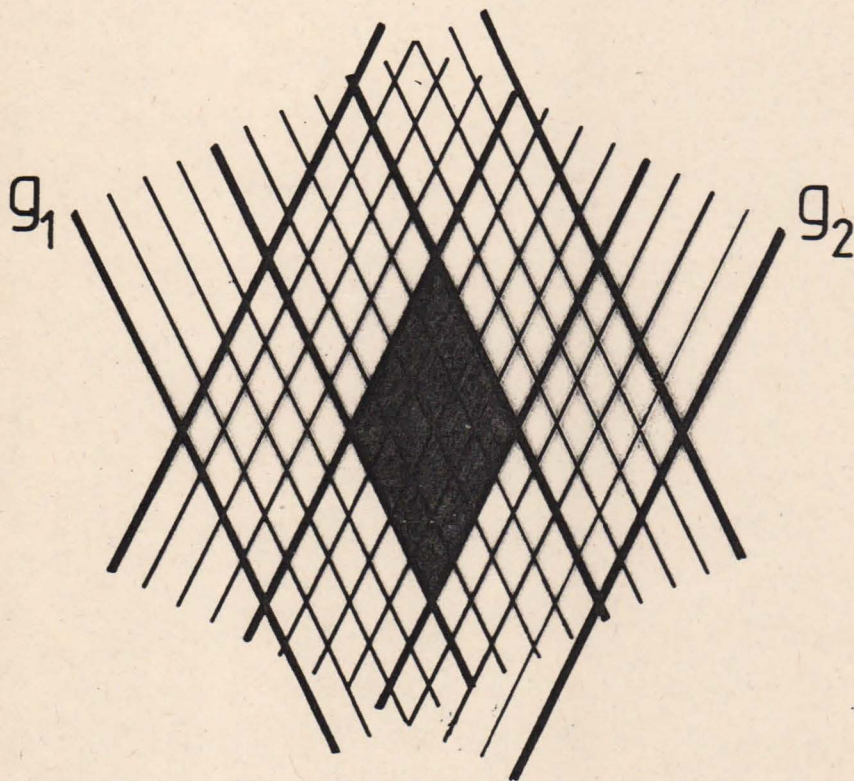
3.1 Analysis of Joint Self-Imaging Effect



Joint self-image output differs from input of
two gratings due to mutual coupling phenomenon

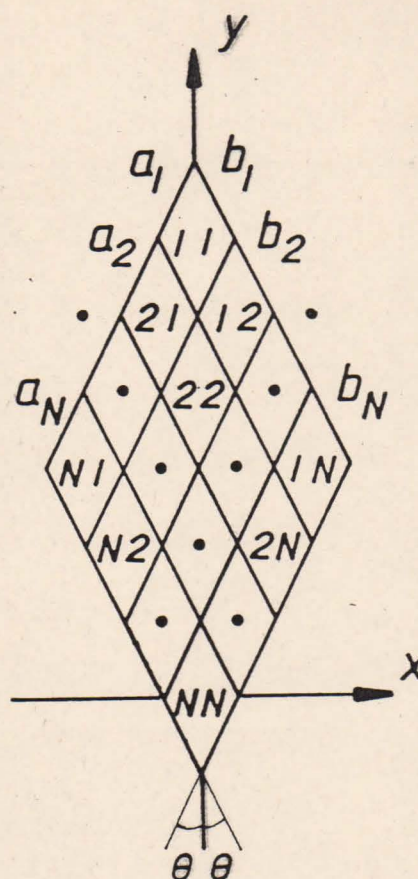
$$[z_2, \theta, \tau, \lambda]$$

N subperiodic stripes in a period
($N=4$)



A basic cell

A basic cell



Truth Table of Multiple-Valued Logic Operation

direct overlapping
of two gratings
(input)

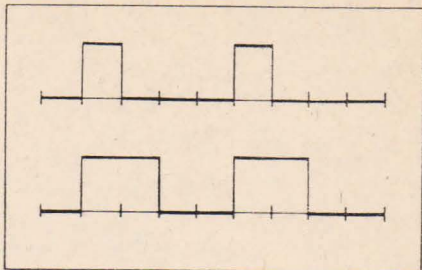
C_{nm}	b_1	b_2	b_N
a_1	C_{11}	C_{12}	...	C_{1N}
a_2	C_{21}	C_{22}	...	C_{2N}
\vdots			
a_N	C_{N1}	C_{N2}	...	C_{NN}

joint self-image
(output)

I_{nm}	b_1	b_2	b_N
a_1	I_{11}	I_{12}	...	I_{1N}
a_2	I_{21}	I_{22}	...	I_{2N}
\vdots			
a_N	I_{N1}	I_{N2}	...	I_{NN}

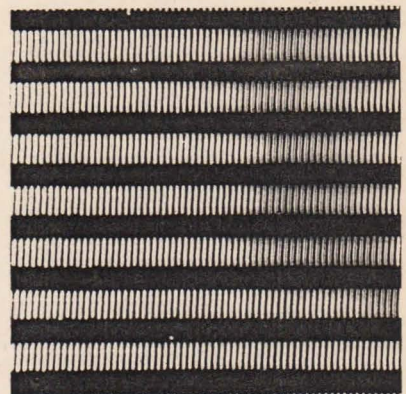
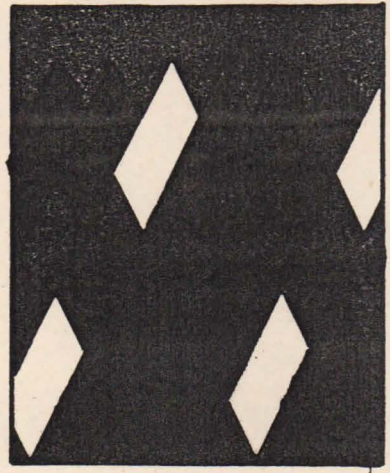
3.2 Logic-Operated Moiré

Quarternary Logic (N=4)



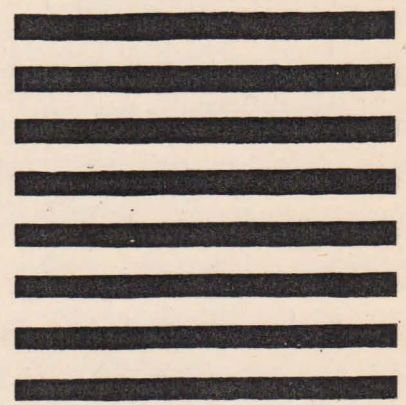
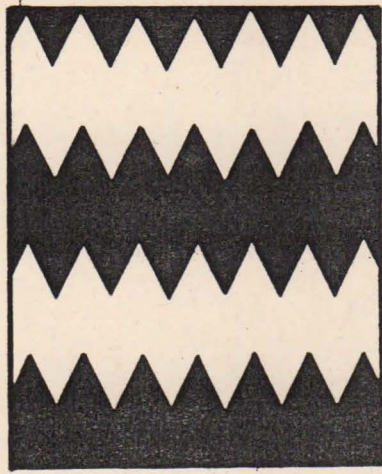
Normal moire pattern

C_{nm}	1	0	0	0
1	1	0	0	0
1	1	0	0	0
0	0	0	0	0
0	0	0	0	0



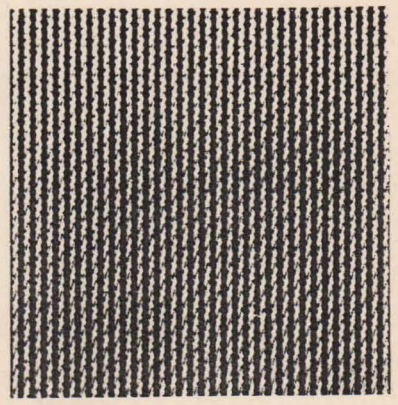
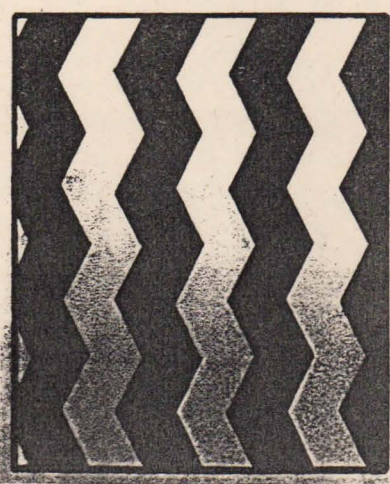
Quarternary logic difference-beat

I_{nm}	1	0	0	0
1	1/4	1/4	0	0
1	1/4	0	0	1/4
0	0	0	1/4	1/4
0	0	1/4	1/4	0



Quarternary logic sum-beat

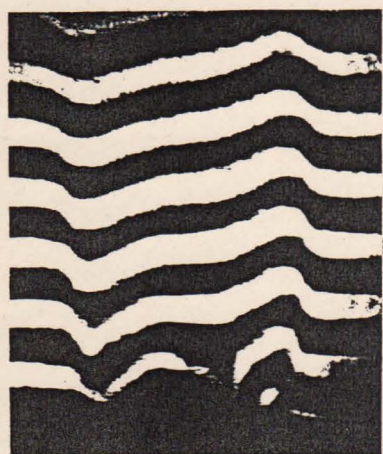
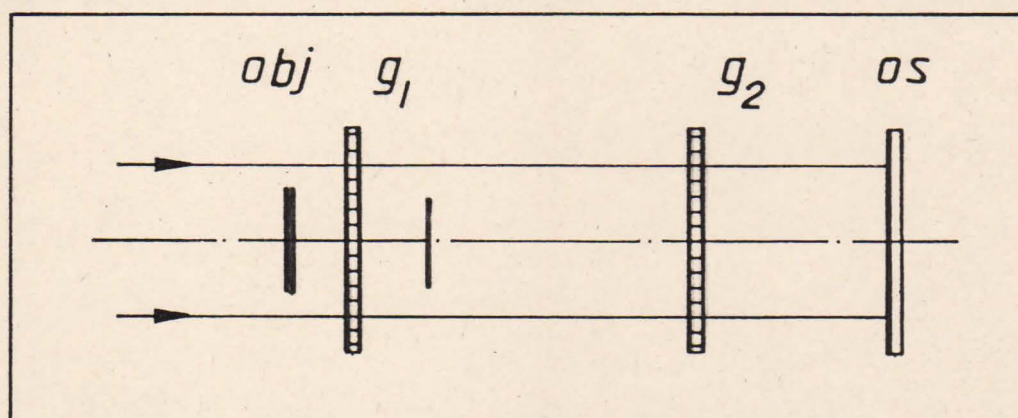
I_{nm}	1	0	0	0
1	1/4	1/4	0	0
1	0	1/4	1/4	0
0	0	0	1/4	1/4
0	1/4	0	0	1/4



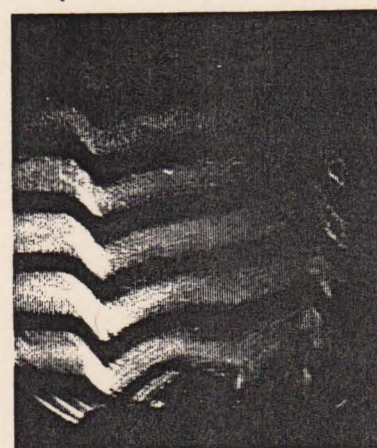
normal moire	performance	logic-operated moire
grating carrier	contrast	carrier suppression
two beats	beat-interference	beat selection
rough fringes	resolution	fringe sharpening

3.3 TALBOT-INTERFEROMETRY USING LOGIC

OPERATED MOIRÉ PHENOMENON



Logic-operated moire
interferogram of a flame
(N=4)



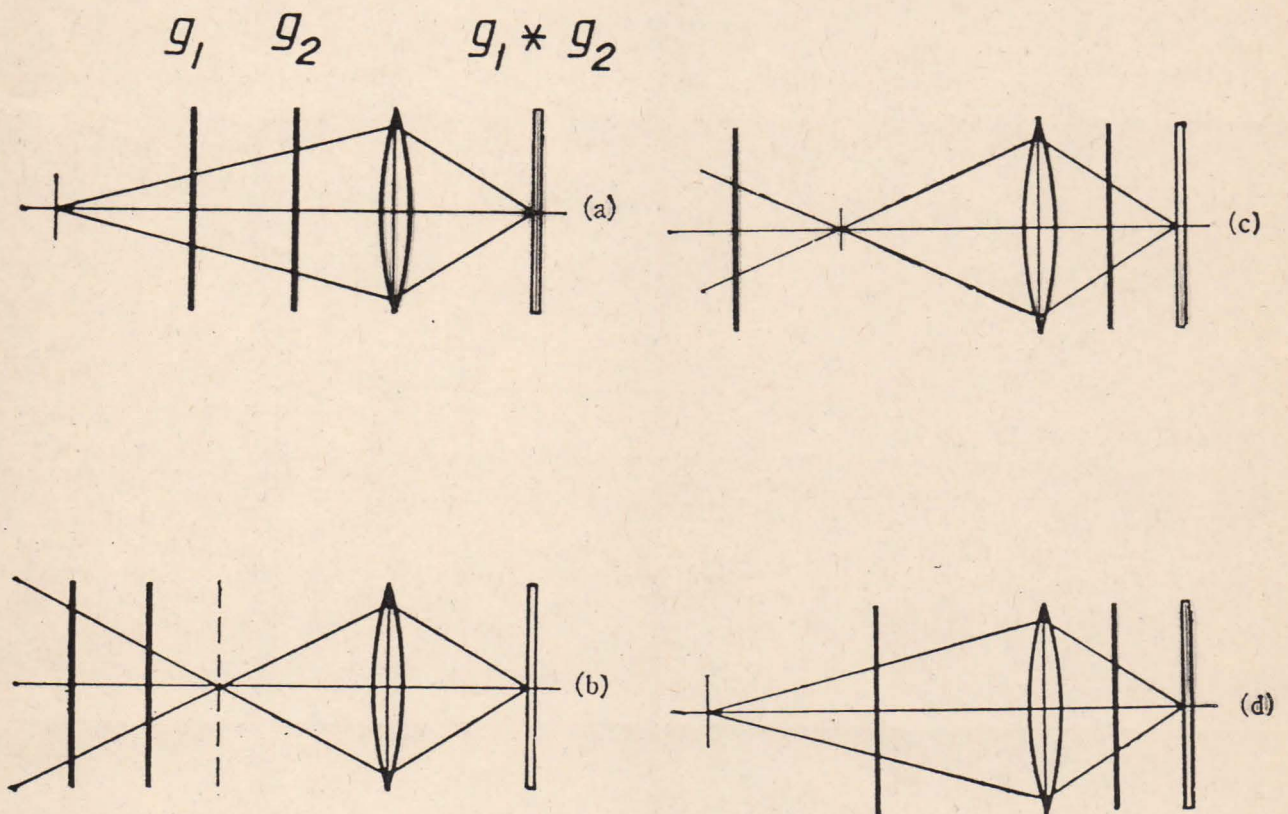
Moire interferogram
(N=4)

17

5. QUASI-INTERFEROMETRY

5.1 Overview

normal Lau effect	<u>general Lau effect</u>
line gratings	2-D patterns
focal plane observation	defocused systems

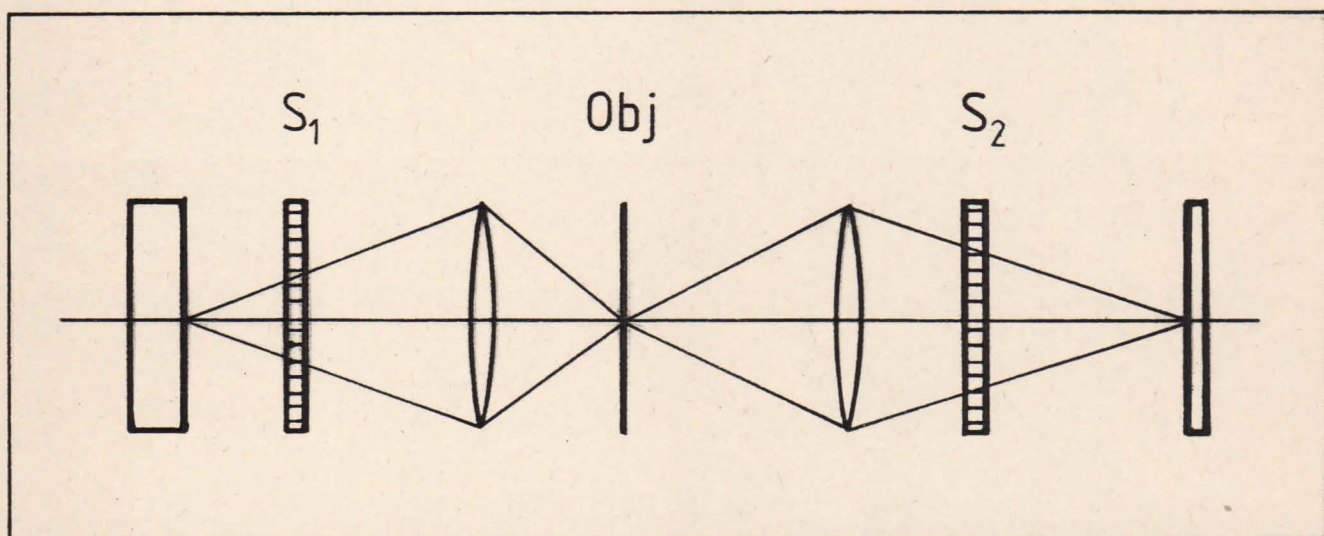


Possibility of contour-mapping:

1. Fringe pattern of correlation,
2. Change of this pattern by tested phase variation.

5.2 Optical-Correlation Quasi-Interferometry

Contour-mapping of strongly varying
or distorted phase objects



Necessary Conditions

object	<ul style="list-style-type: none"> -imaging -between two masks 	
masks	<ul style="list-style-type: none"> -correlation pattern of high contrast and of periodicity 	
arrangement between masks	-geometric projection	<u>quasi-interferometer</u>
	-imaging optics	
	-diffraction	<u>Fourier-transform interferometer</u>

Output intensity distribution:

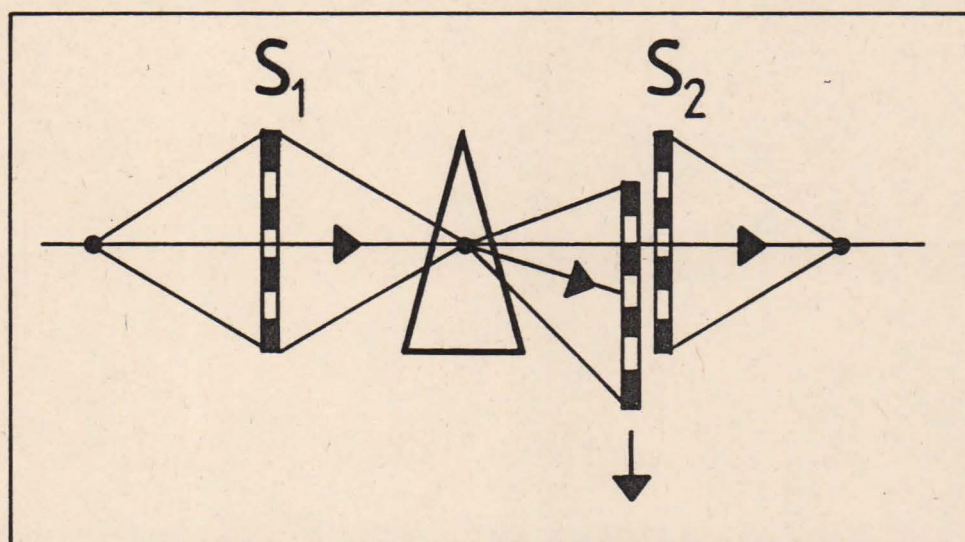
$$I(-x_d, -y_d) = \iint_{-\infty}^{\infty} S_1(x, y) S_2 \left\{ K \left[x - Cx_d - \right. \right. \\ \left. \left. - R \tan \beta_x \left(-\frac{x_d}{N}, -\frac{y_d}{N} \right) \right], \right. \\ \left. K \left[y - Cy_d - R \tan \beta_y \left(-\frac{x_d}{N}, -\frac{y_d}{N} \right) \right] \right\} dx dy.$$

K determines the scaling factor between two masks,

R determines the sensitivity for measurement,

C determines the background fringes,

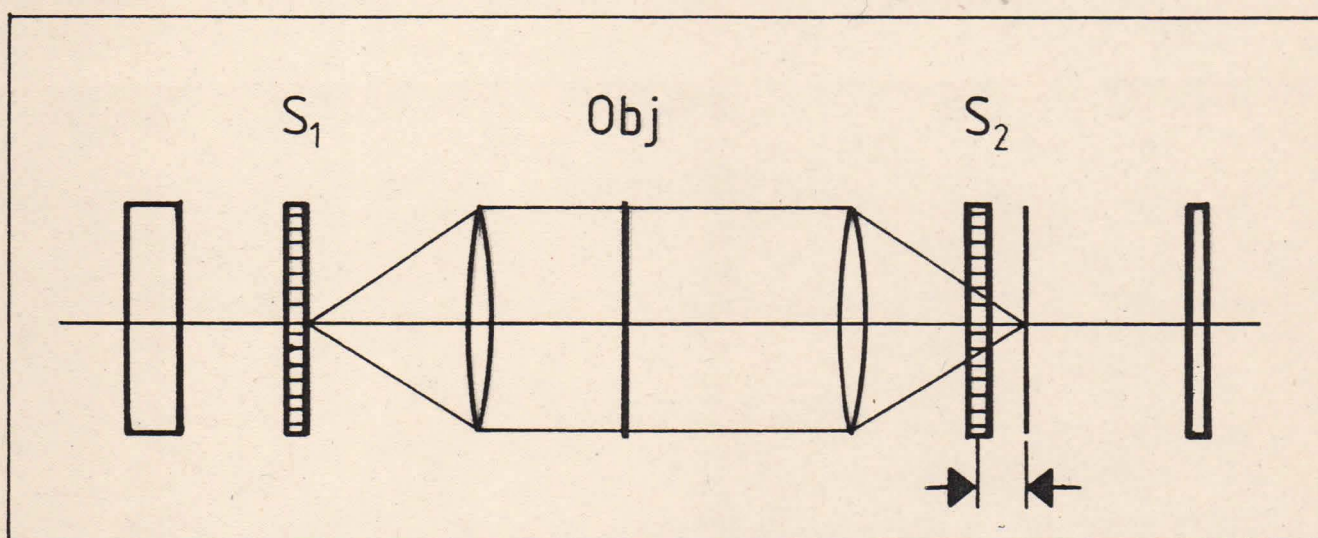
N is the imaging magnification factor of the object.



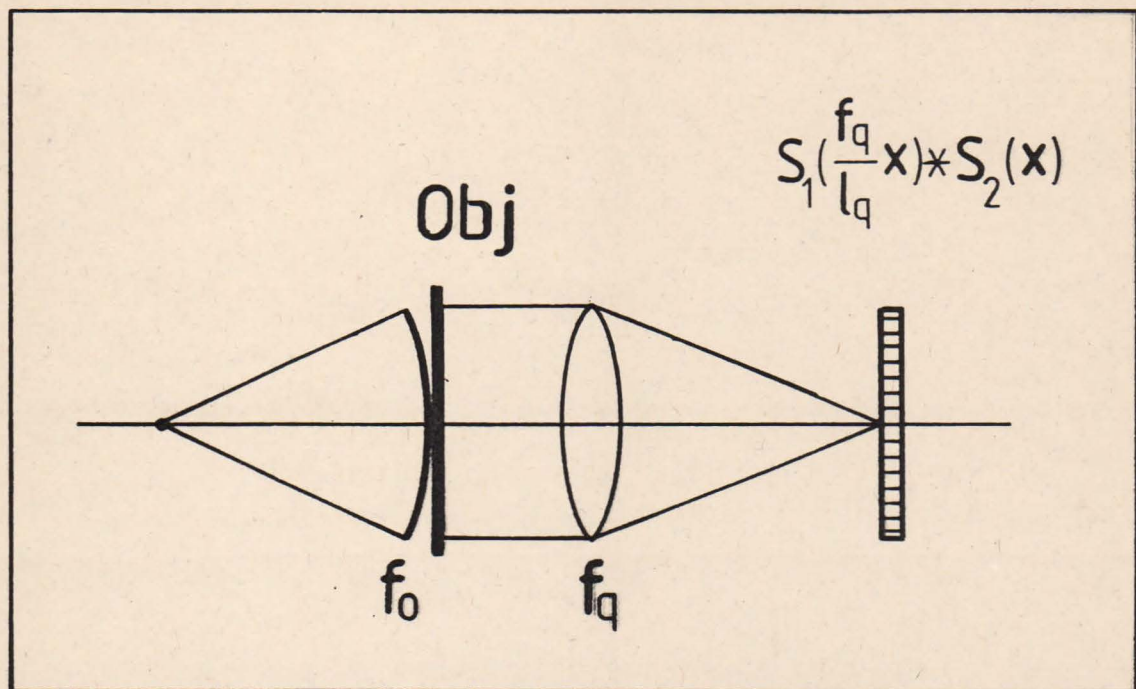
"quasi-" ——— no interference

5.3 Quasi-Interferometry with Coded Correlation Filtering

space domain	<u>optical correlation</u>
spatial frequency domain	<u>optical filtering</u>



1. An equivalent filter function of correlation between two mask functions,
2. A quadratic phase factor biased by defocusing of a mask.



f_0 is the focal length of quadratic phase biasing,

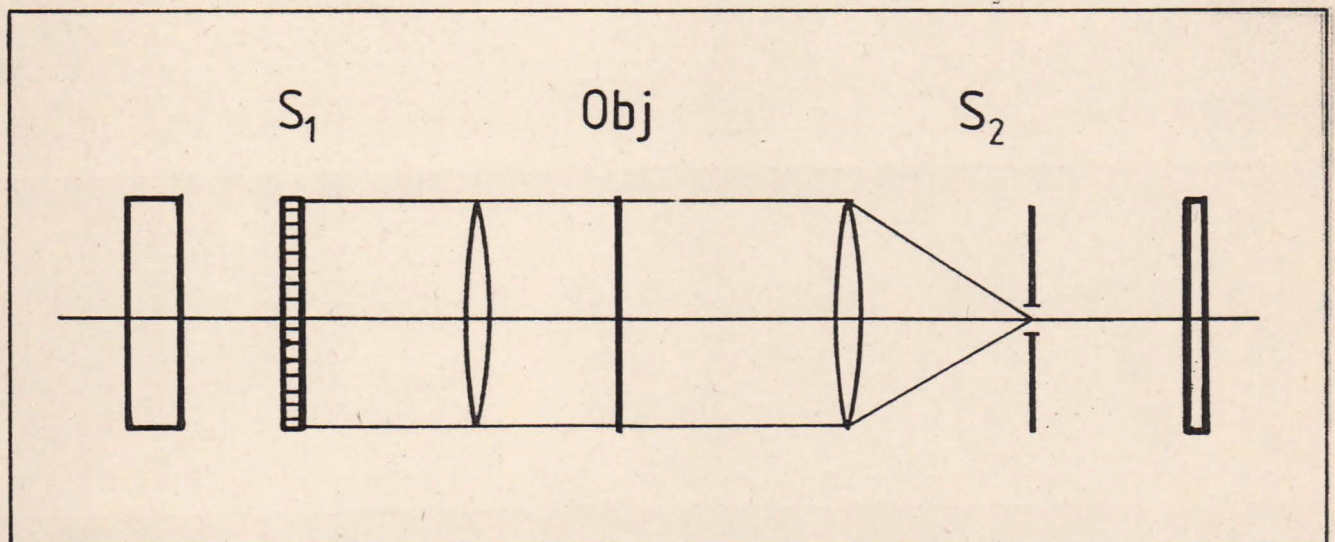
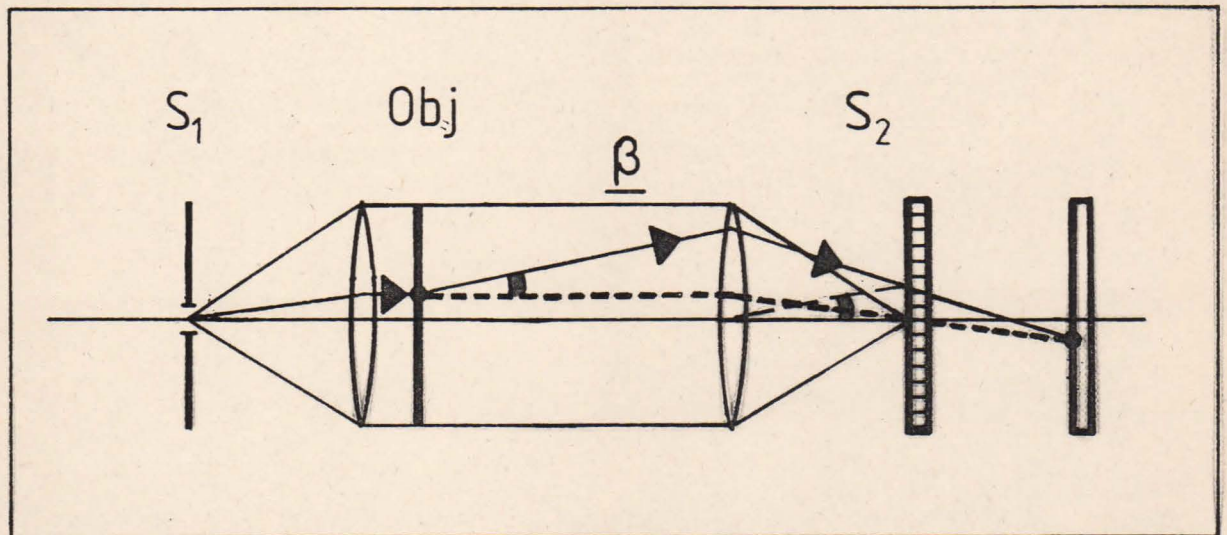
f_q is the equivalent transforming focal length,

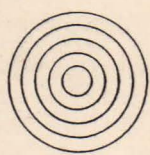
l_q/f_q is the scaling factor between two masks.

$$f_q = KR, \quad l_q = R, \quad f_0 = -(R/(NC))$$

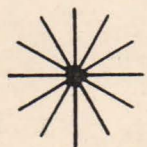
Simplified Systems

with a point source or a pinhole filter

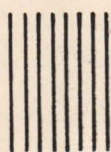




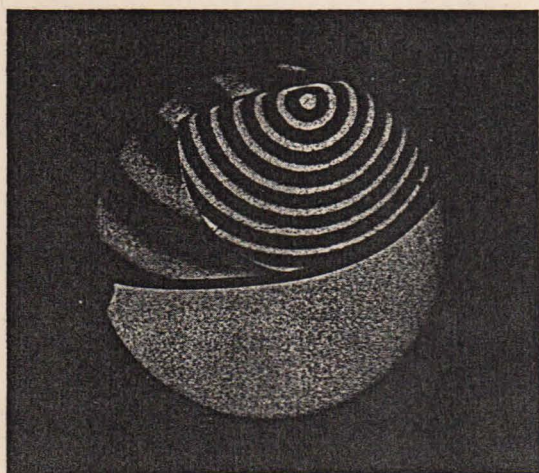
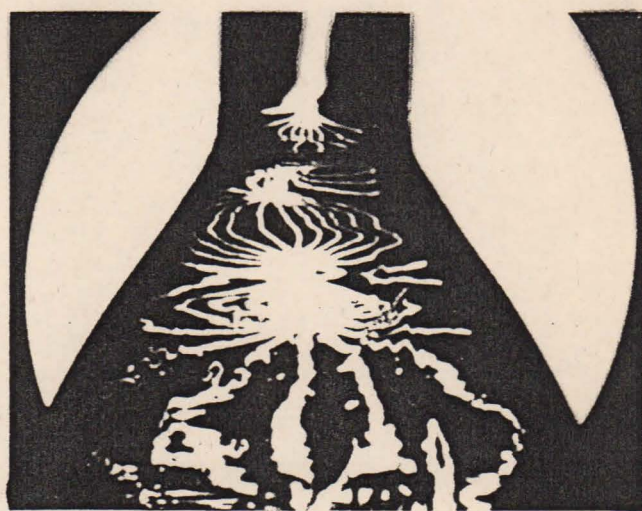
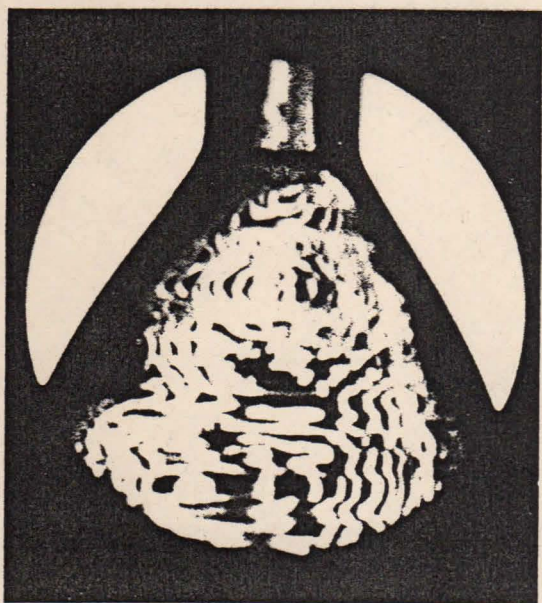
magnitude of phase gradient



directions of gradient

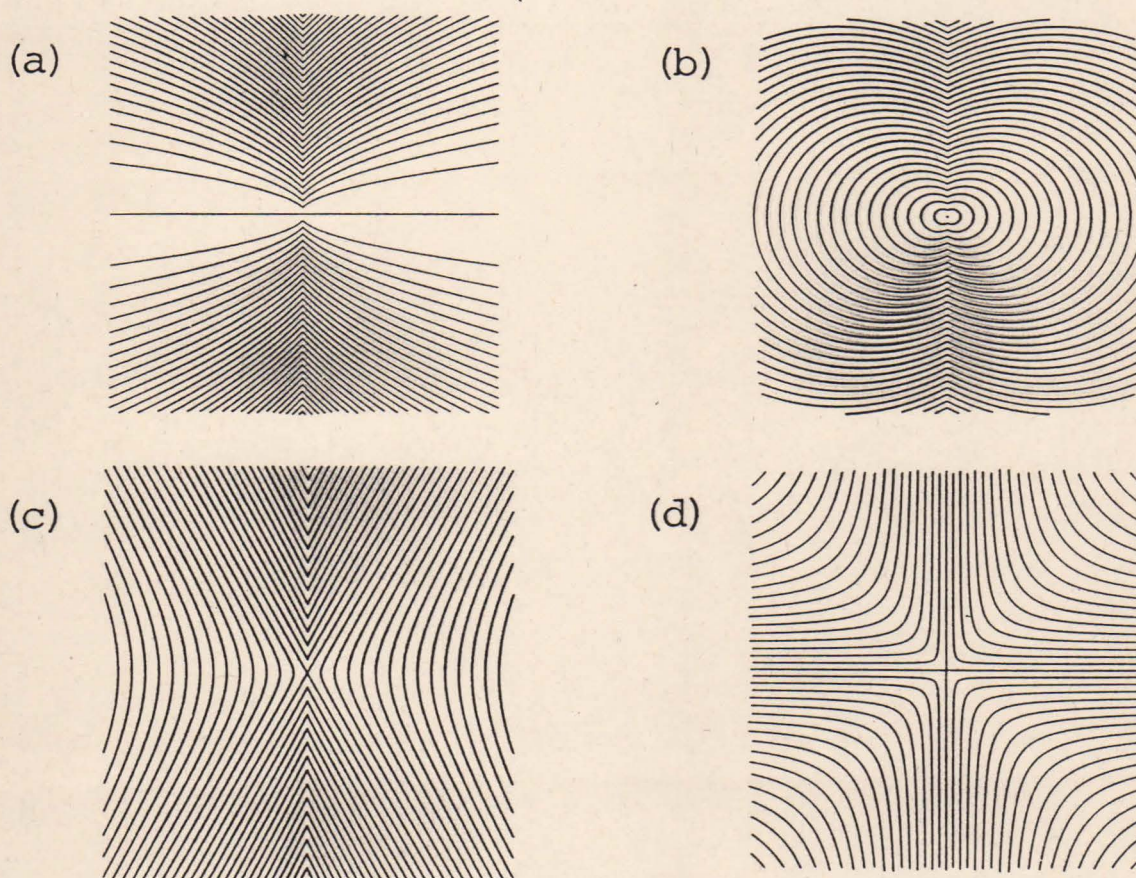


lateral shearing

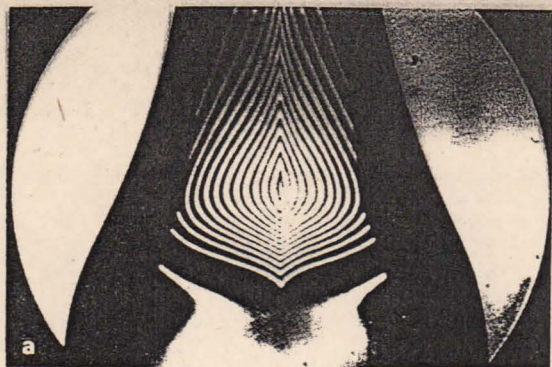


5.4 Mask Functions in Quasi-Interferometry

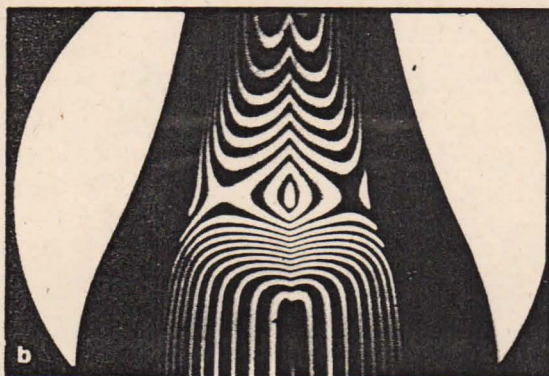
Contour-Mapping Systems



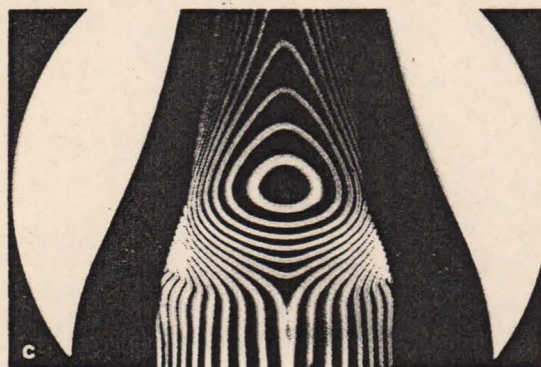
$$\begin{aligned}
 (a) \quad & g - M |g_x| = nK, \quad M = 1 \\
 (b) \quad & \quad \quad \quad M = 0.5 \\
 (c) \quad & \quad \quad \quad M = 1.5 \\
 (d) \quad & g + |g_x| + |g_y| = nK
 \end{aligned}$$



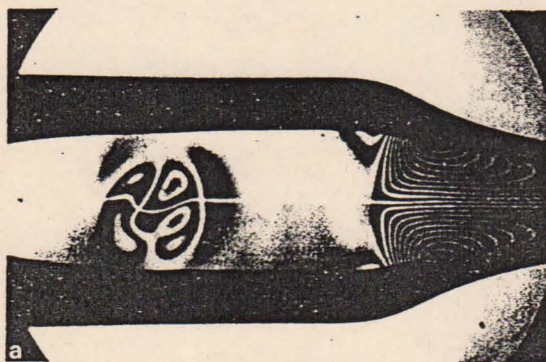
mask (a)



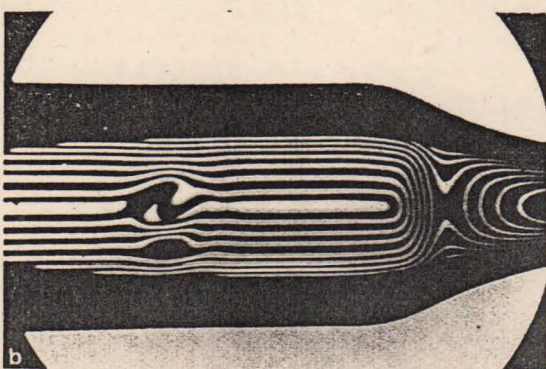
mask (b)



mask (c)



mask (d)

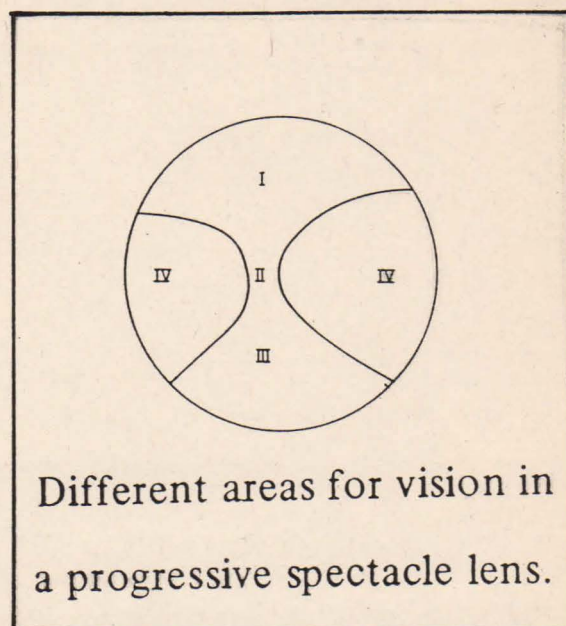


rings-mask

5.5 Moiré Quasi-Interferometry

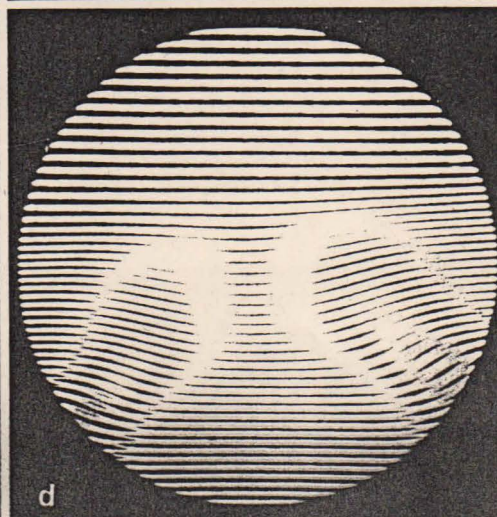
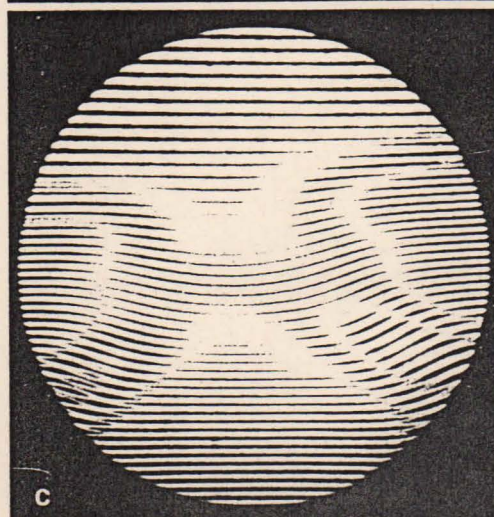
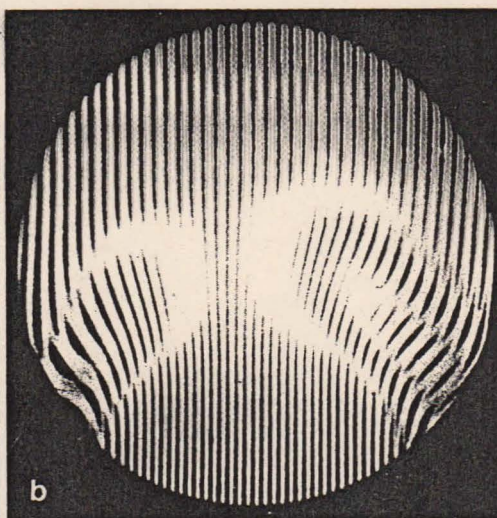
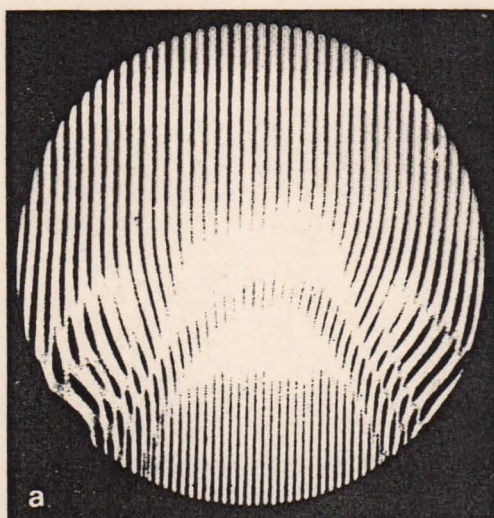
Moiré technique with
two exposures

Contour-mapping of the
second-order phase
derivatives



$$\partial^2 \phi / \partial x^2$$

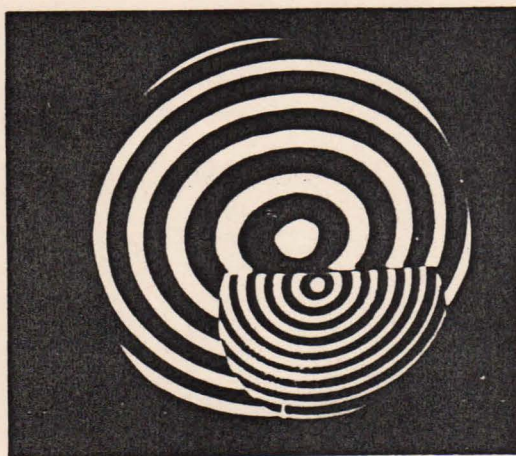
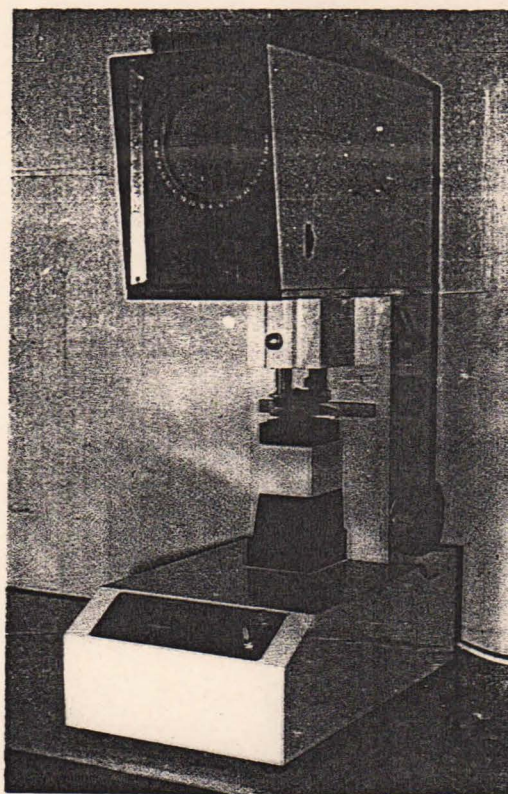
$$\partial^2 \phi / \partial y \partial x$$



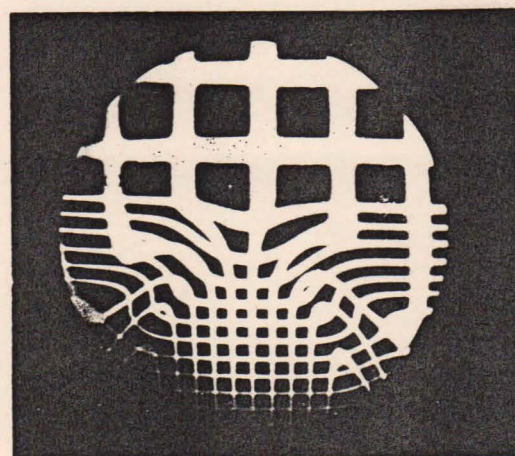
$$\partial^2 \phi / \partial y^2$$

$$\partial^2 \phi / \partial x \partial y$$

5.6 Lensmeter



A bifocal lens

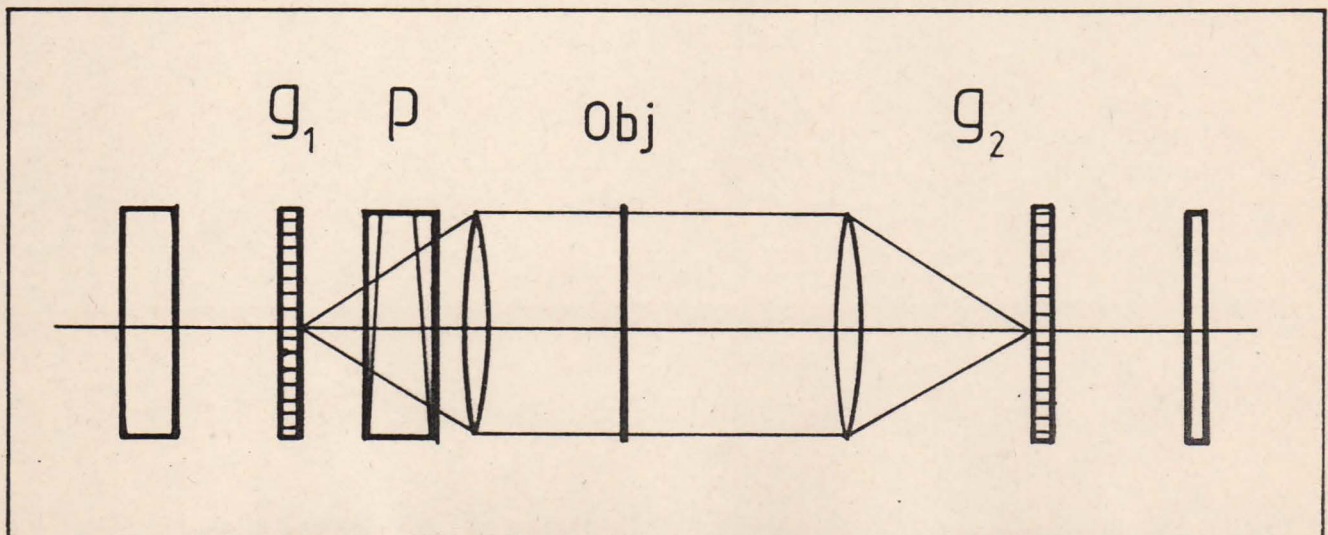
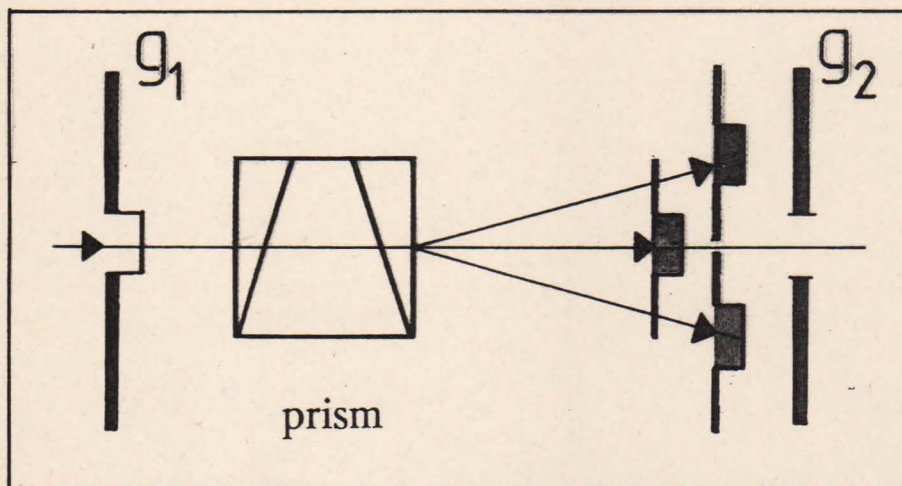


A progressive lens

5.7 Color Coding of Fringes

colors \gg grey level

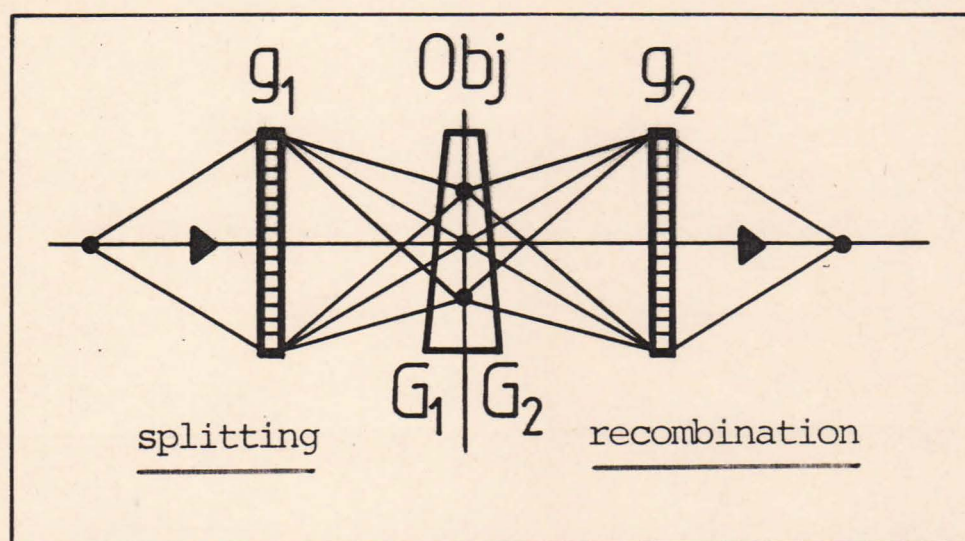
1. Dispersion prisms,
2. Gratings of diffractive dispersion,
3. Colored masks.



6. FOURIER-TRANSFORM DIFFRACTION

INTERFEROMETRY

COMPARISON	mask diffraction	phase variation
quasi-interferometry	small	strong
diffraction interferometry	large	weak

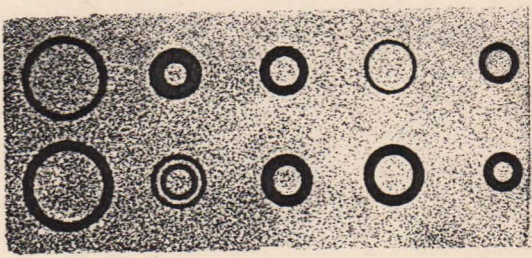


$$E(x, y; \lambda) = \iint_{-\infty}^{\infty} G_1\left(\frac{\alpha}{\lambda f}, \frac{\beta}{\lambda f}\right) G_2\left(\frac{\alpha}{\lambda f}, \frac{\beta}{\lambda f}\right) O\left(\alpha - \frac{x}{M}, \beta - \frac{y}{M}\right) d\alpha d\beta.$$

6.1 Character Recognition of Phase Objects

COMPARISON	holographic complex filter	diffraction correlation
phase objects	no	yes
output	-correlation peaks -no image	-correlation peaks -visuable image

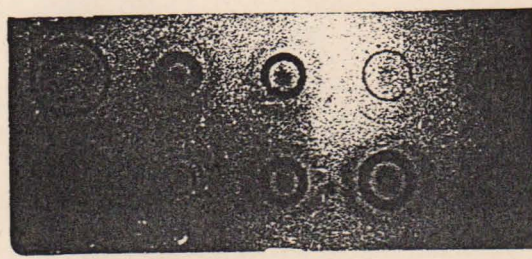
phase specimen



phase character



output image



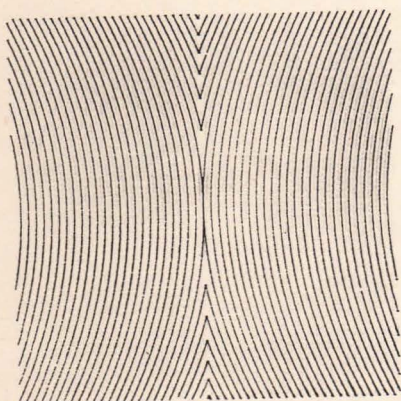
6.2 Interferometrical Experiments

Forms of Interference

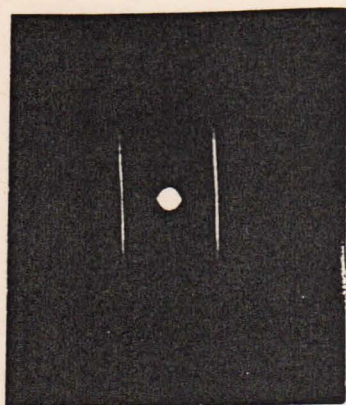
1. optical holograms
2. computer-generated gratings

Experiment 1

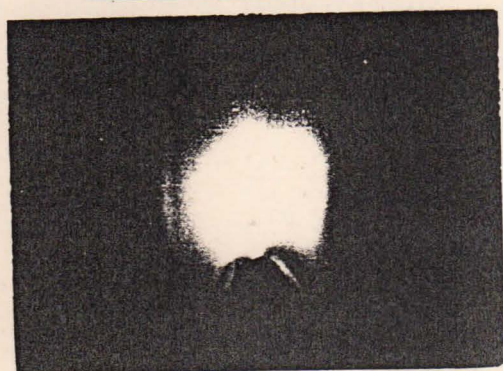
$$G_1 = G_2$$



computer-generated
mask



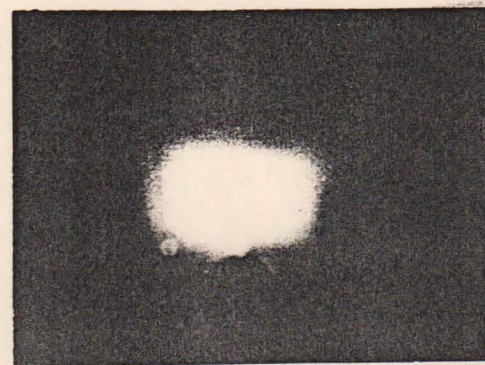
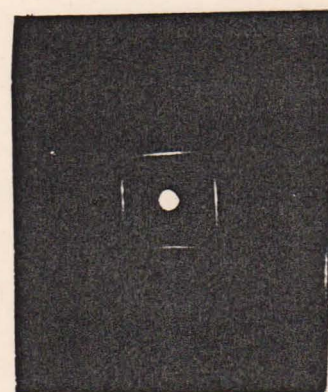
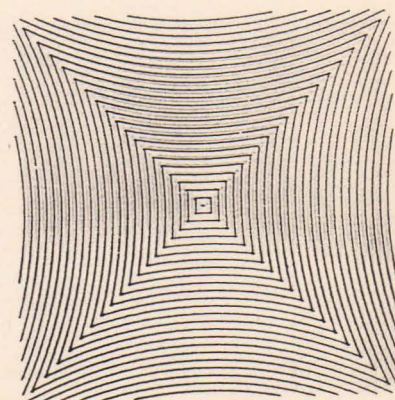
Fourier-transform
of mask



flame interferogram

Experiment 2

$$G_1 = G_2$$



CONCLUSIONS

The advantages of the optical correlation interferometry:

- incoherent illumination (incandescent lamps)
- various kinds of contour-mapping systems
- changeable sensitivity (from strongly distorted phase objects to weak phase objects)
- background-fringe biasing
- moire technique
- color-coding of fringes
- image processing

Future Prospects

Applications:

- interferometry
- image processing
- laser technique
- optical computing
- optical interconnection
- binary optics
- photorefractive grating
- acoustic grating

