Evidence for the droplet picture of spin glasses

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collaboration with M. Moore, H. Bokil, A. Bray

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1. Introduction

2. Two competing theories

3. MKA for the Edwards-Anderson spinglass

4. The $\pm J$ spinglass

5. Three-spin model

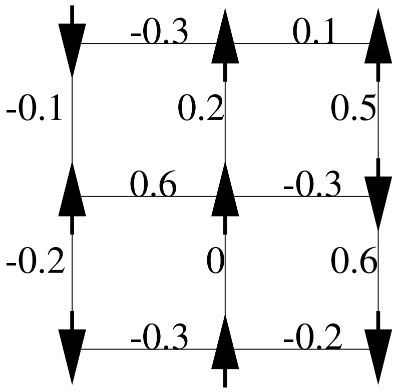
6. Summary

Introduction

Signature of spinglass phase: frozen disorder

Edwards-Anderson-Model:

$$
\begin{aligned}
H &= -\sum_{<ij>} J_{ij} S_i S_j, \\
S_i &= \pm 1 \\
P(J_{ij}) &= \frac{\exp(-J_{ij}^2/2J^2)}{\sqrt{2\pi}J}
\end{aligned}
$$



What does the system look like at low temperatures? 2 macroscopic states? More than 2? Infinitely many?

Consider two replicas (copies) of the same system:

Overlap $q = (1/N) \sum_i S_i^{(1)} S_i^{(2)}$

Two competing theories

A. Droplet picture

- Only 2 macroscopic states
- Zero-temperature fixed point with one-parameter scaling picture
- Scale-dependent coupling strength $J(L) \sim JL^{\theta}$
- Consequences
  1. Typical energy of a domain wall: L^{θ}
  2. Typical energy of a droplet of radius r: r^{θ}
  3. Probability density of having zero overlap between two replicas: $\sim kTL^{-\theta}$.

4. Magnetic field destroys spinglass phase:

   energy gain by aligning local magnetization of droplet of radius r parallel to the field: $\sim hr^{d/2}$

   energy cost: $\sim Jr^{\theta}$

   (Gain $>$ cost) for $r > (h/J)^{-1/(d/2-\theta)}$

B. Replica symmetry breaking

Mean-field theory: every spin coupled to every other spin

Calculate

$$\Big[F[J_{ij}]\Big]_{av} = -T \int dP[J_{ij}] \ln Z[J_{ij}]$$

using

$$\ln Z = \lim_{n \to 0} \frac{Z^n - 1}{n}$$

$$\begin{aligned} \Big[Z^n[J_{ij}]\Big]_{av} &= \int d[J_{ij}] P[J_{ij}] Tr_S e^{\beta \sum_{(i,j)} J_{ij} \sum_\alpha S_i^\alpha S_j^\alpha} \\ &= e^{-\left(\frac{\beta J N}{2}\right)^2 n} \left(\frac{\beta J}{2\sqrt{\pi}}\right)^{n(n-1)} \int d[q^{\alpha\beta}] e^{-NnG} \end{aligned}$$

with

$$nG = \sum_{\alpha \neq \beta} \frac{\beta^2 J^2}{N} \left(q^{\alpha\beta}\right)^2 - \ln Tr_S e^{\frac{\beta^2 J^2}{2N} \sum_{\alpha \neq \beta} S^\alpha S^\beta q^{\alpha\beta}}$$

Assume T close to T_c and expand in the overlaps $q^{\alpha\beta}$

$$
\begin{aligned}
G[q] &\simeq \lim_{n\to 0}[\theta \sum_{\alpha\beta}(q^{\alpha\beta})^2 - \sum_{\alpha\beta\gamma} q^{\alpha\beta}q^{\beta\gamma}q^{\gamma\alpha}/3 - \\
&- \sum_{\alpha\beta}(q^{\alpha\beta})^4/6]/2n \\
&\simeq \int_0^1 dx[|\theta|q^2(x) + q^4(x)/6 - xq^3(x)/3 - \\
&- q(x)\int_0^x q^2(y)dy]/2
\end{aligned}
$$

Matrix $q^{\alpha\beta}$:

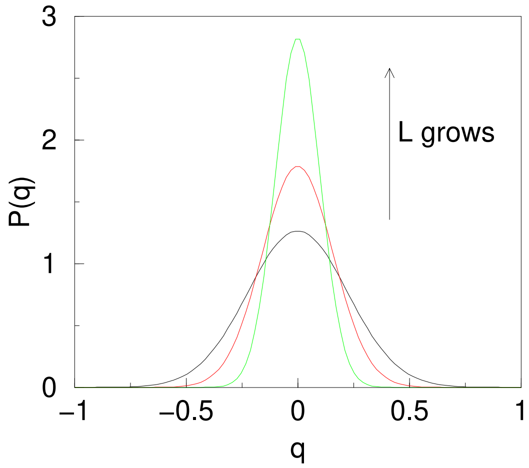
Saddle point method gives $q(x) = x/2$ for $q < q_{max} = |\theta|$. Non-trivial overlap distribution, and no self-averaging: $(\Delta q)^2 > 0$.

MKA for the Edwards-Anderson spin-glass

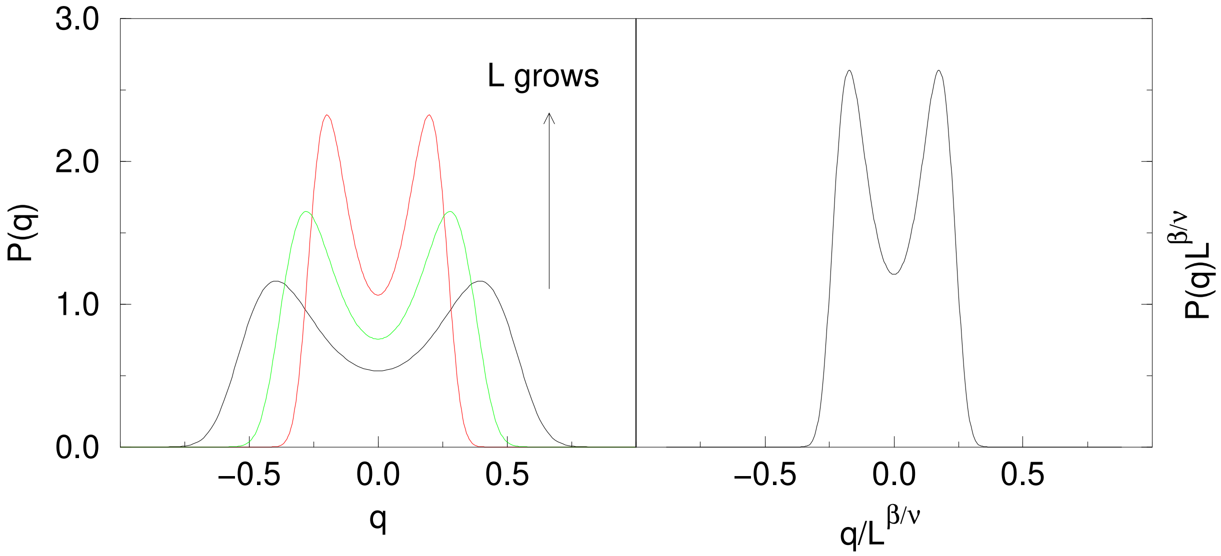
in order to evaluate the overlap distribution $P(q)$

- $T > T_c$:

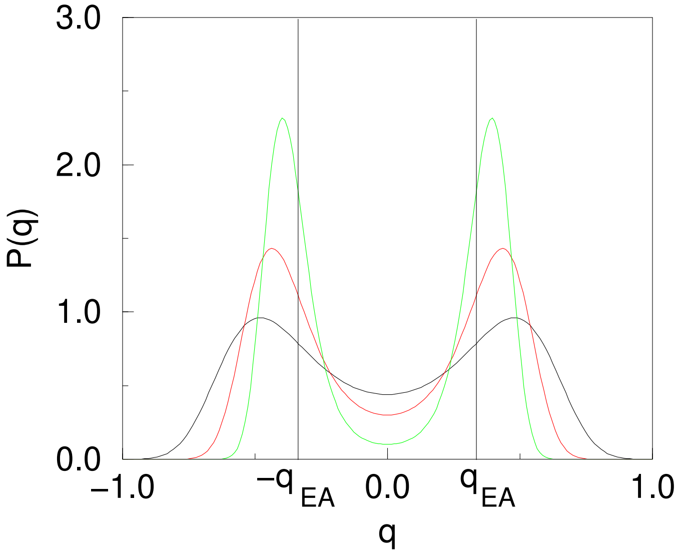
$$\lim_{L\to\infty} P(q) = \delta(0)$$



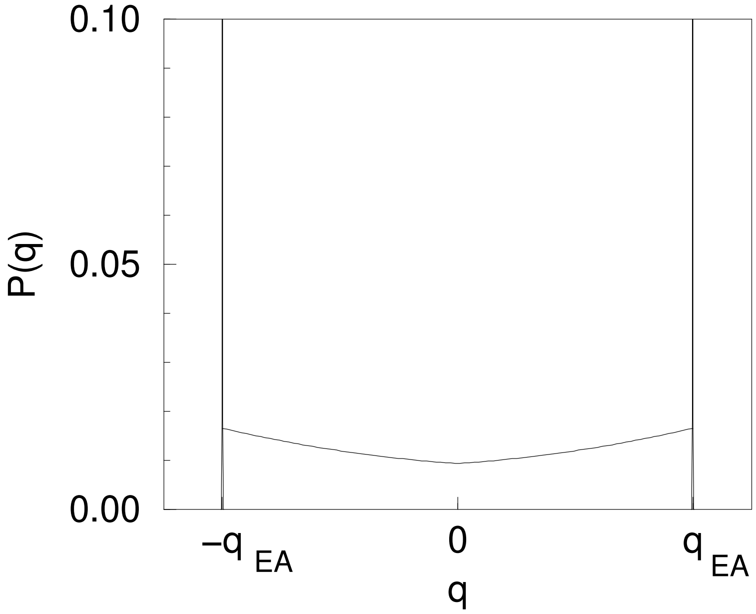
- $T = T_c, d = 3$:



- $T < T_c$:



or

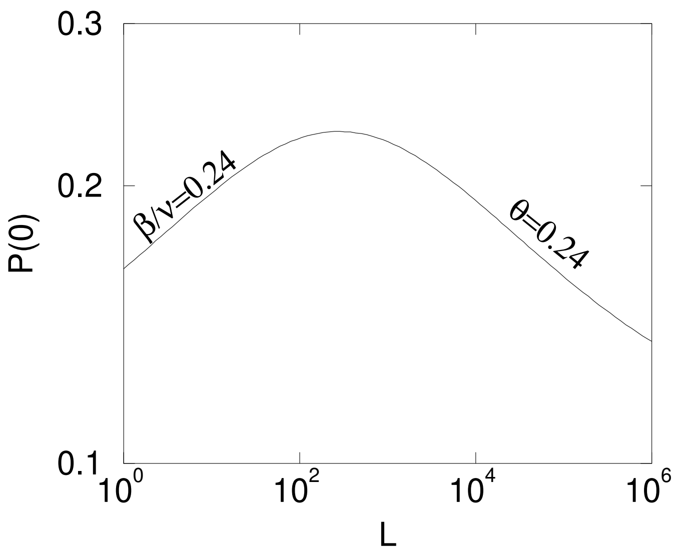


Droplet picture or replica symmetry braking?

Parisi et al: Monte Carlo simulations in $3d$ at $T \simeq 0.7T_c$: good agreement with RSB

Newman/Stein: “proof” against RSB

Prediction droplet model:



for not too small T: are Monte Carlo simulations close to the plateau?

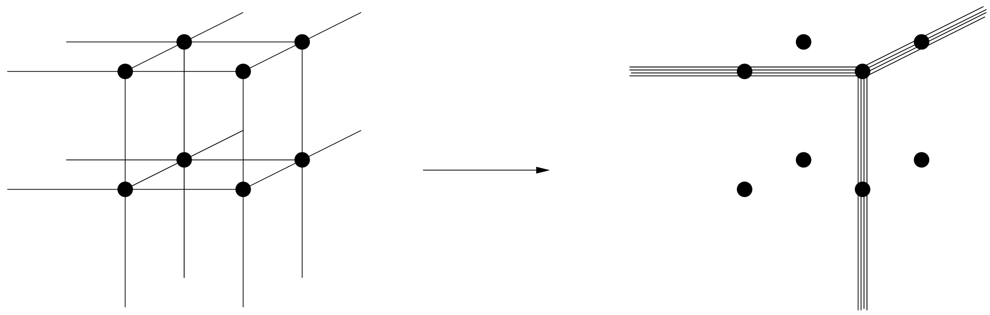
Migdal-Kadanoff approximation (MKA)

$$P(q) = \left[\frac{\sum_{\{S_i^{(1)}\},\{S_i^{(2)}\}} \exp\left[-(H_1+H_2)/kT\right]\delta(\frac{1}{N}\sum_i S_i^{(1)} S_i^{(2)} - q)}{\sum_{\{S_i^{(1)}\},\{S_i^{(2)}\}} \exp\left[-(H_1+H_2)/kT\right]} \right]$$

Calculate first

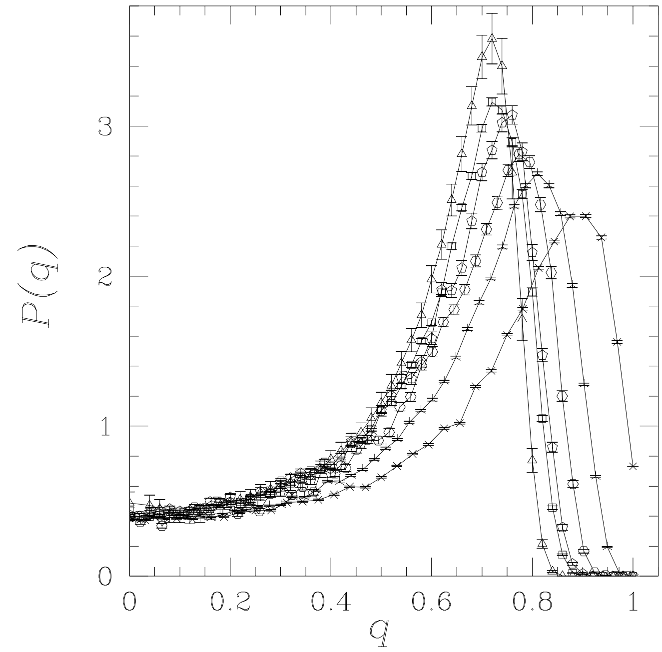
$$F(y,L) = \left[\left\langle \exp\left(iy \sum_{\langle ij \rangle} \frac{(S_i^{(1)} S_i^{(2)} + S_j^{(1)} S_j^{(2)})}{2N_L} \right) \right\rangle \right]$$

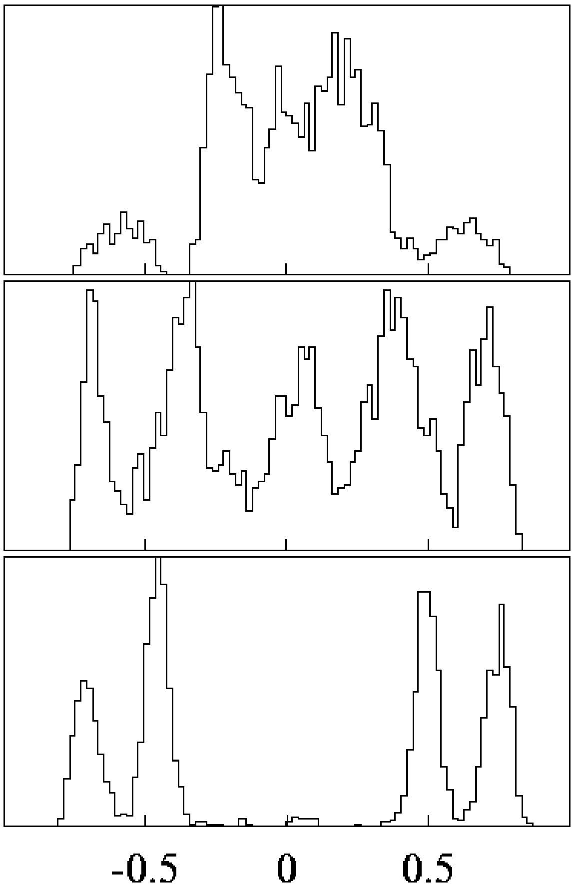
Take trace such that lattice constant is doubled at every step



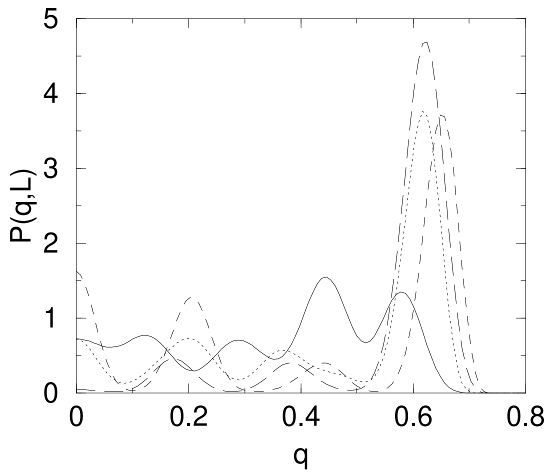
Uncontroled approximation, but very successfull in many situations

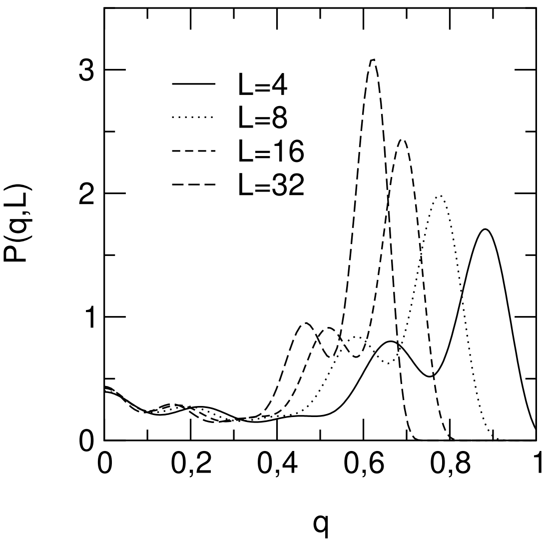
Results of Monte-Carlo simulations:

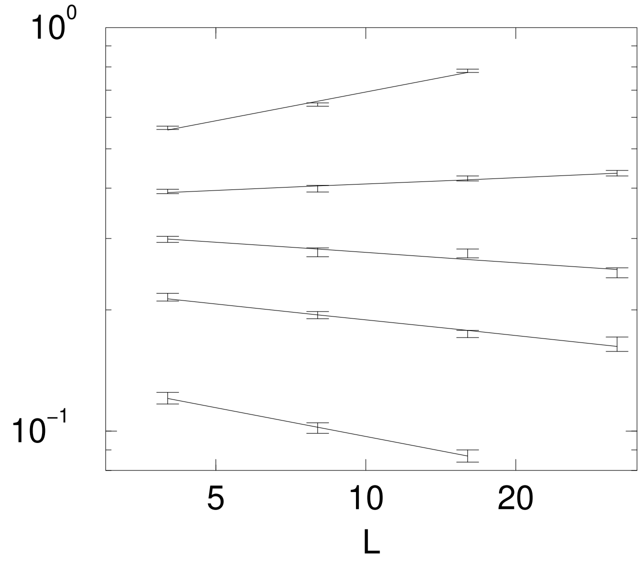




Results of MKA:





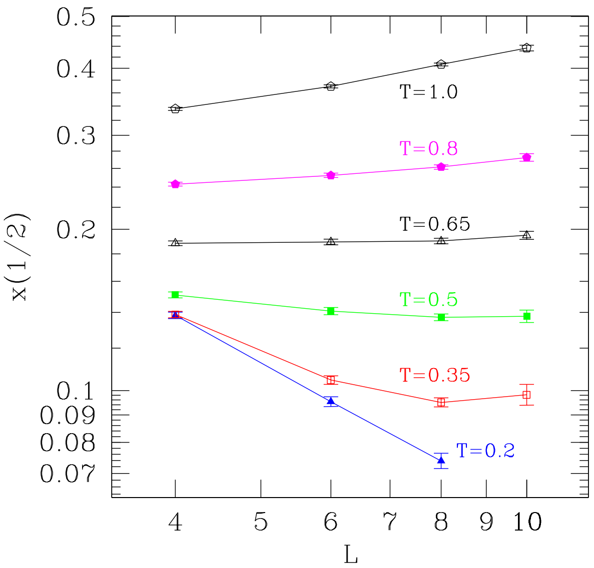


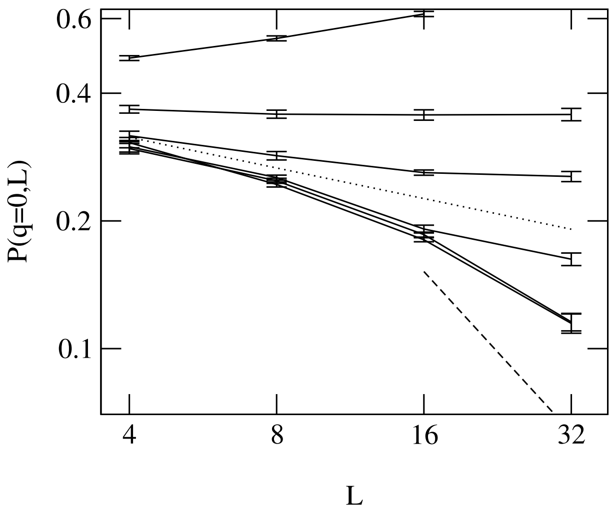
Exponents 0.24,0.053,−0.084,−0.13,−0.24

⇒ MC simulations might not be in the asymptotic scaling regime.

The $\pm J$ spinglass: $J_{ij} = \pm 1$

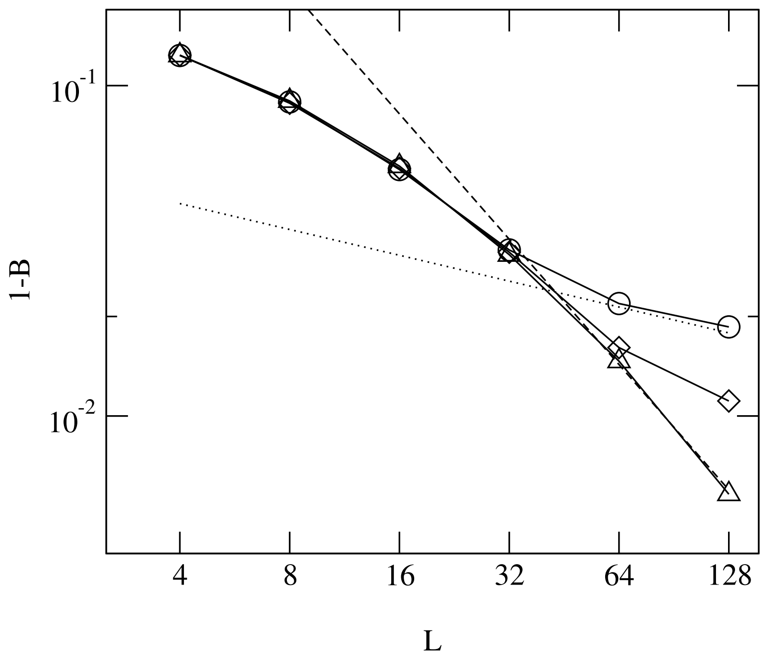
Ground state degenerate



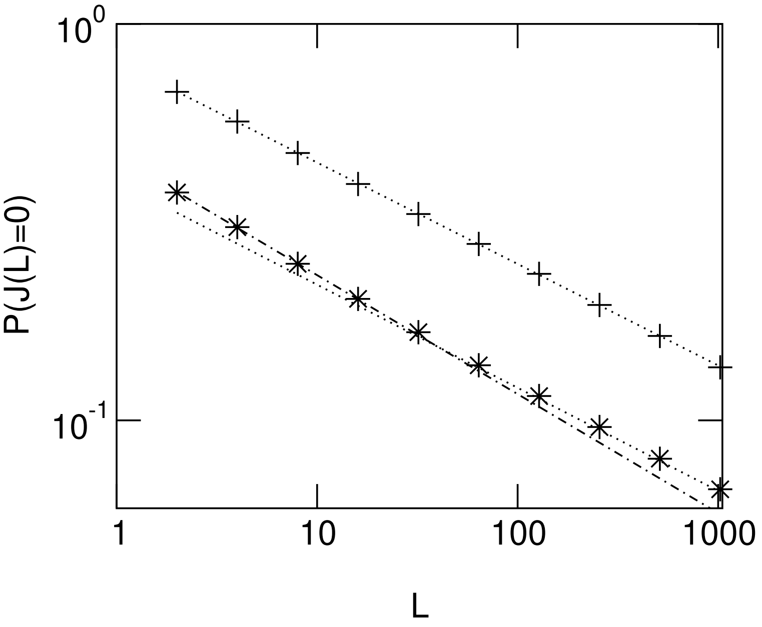


Binder parameter

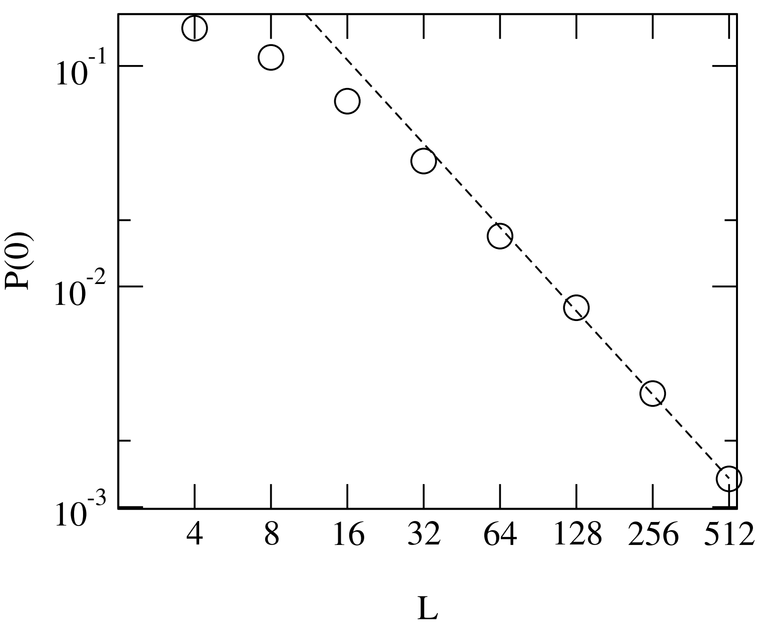
$$B = \frac{3}{2}\left(1 - \frac{[\langle q^4 \rangle]}{3[\langle q^2 \rangle]^2}\right)$$



Probability of having a domain wall with no energy cost:



Probability that the two replicas differ by a domain wall



- $T = 0$: limit slope $-d_s/2 - \theta$ only for $L > 64$ visible.

- low T: droplet picture not visible for $L < 16$.

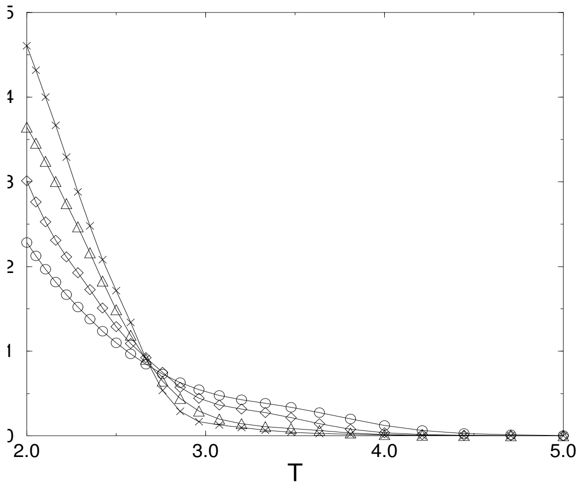
- Crossover: curves branch off from $T = 0$ at $L \sim T^{-2/d_s}$.

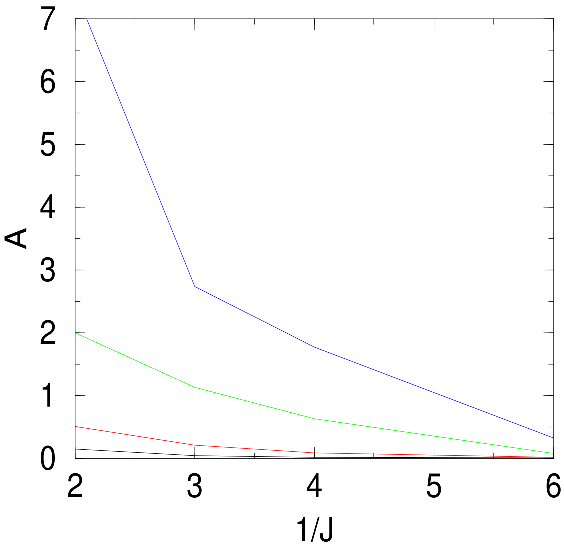
Three-spin model

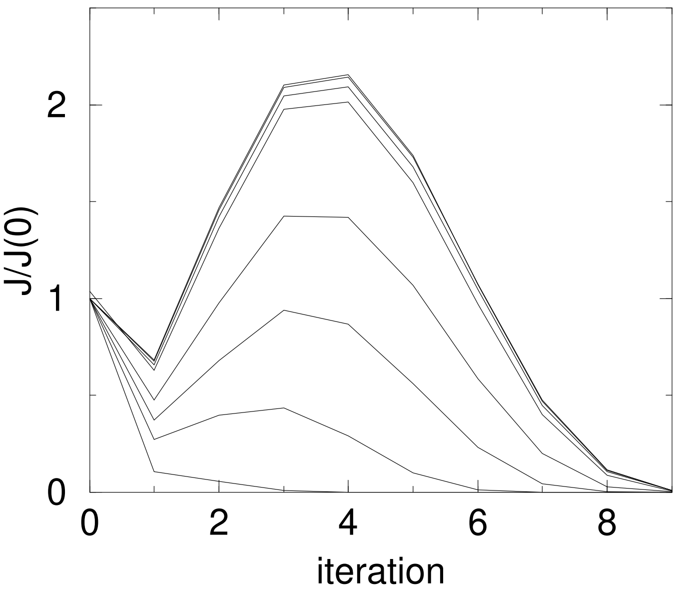
$$-\beta\mathcal{H}(\sigma) = \sum_{ij}(J_{ij}^{(1)}\sigma_i^{(1)}\sigma_i^{(2)}\sigma_j^{(1)} + J_{ij}^{(2)}\sigma_i^{(1)}\sigma_i^{(2)}\sigma_j^{(2)}$$
$$+ J_{ij}^{(3)}\sigma_i^{(1)}\sigma_j^{(1)}\sigma_j^{(2)} + J_{ij}^{(4)}\sigma_i^{(2)}\sigma_j^{(1)}\sigma_j^{(2)})$$

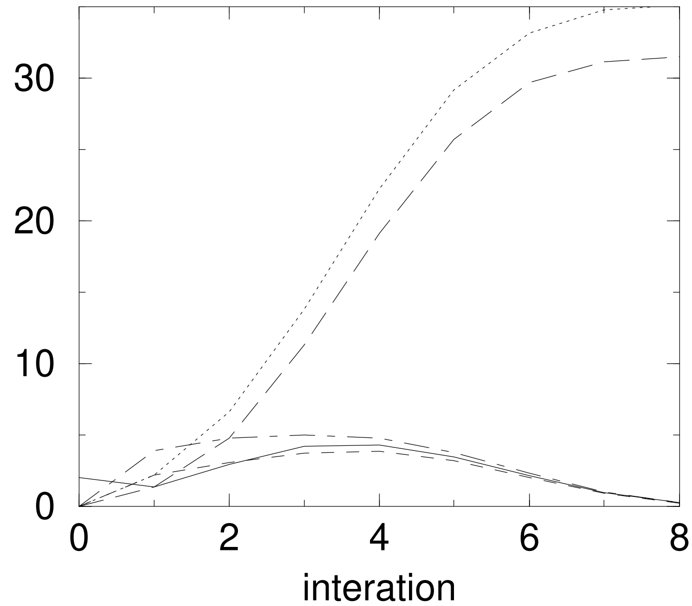
Degree of non-self-averaging

$$A = \frac{\left[\langle(q-\langle q\rangle)^2\rangle^2\right]}{\left[\langle(q-\langle q\rangle)^2\rangle\right]^2} - 1$$






J/J(0)
0
1
2
0
2
4
6
8
iteration


0
10
20
30
0
2
4
6
8
interation

Summary

- Results of MC simulations are influence by the critical point. The critical correlation length is not small even at $T \simeq 0.7T_c$.

- For sufficiently low temperatures agreement with the droplet picture possible. Similar: link overlap.

- In the $\pm J$ model the true scaling behaviour is invisible below $L \simeq 16$.

- Three-spin model and spin glass in external field show apparent non-self-averaging in the parameter range used in MC simulations.