Oldenburg
University
10-10-08

Exploring Spin Glass Ground States with Extremal Optimization

Stefan Boettcher

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EMORY

Find at: www.physics.emory.edu/faculty/boettcher

Collaborator:

- Allon Percus (Los Alamos/UCLA)

Funding:

- NSF-DMR, Los Alamos-LDRD, Emory-URC

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  - Spin Glasses (MAX-CUT)
- Spin Glass Ground States with τ-EO
  - Dilute **E**dwards-**A**nderson in $d=3,...,7$
  - Mean-Field: **S**herrington-**K**irkpatrick & **B**ethe **L**attice
  - A Comprehensive View
  - **SK** with Power-Law Bonds $P(J)\sim 1/|J|^{1+\mu}$

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- How can we use it to optimize?
  - Extremal Driving:
    - Select and eliminate the “bad”,
    - Replace it *at random*,
    - Eventually, only the “good” is left!

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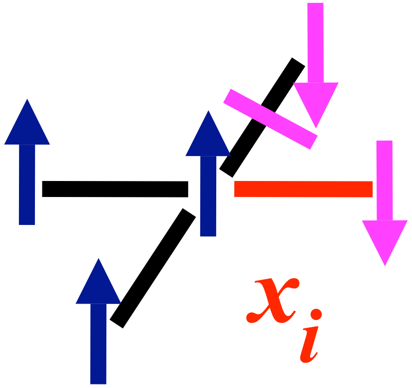
$$\lambda_i = x_i \sum_j J_{i,j} x_j$$



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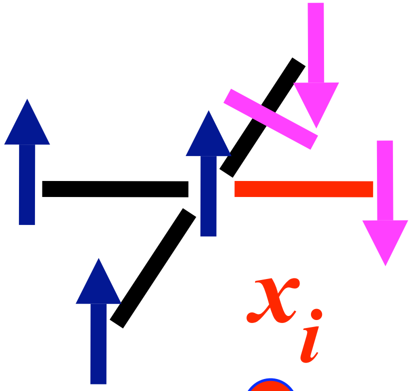

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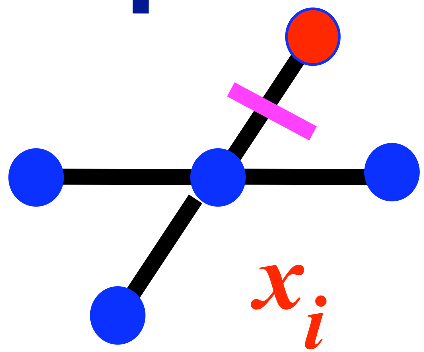

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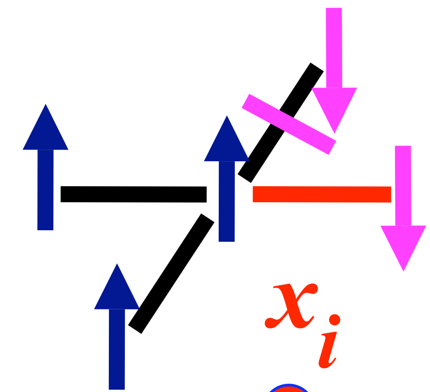

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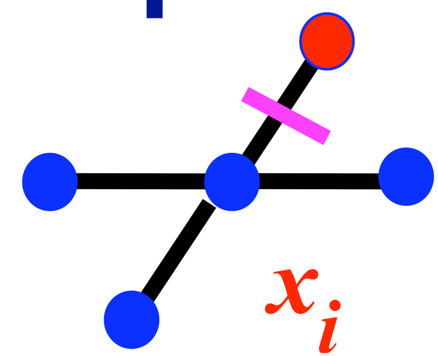

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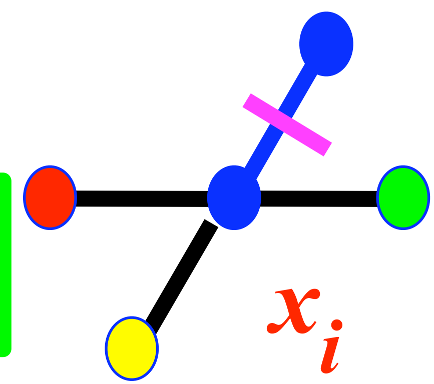

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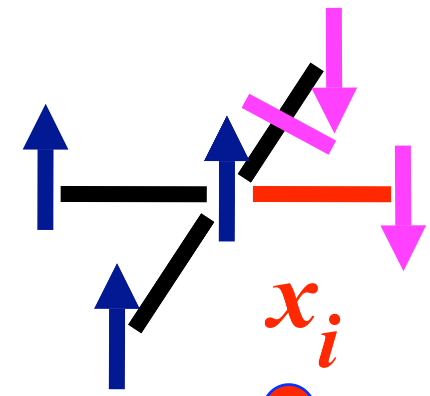

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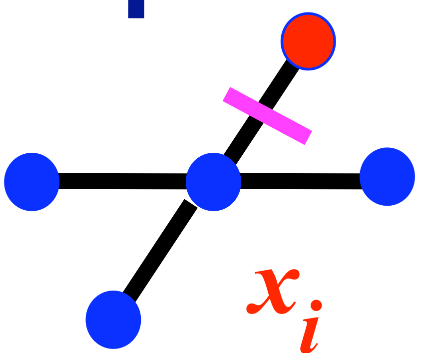

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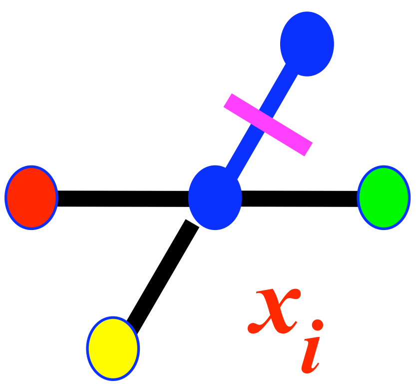

$$Cost \propto H = -\sum_i \lambda_i$$

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(7) Return: Best $C(S)$ found along the way!

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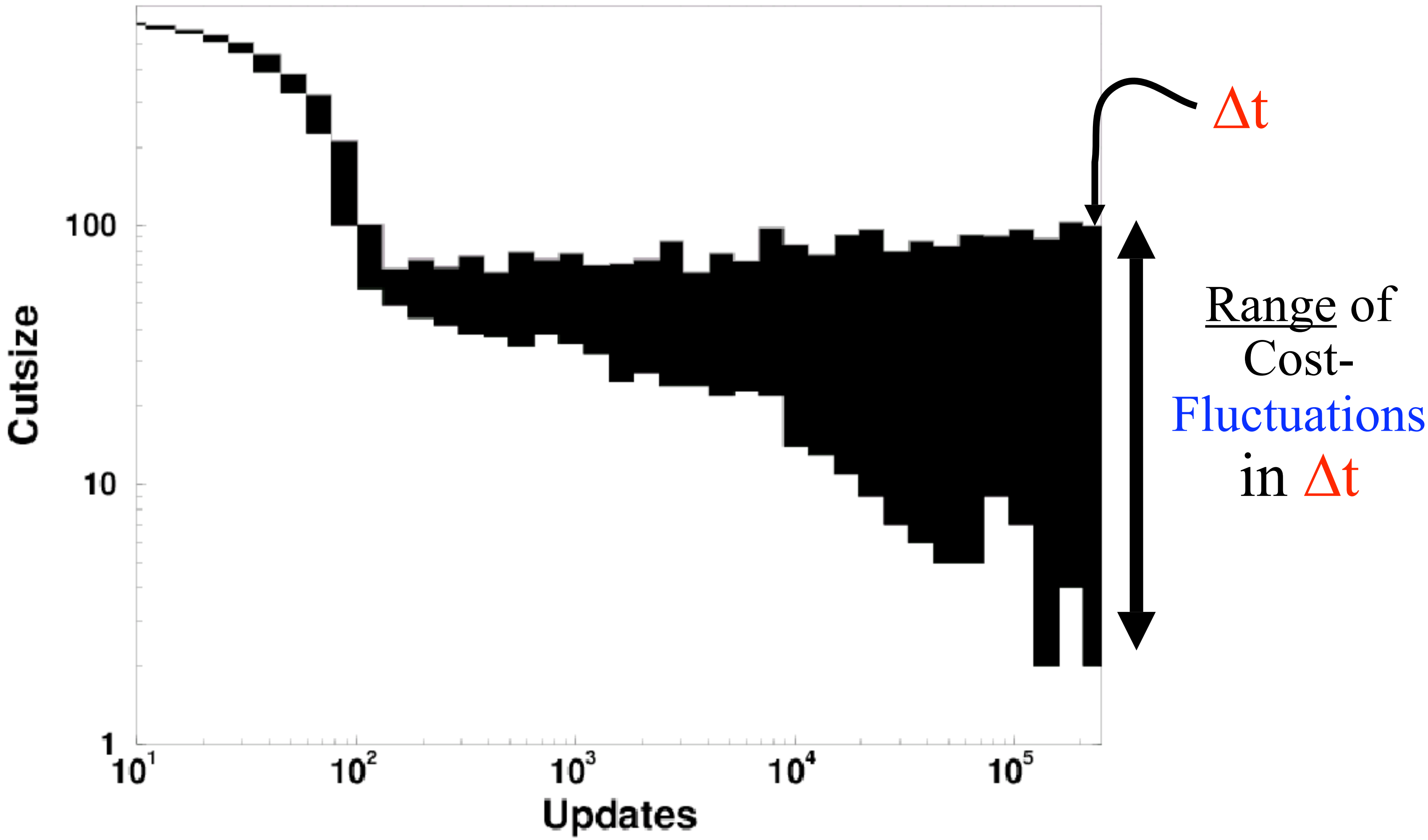

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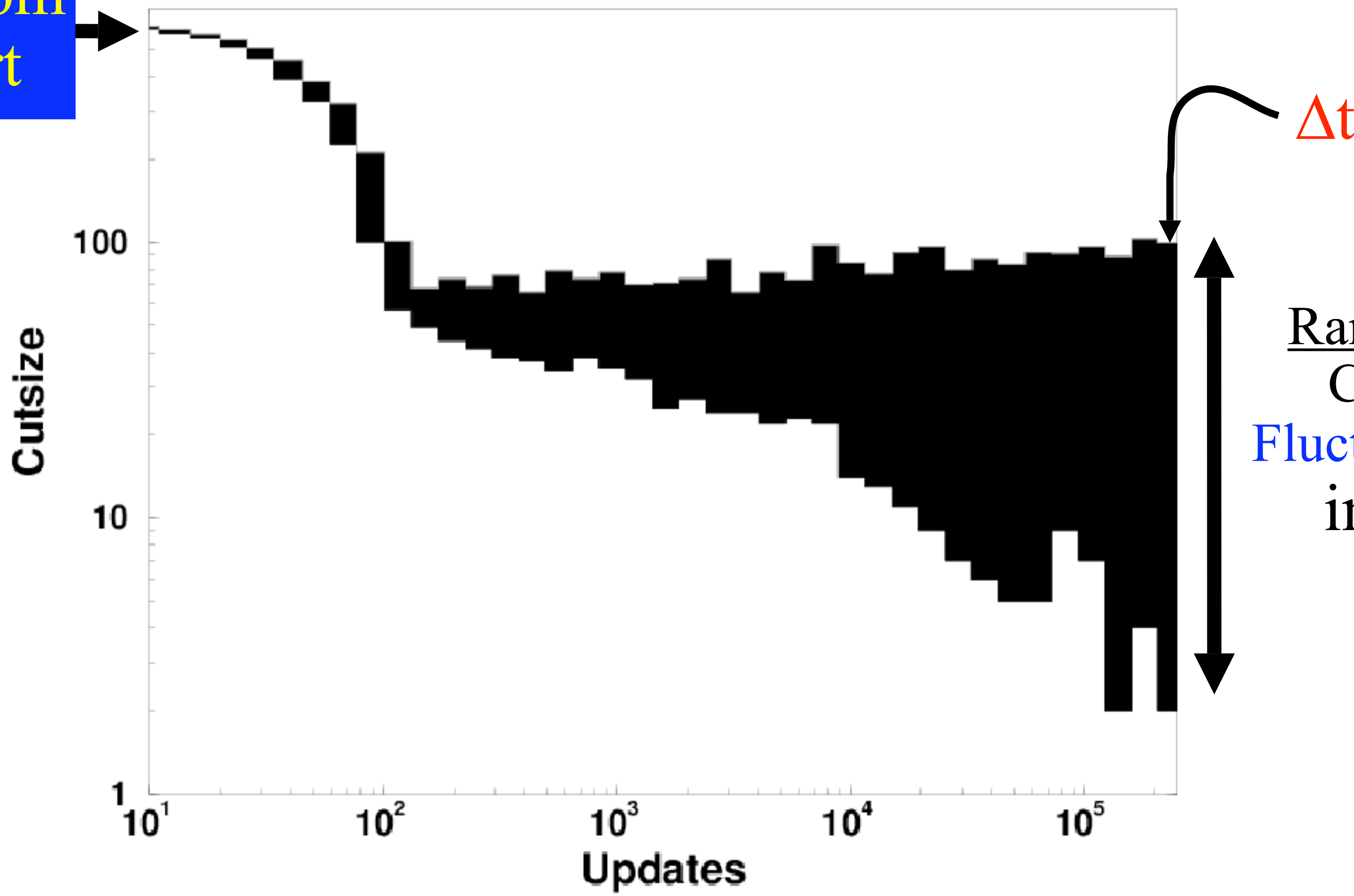
random start



Range of Cost-Fluctuations in Δt

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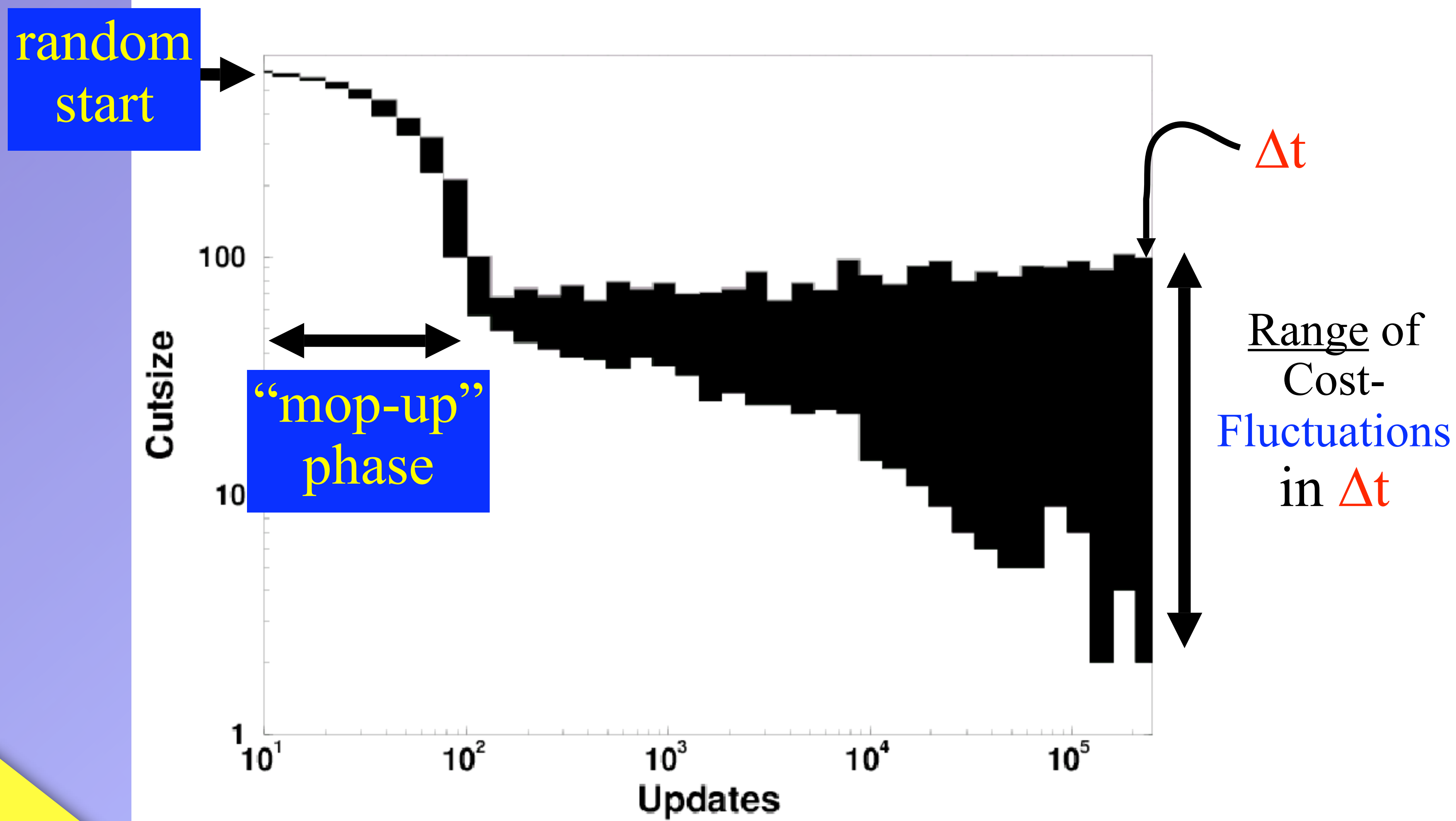

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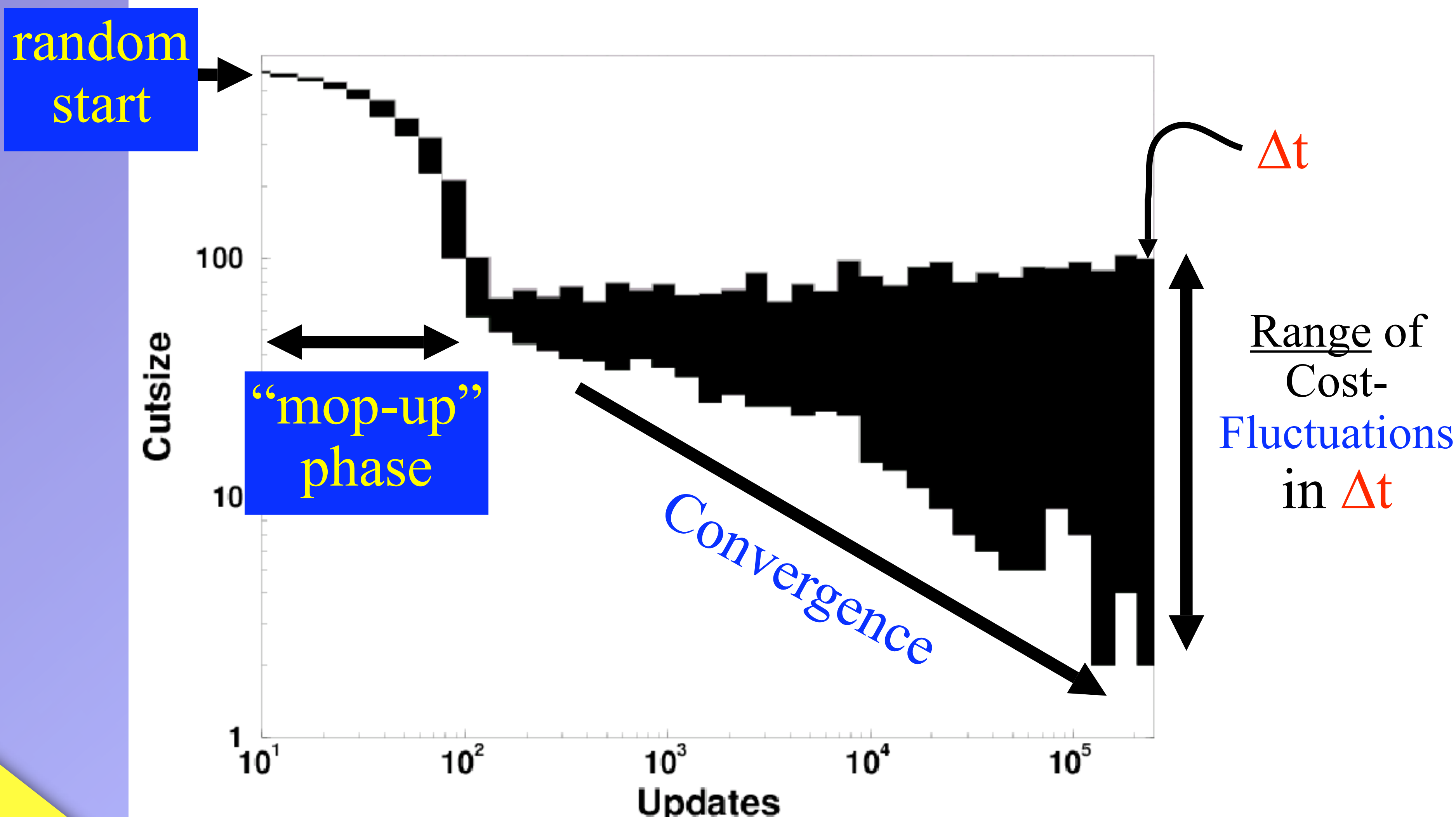

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For Ranks $\lambda_{\Pi(1)} \leq \ldots \leq \lambda_{\Pi(n)}$, update $i=\Pi(k)$ with scale-free, power-law distribution

$$P(k) \propto k^{-\tau}$$

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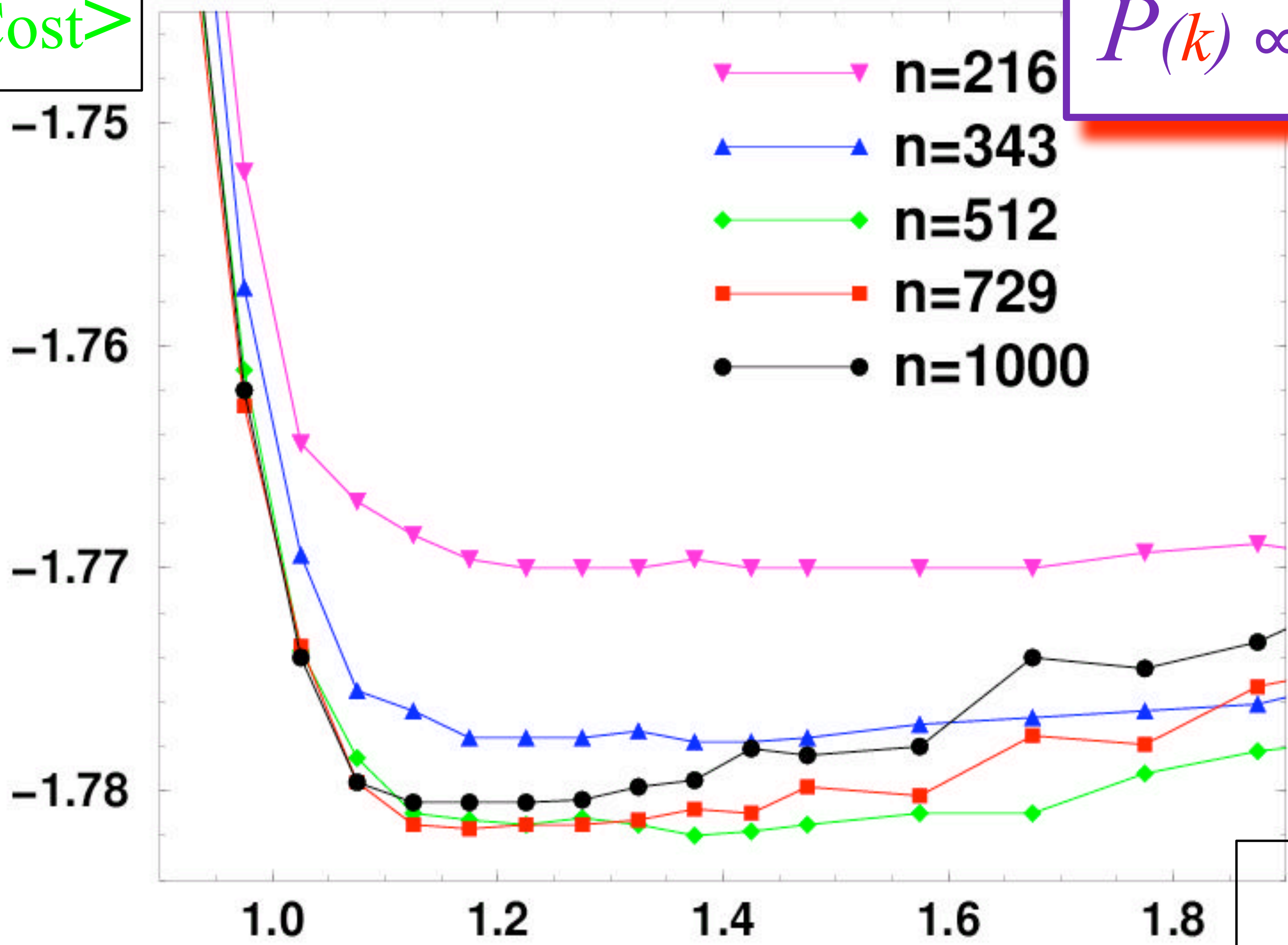

τ

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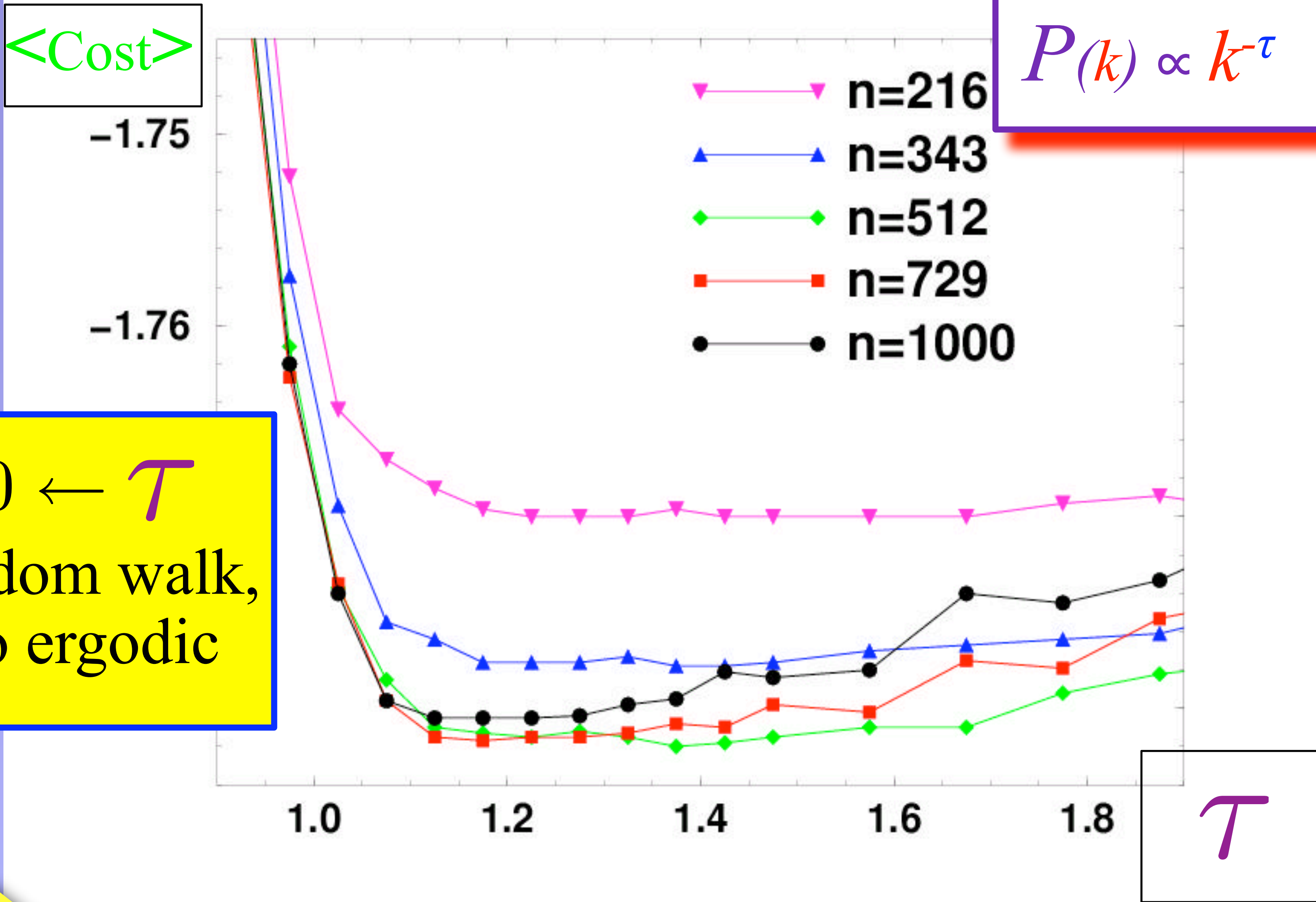

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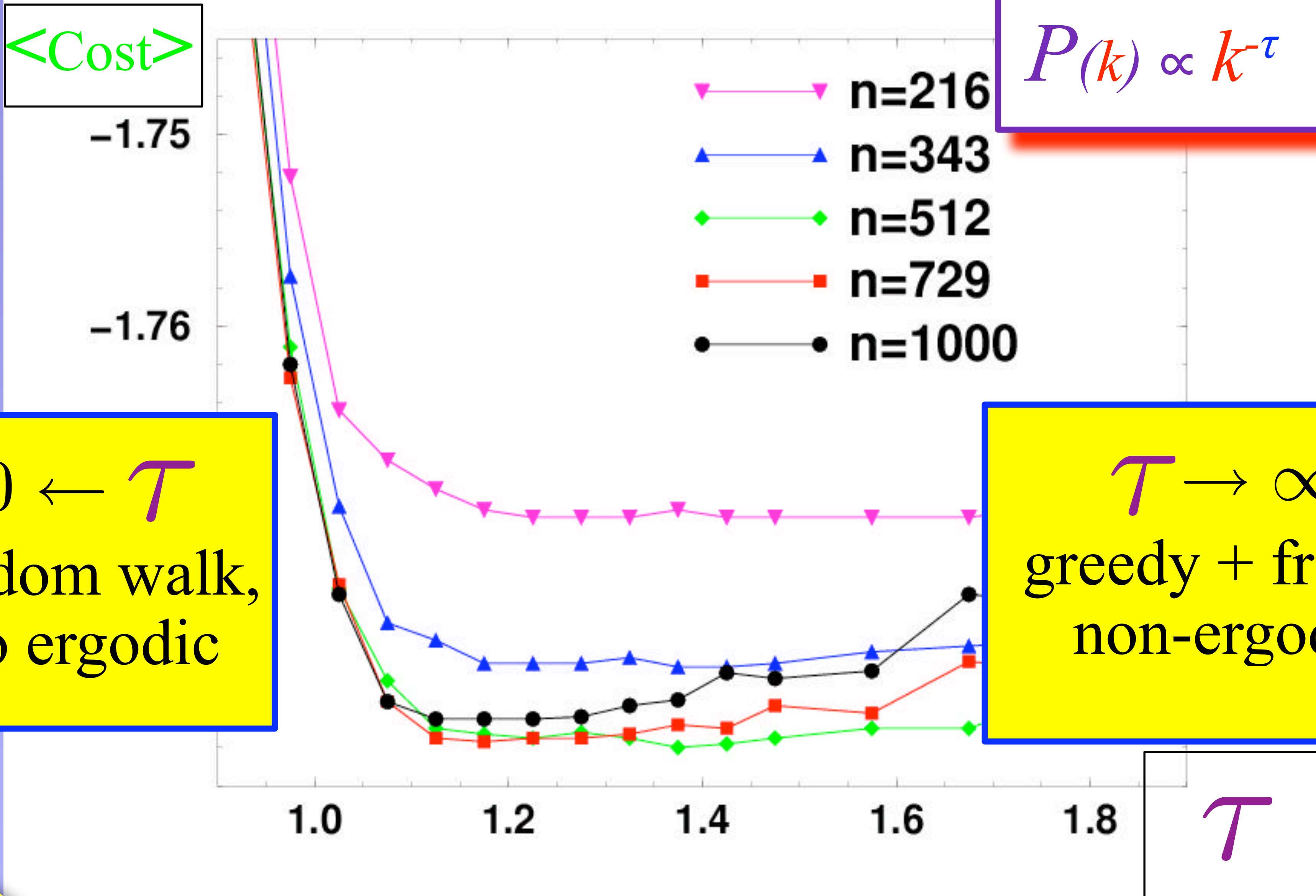

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$\tau \rightarrow \infty$
greedy + frozen,
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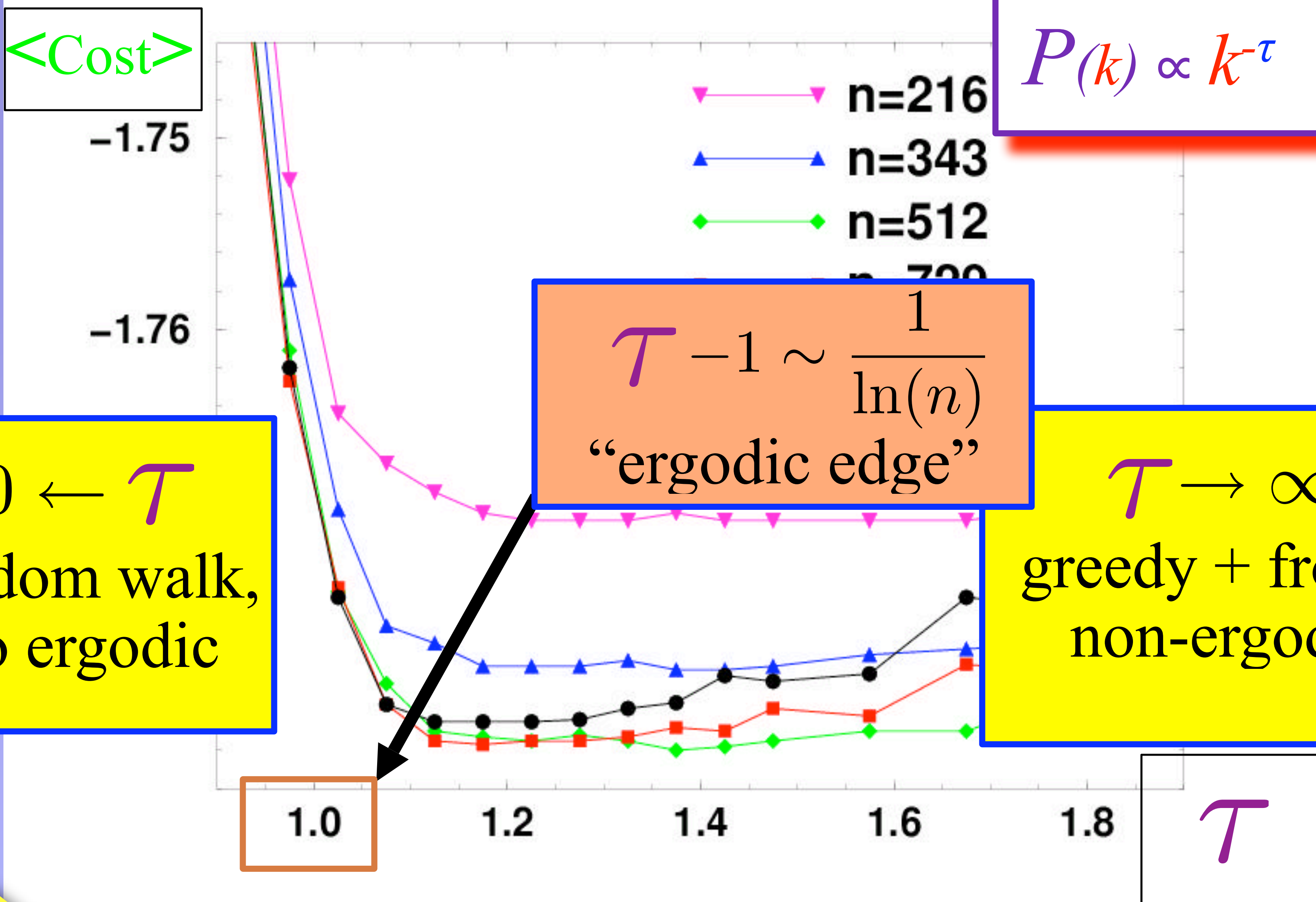

$$\tau - 1 \sim \frac{1}{\ln(n)}$$

"ergodic edge"

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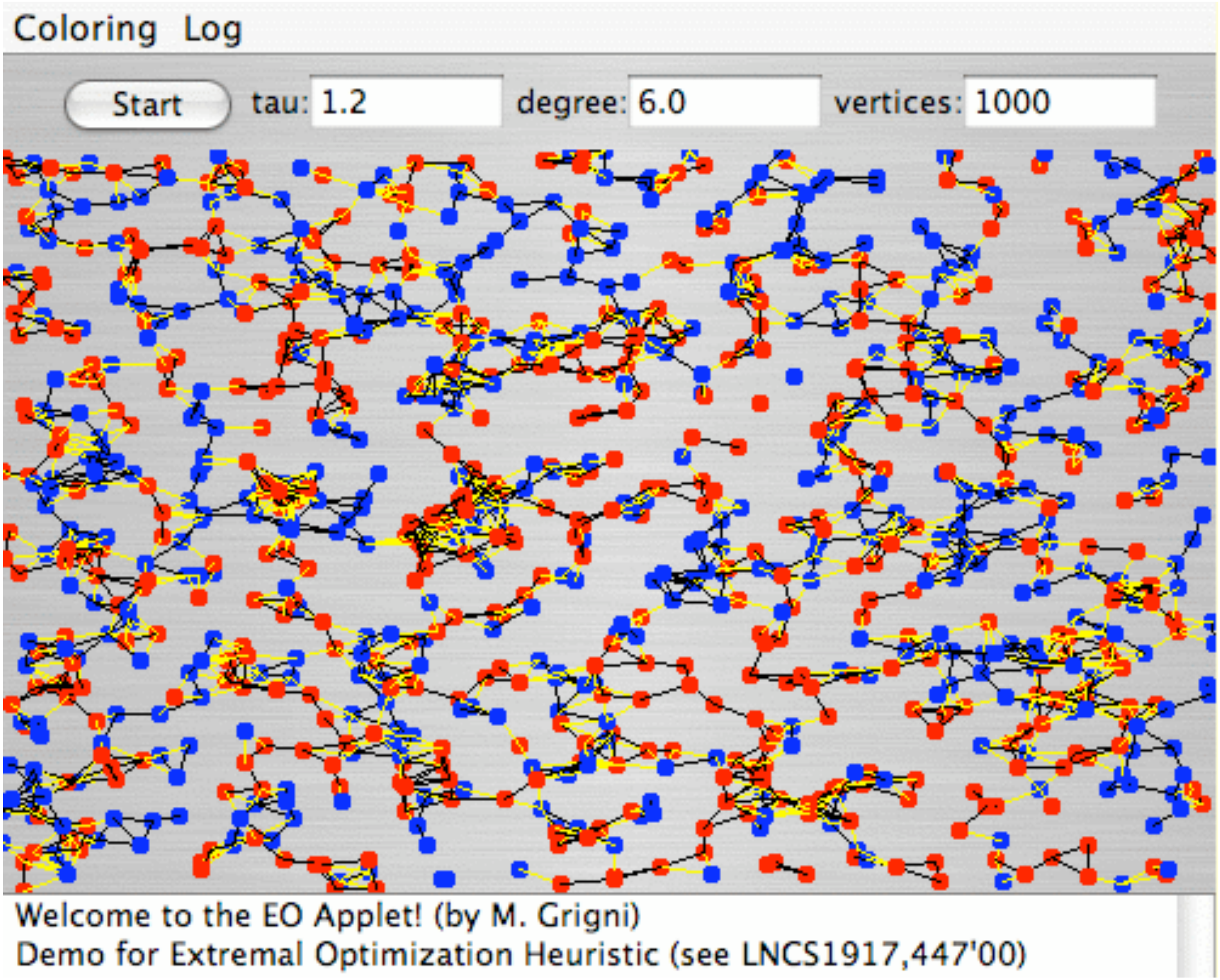
$\tau \rightarrow \infty$
greedy + frozen, non-ergodic

Animation of τ-EO for Graph-Partitioning



Results for τ-EO:

- For Graph Bi-Partitioning:

Graph	Size n	EO	GA	heuristics
Hammond	4720	90 (42s)	90 (1s)	97 (8s)
Barth5	15606	139 (64s)	139 (44s)	146 (28s)
Brack2	62632	731 (12s)	731 (255s)	—
Ocean	143437	464 (200s)	464 (1200s)	499 (38s)

Comparison on Testbed of Graphs [AI**119**('00)275],

- GA by Merz et al. [LNCS**1498**('98)765],
- Spectral Heuristic by Hendrickson et al. [Supercomputing '95].

Results for τ-EO:

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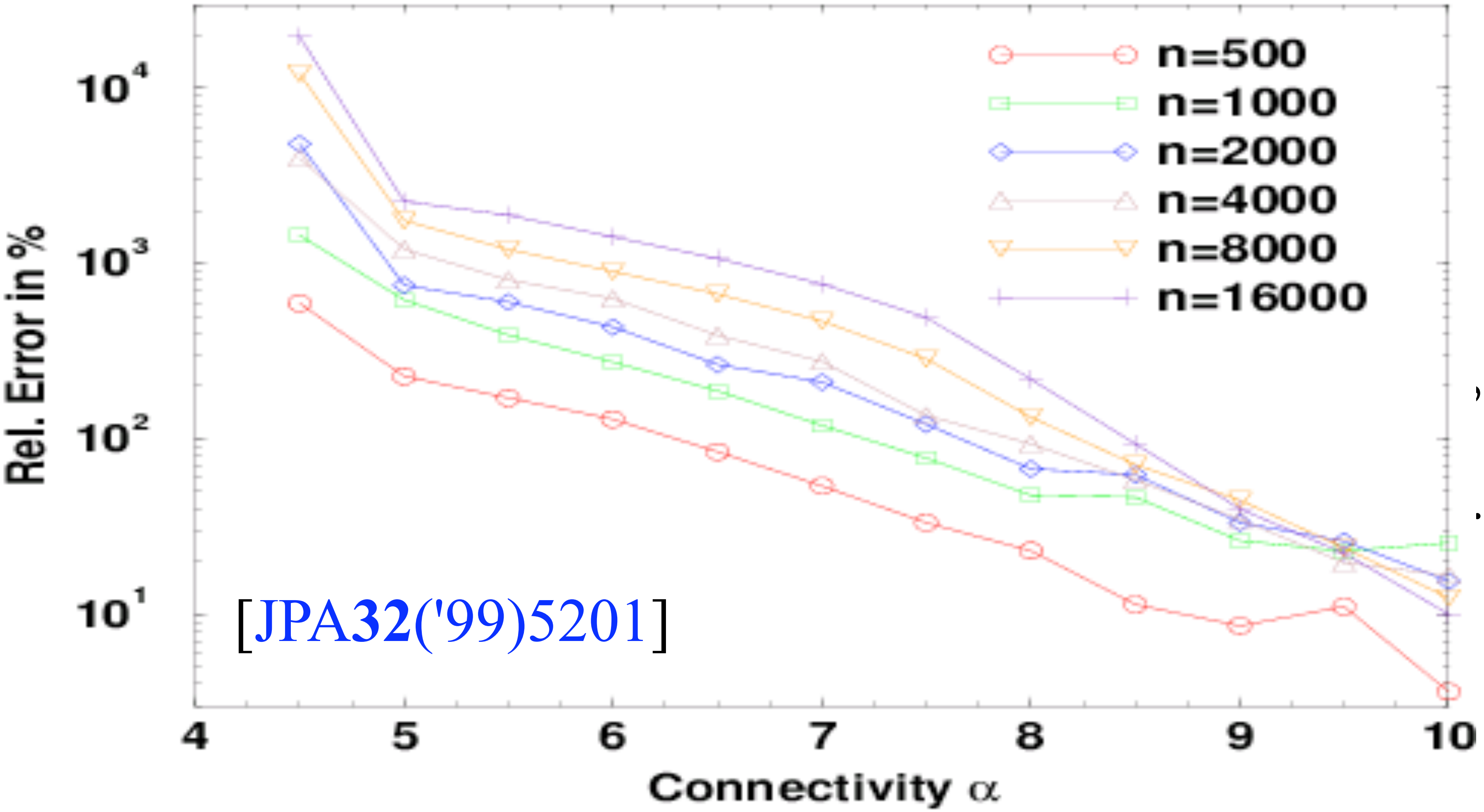
EOvsSA near Percolation for geom. Graphs:



[JPA**32**('99)5201]

Results for τ-EO:

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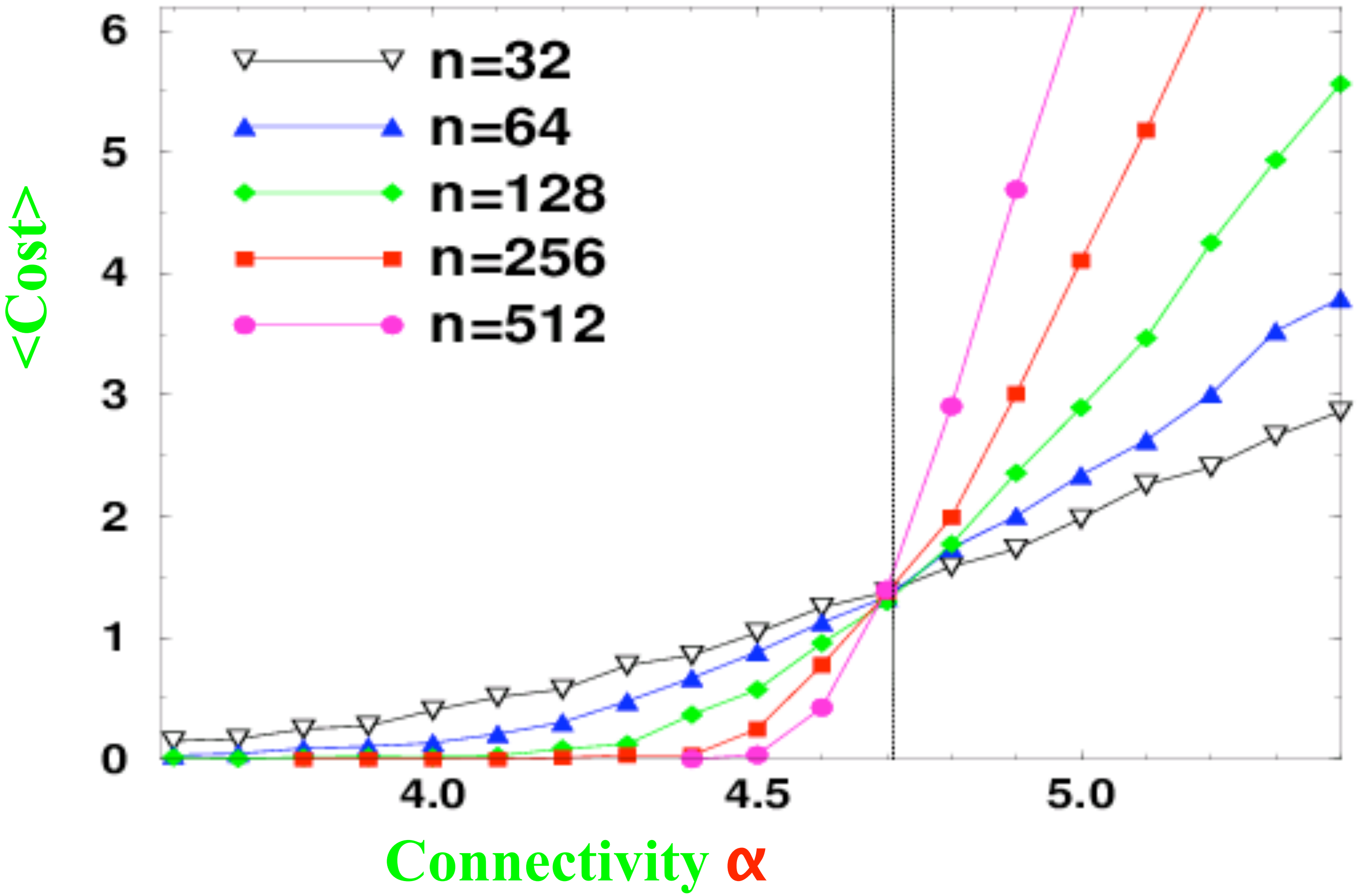

Results for T-EO:

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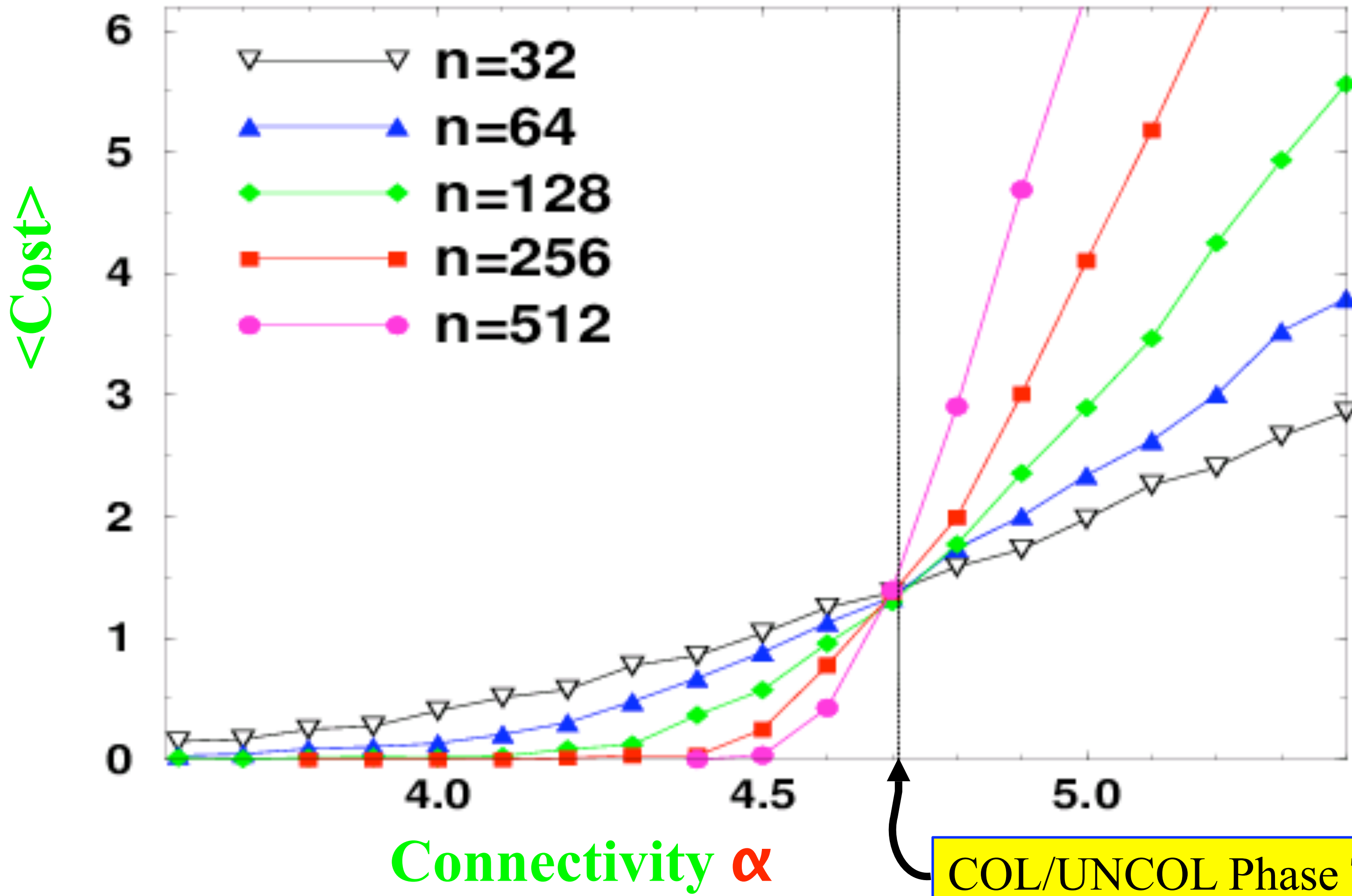

COL/UNCOL Phase Transition:
Theory: $\alpha_c = 4.69$
Here: $\alpha_c = 4.70$

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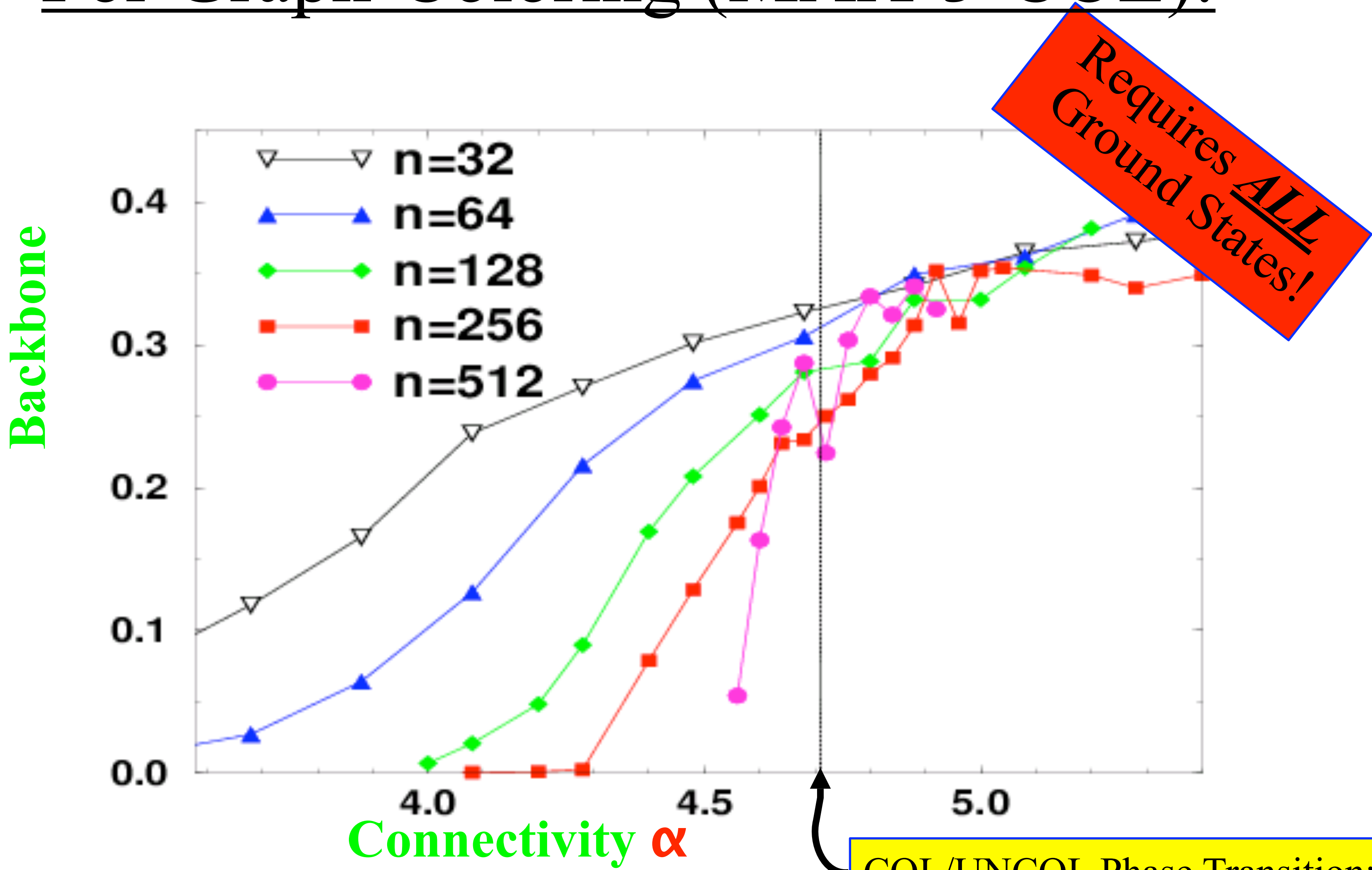

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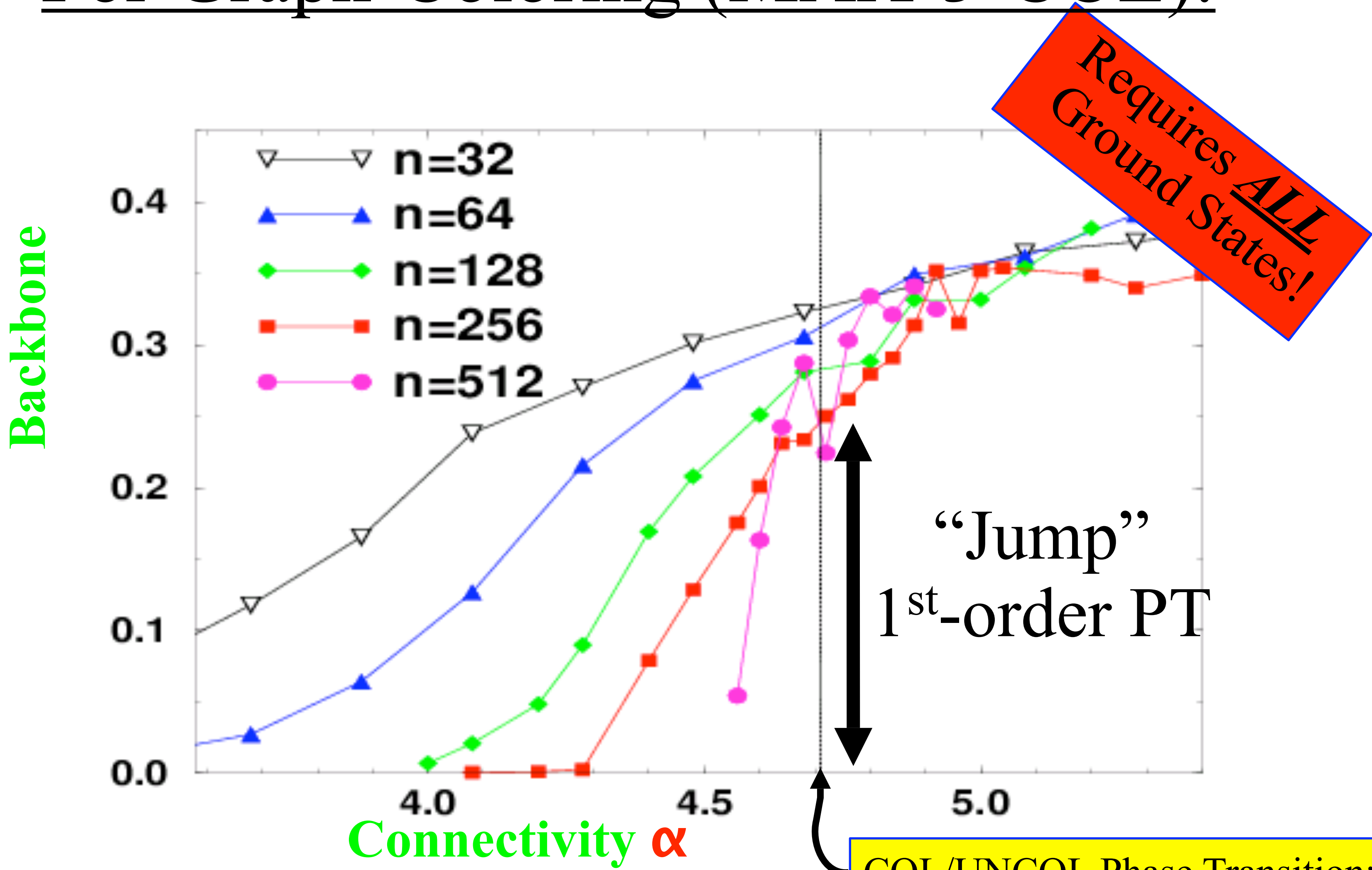

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Results for T-EO:

•For Spin Glasses:

EO for *3d*-Lattice Spin Glasses [PRL**86**('01)5211]

L	t	E/L^3	Pal96	Hartmann97
3	0.0006	-1.6712(6)	-1.67171(9)	-1.6731(19)
4	0.0071	-1.7377(3)	-1.73749(8)	-1.7370(9)
5	0.0653	-1.7609(2)	-1.76090(12)	-1.7603(8)
6	0.524	-1.7712(2)	-1.77130(12)	-1.7723(7)
7	3.87	-1.7764(3)	-1.77706(17)	
8	22.1	-1.7796(5)	-1.77991(22)	-1.7802(5)
9	100	-1.7822(5).		
10	424.	-1.7832(5)	-1.78339(27)	-1.7840(4)
12	9720.	-1.7857(16)	-1.78407(121)	-1.7851(4)
∞	$O(n^4)$	-1.7865(3)	-1.7863(4)	-1.7868(3)

Genetic Algorithms by Pal [PhysicaA**223**('96)283]
and by Hartmann [EPL**40**('97)492]

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Defect-Energy:

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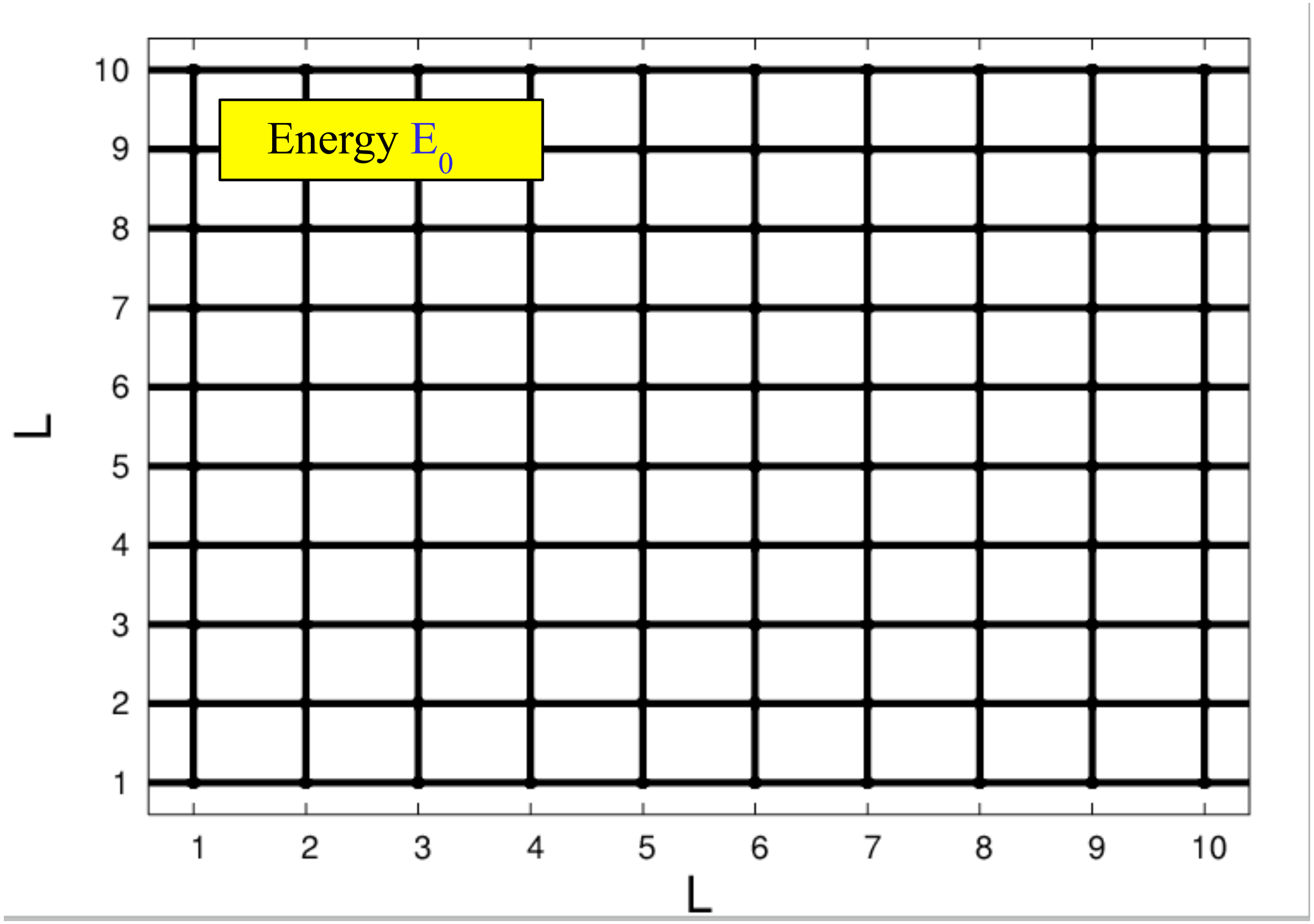

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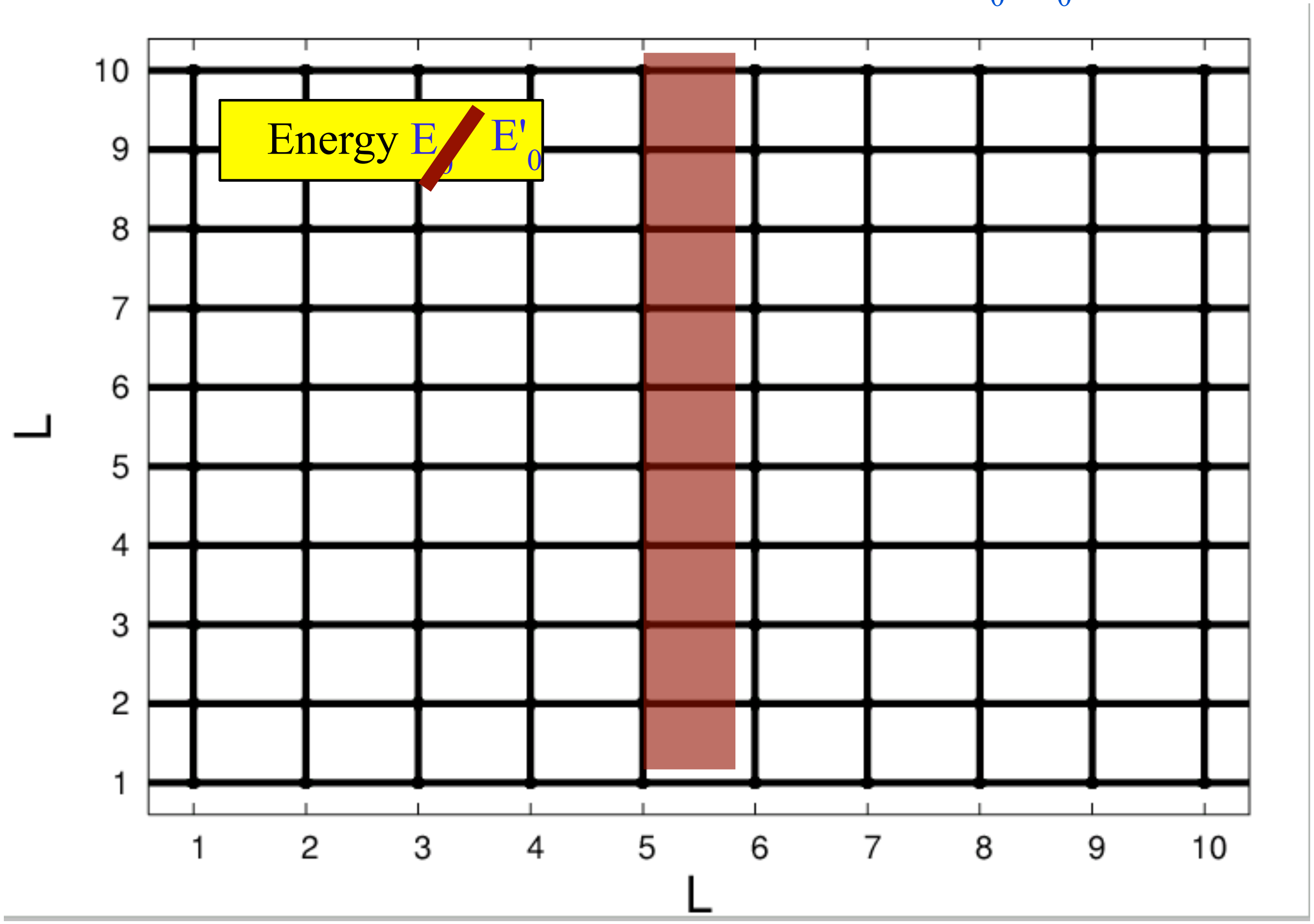

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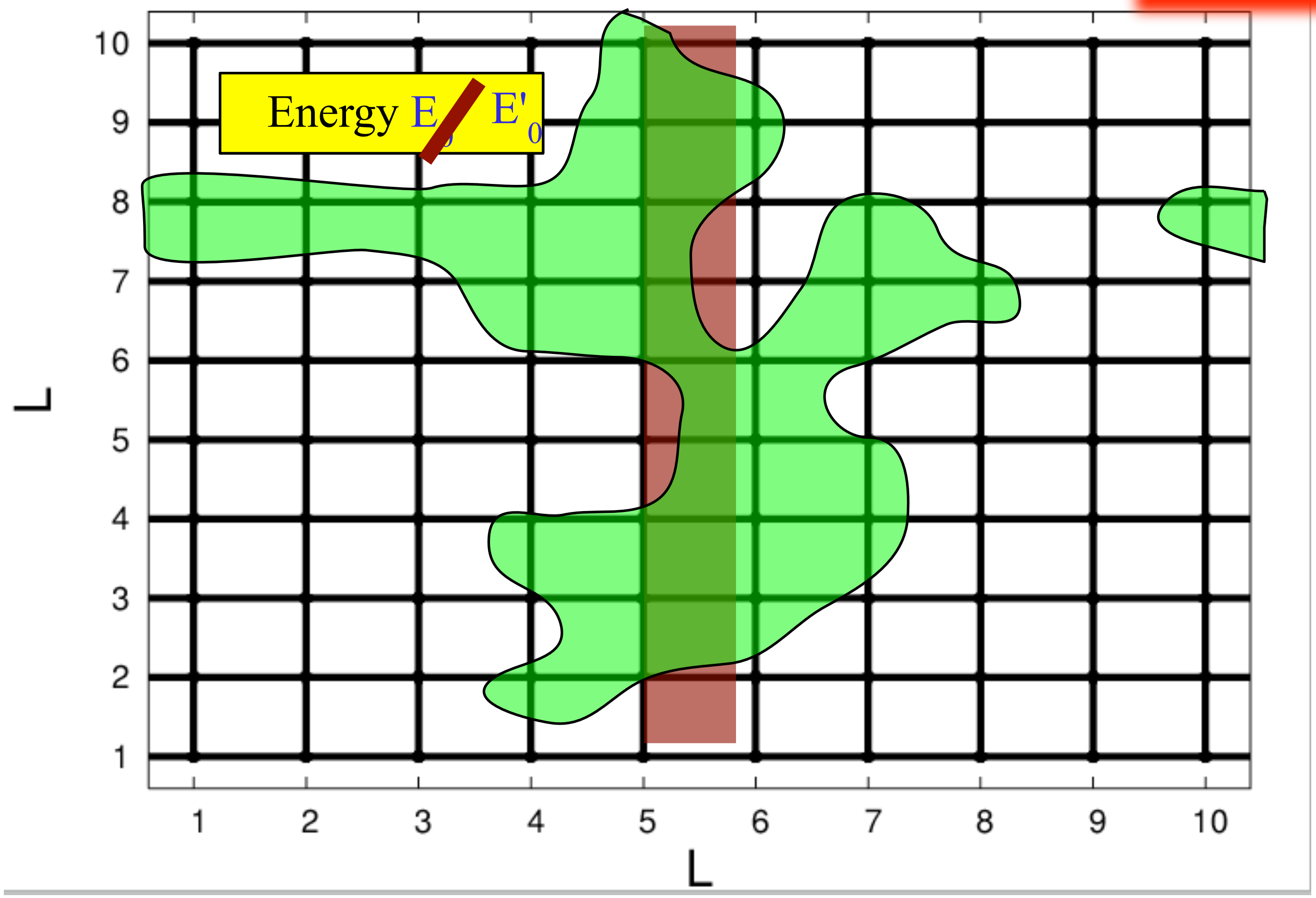

⇒ Low Energy Excitations (like "small oscillations")

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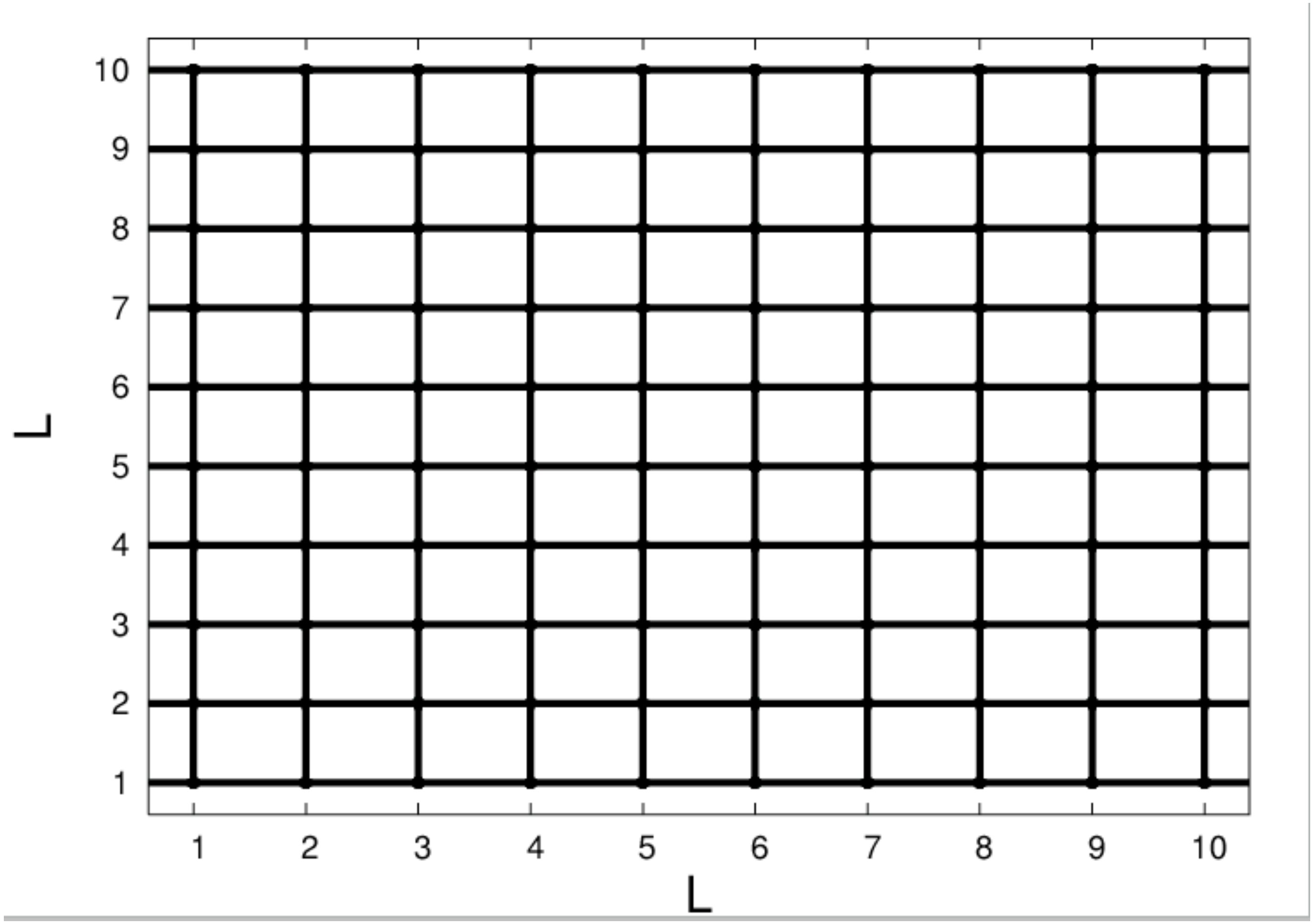

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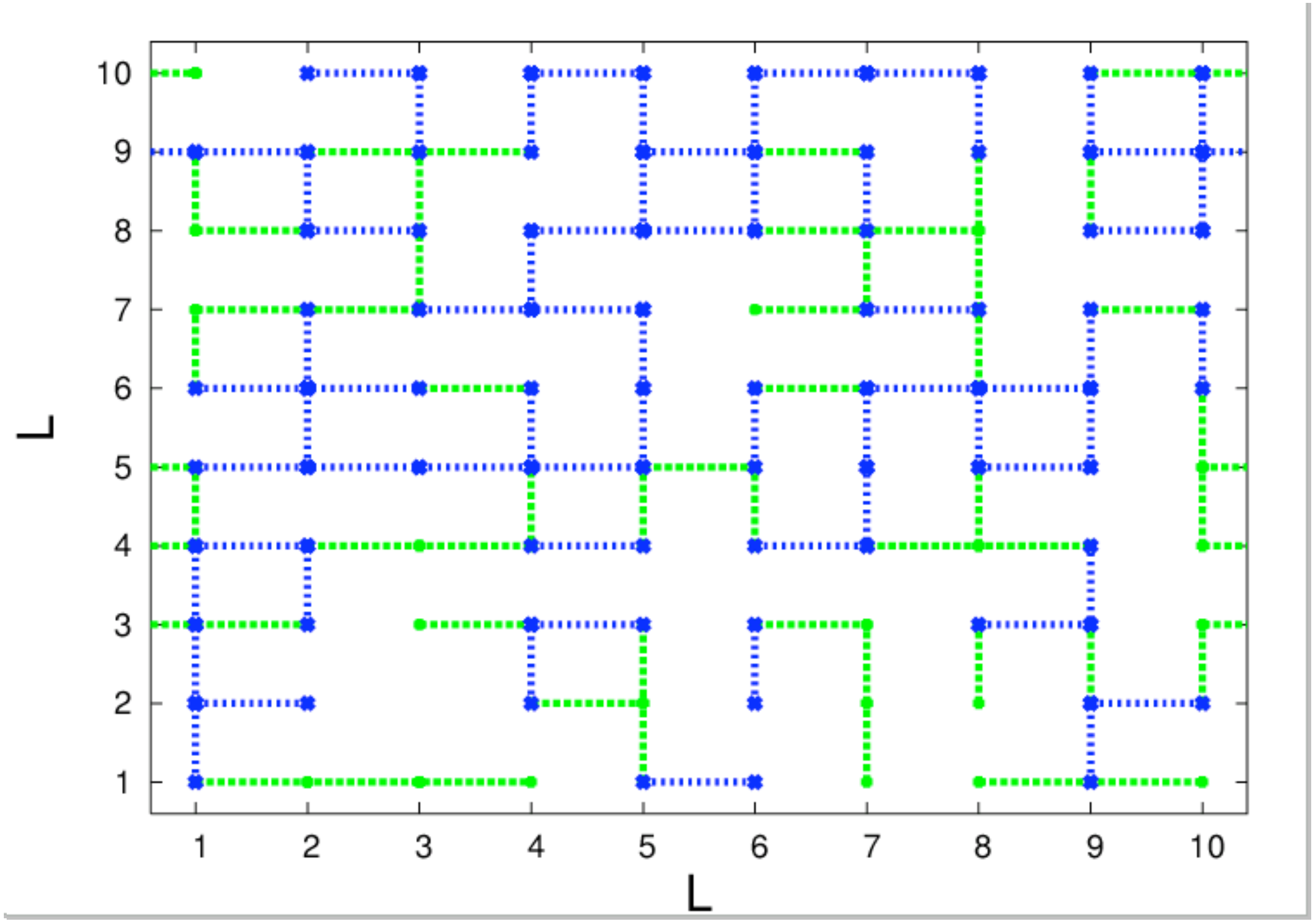

Before:
100 Spins

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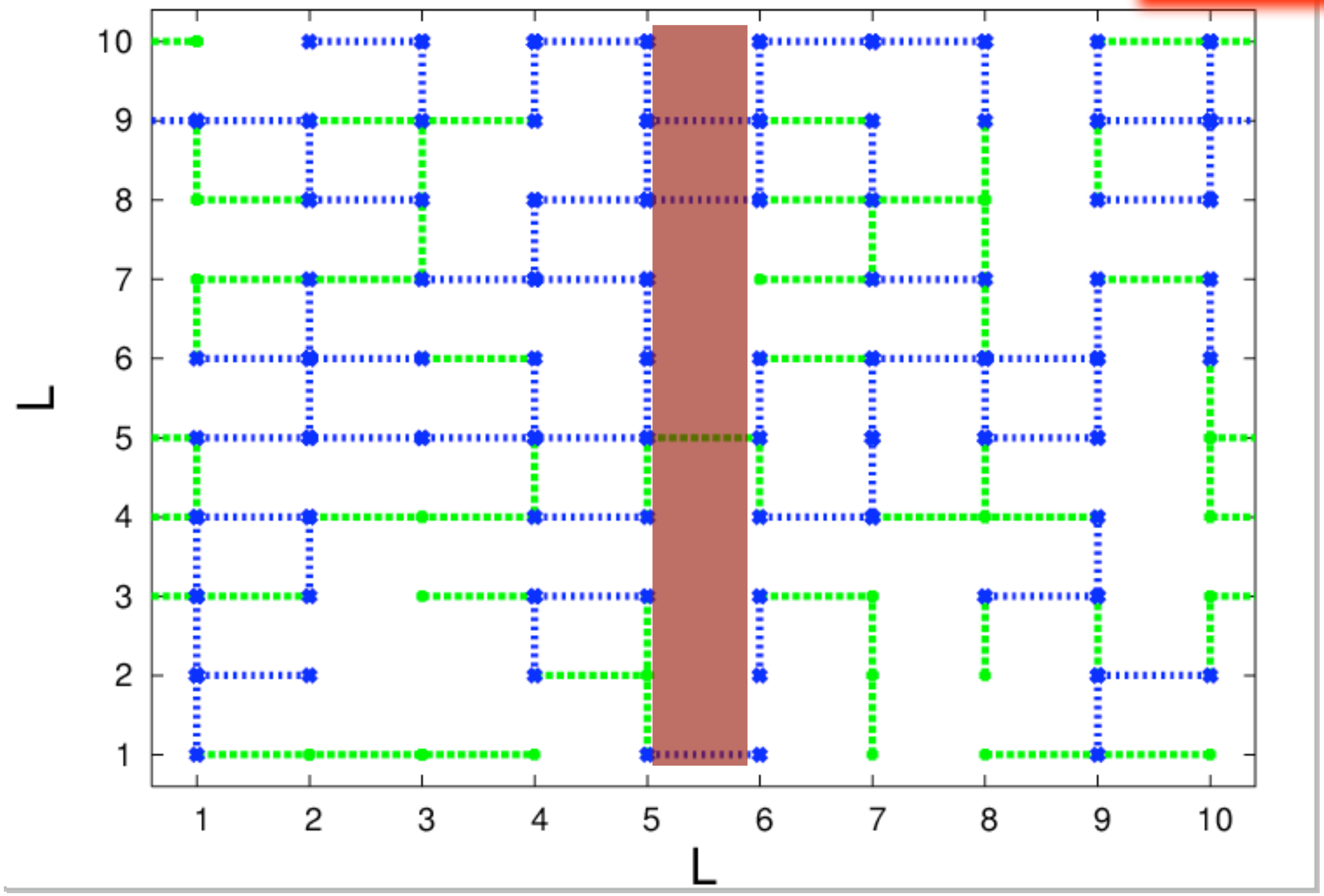

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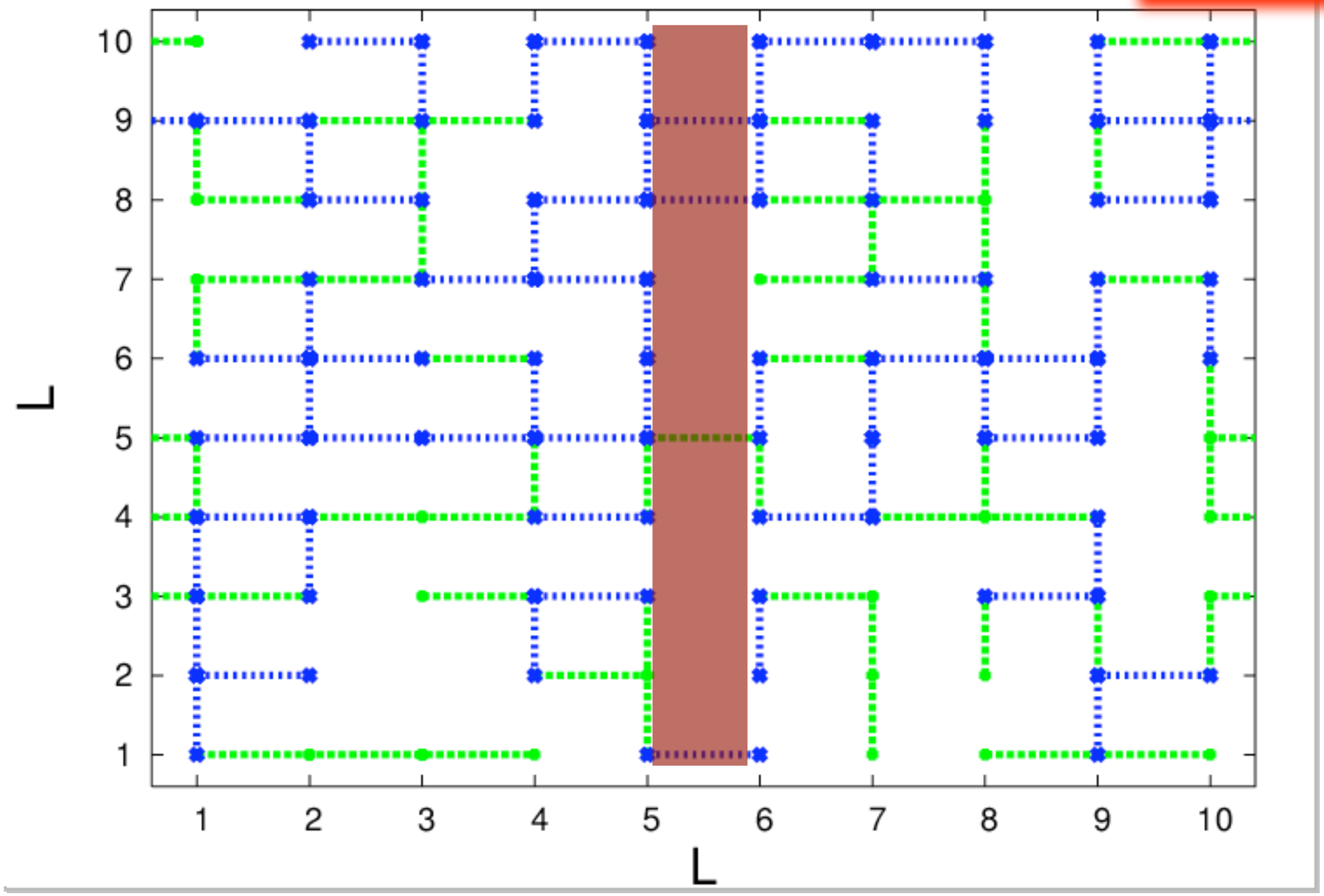

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⇒Low Energy Excitations of bond-diluted Lattices

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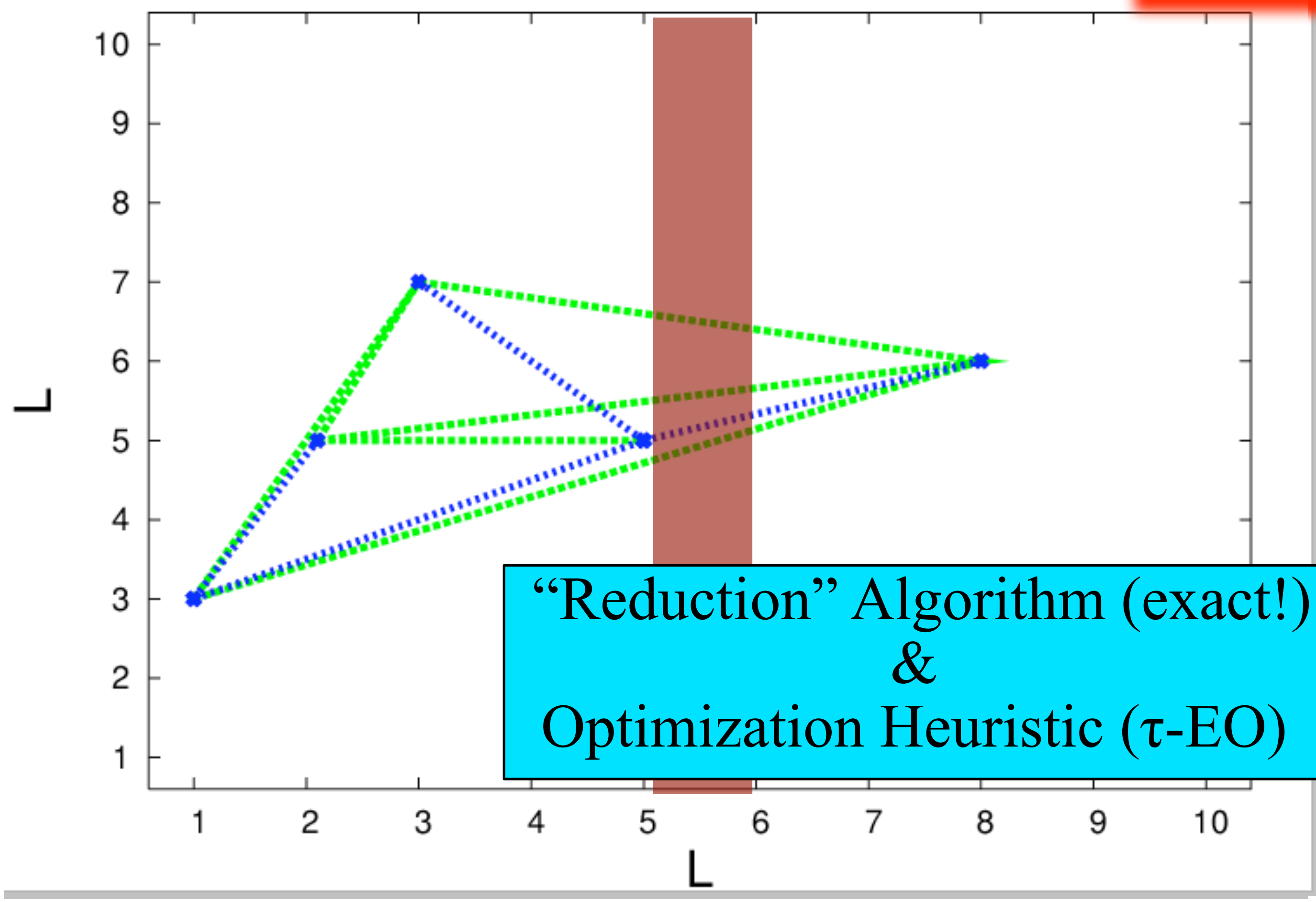

"Reduction" Algorithm (exact!)
&
Optimization Heuristic (τ-EO)

Before:
100 Spins

After:
5 Spins

⇒Low Energy Excitations of bond-diluted Lattices

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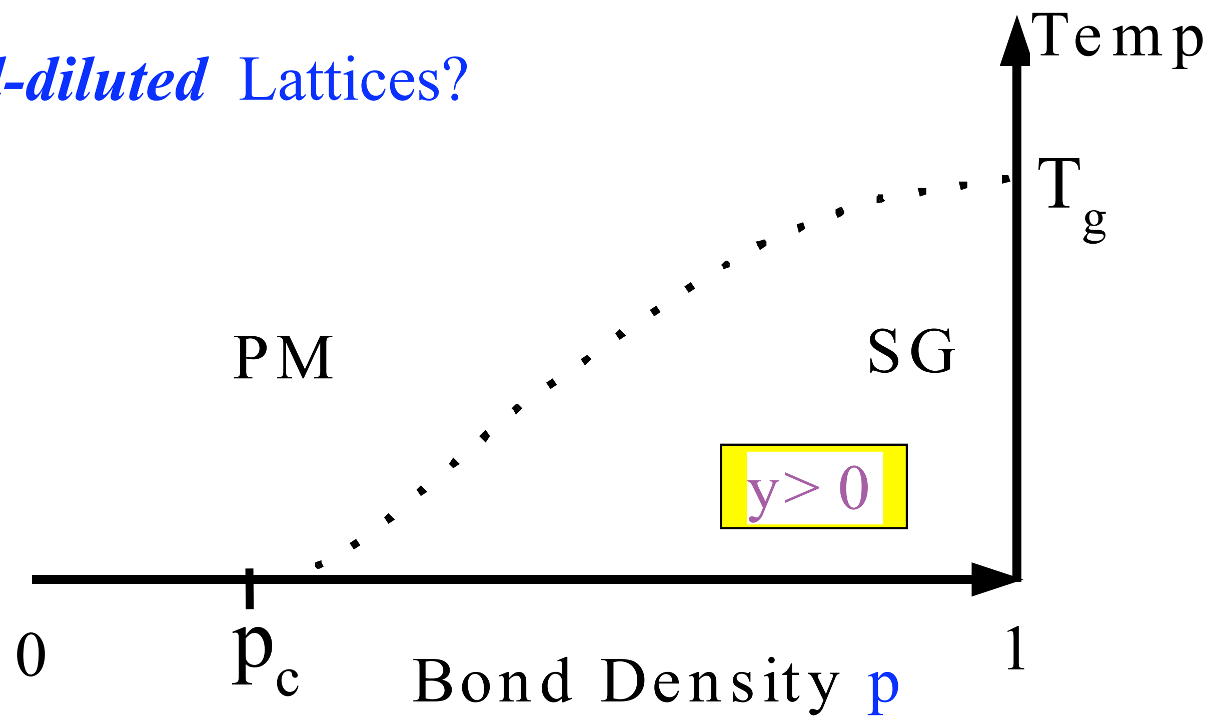

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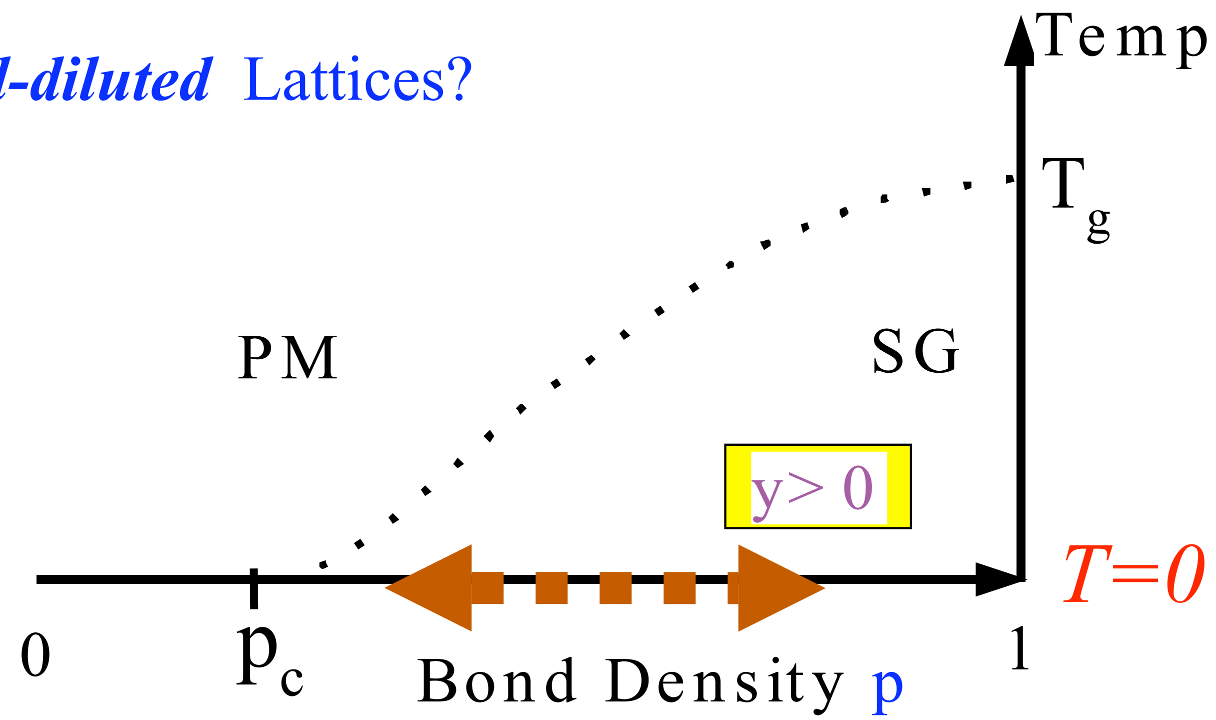

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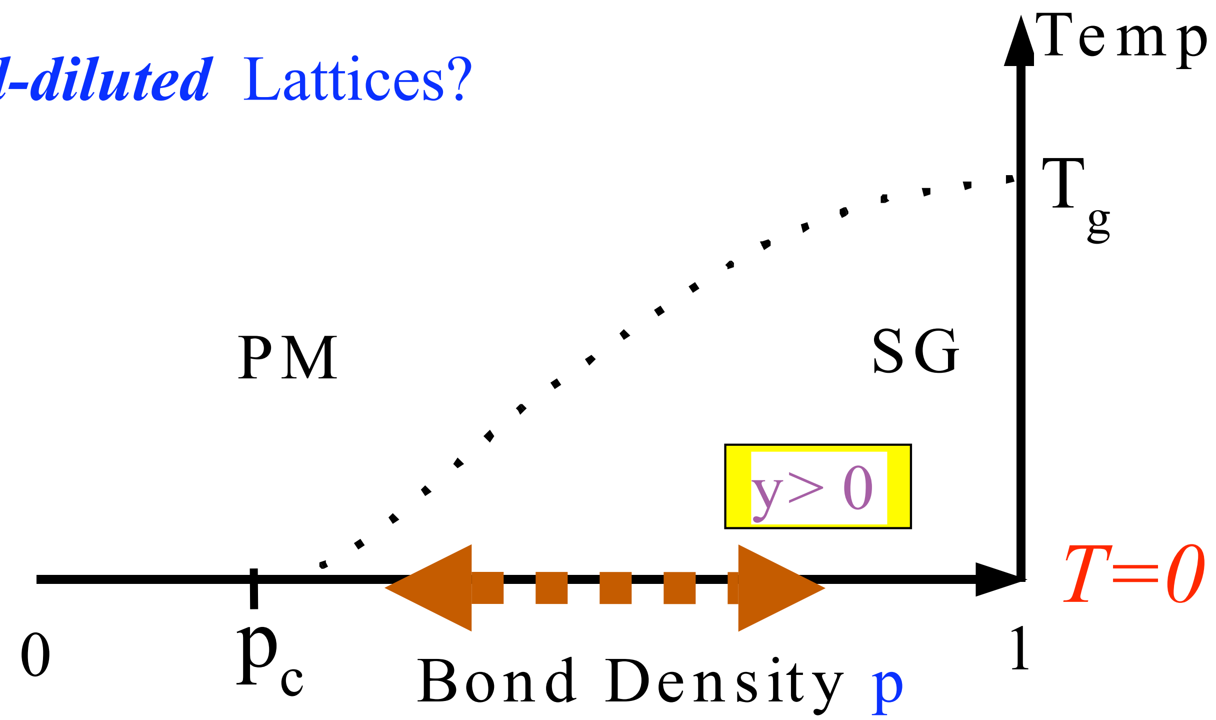

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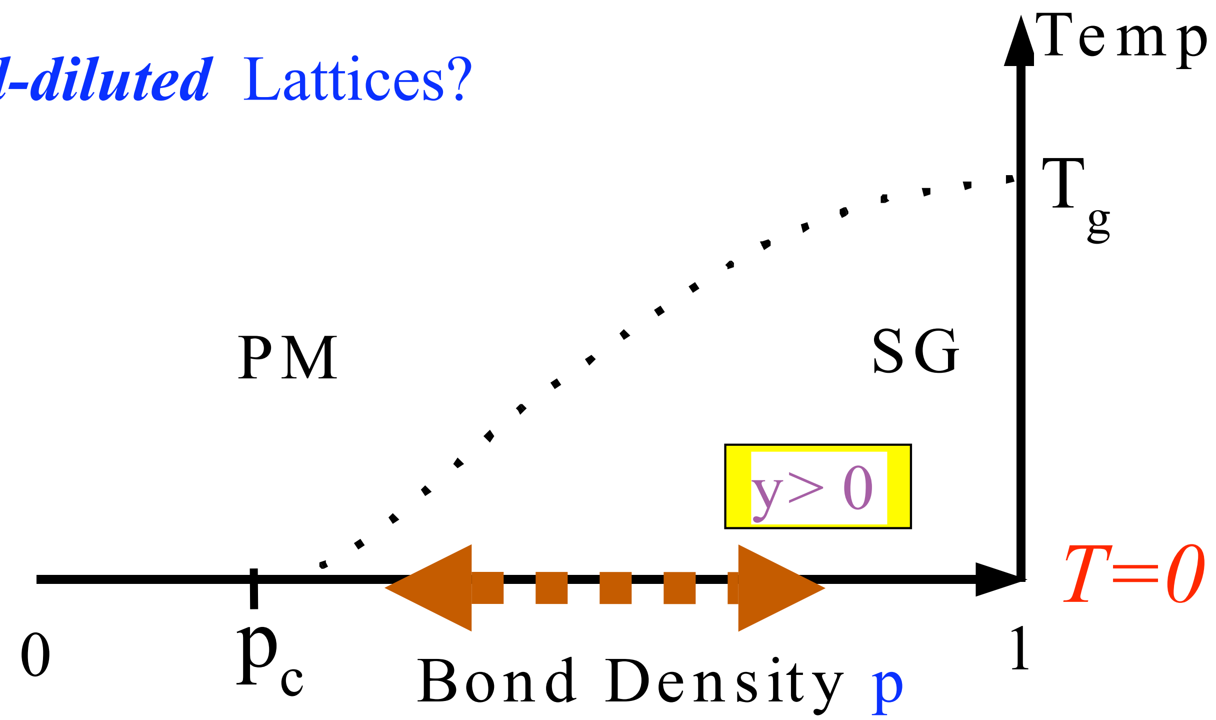

***Why** bond-diluted* Lattices?

- Simpler Problem
- Larger Sizes L
- Better Scaling

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- ±J-Glasses on Lattices of size L and density p.
- Defect-Energy $\sigma(\Delta E)$ with Reduction & Heuristic (τ-EO).

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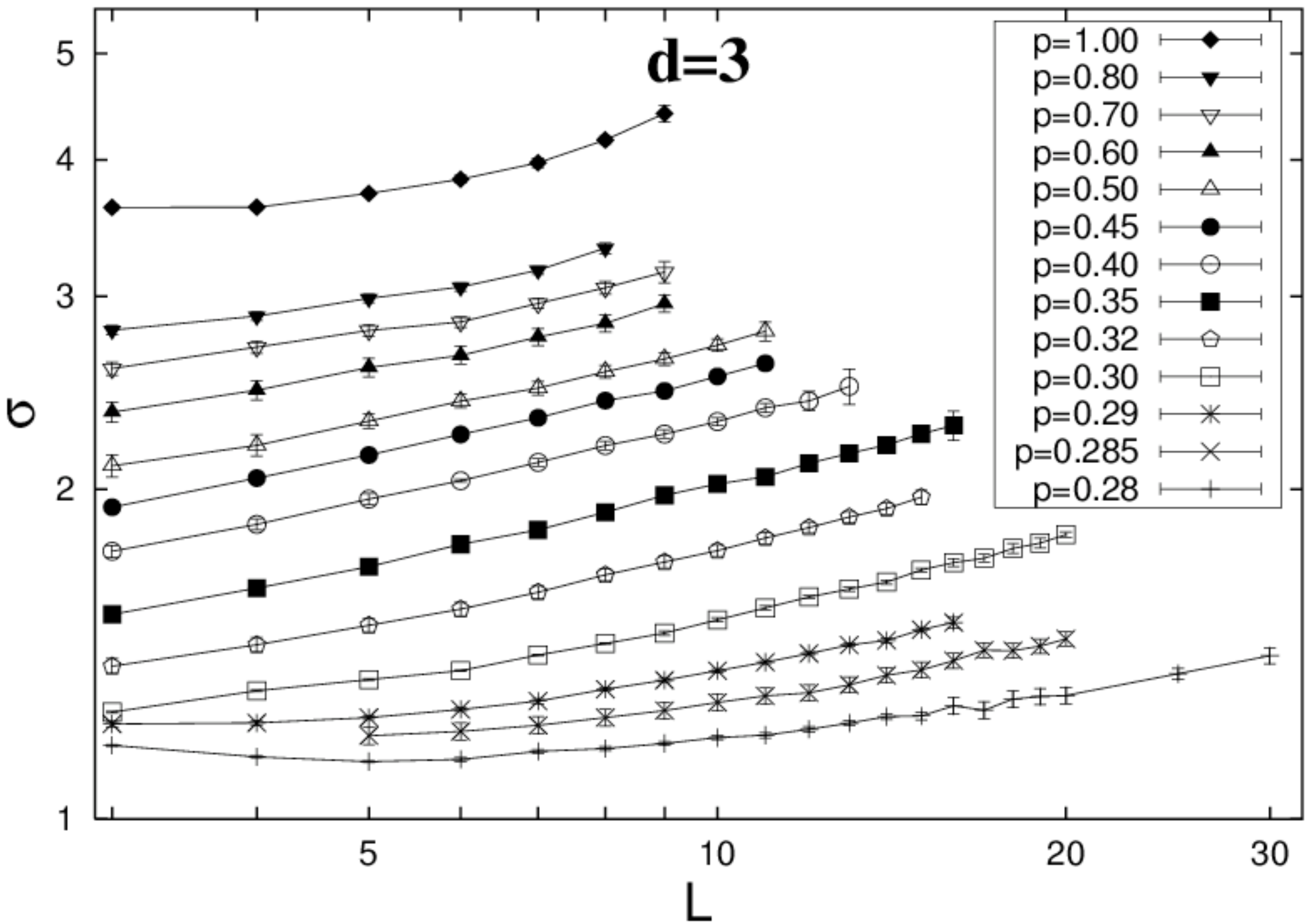
- ±J-Glasses on Lattices of size L and density p.
- Defect-Energy σ(ΔE) with Reduction & Heuristic (τ-EO).



Stiffness Exponent *y* for Lattice Glasses:

"Stiffness": $\sigma(\Delta E) \sim L^{y}$

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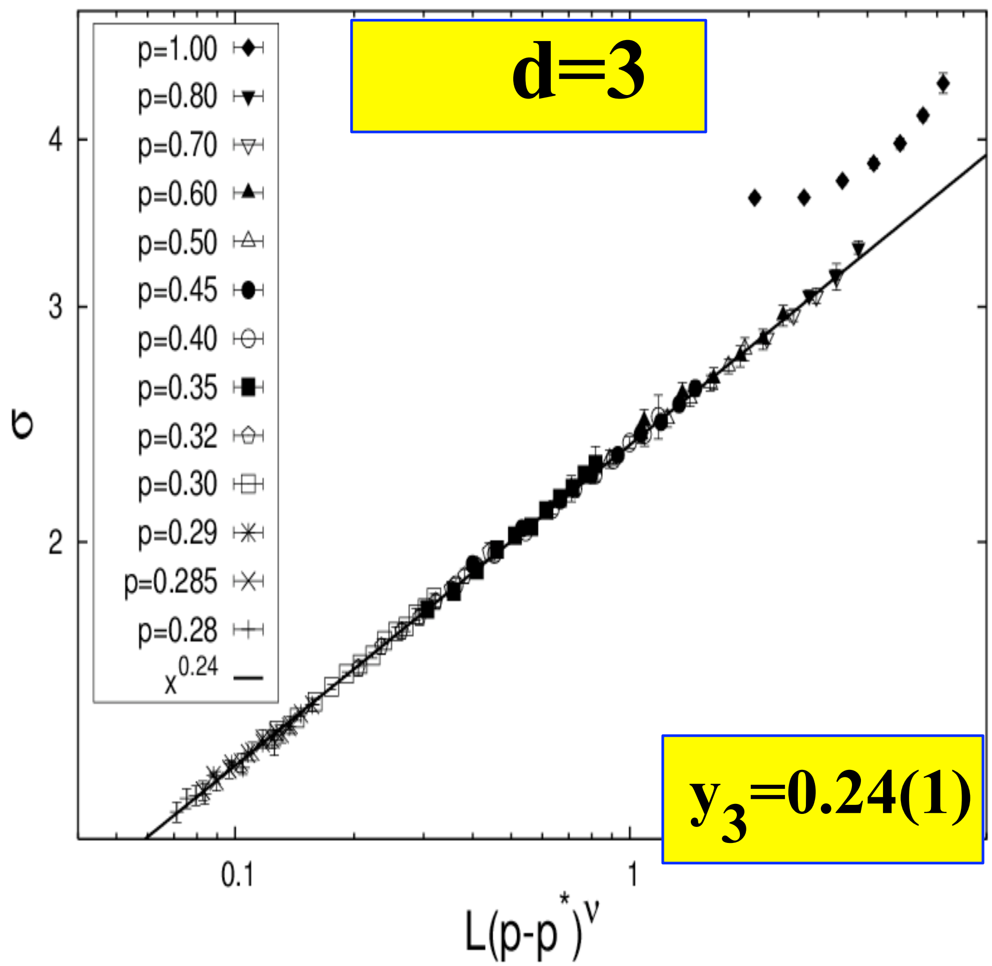

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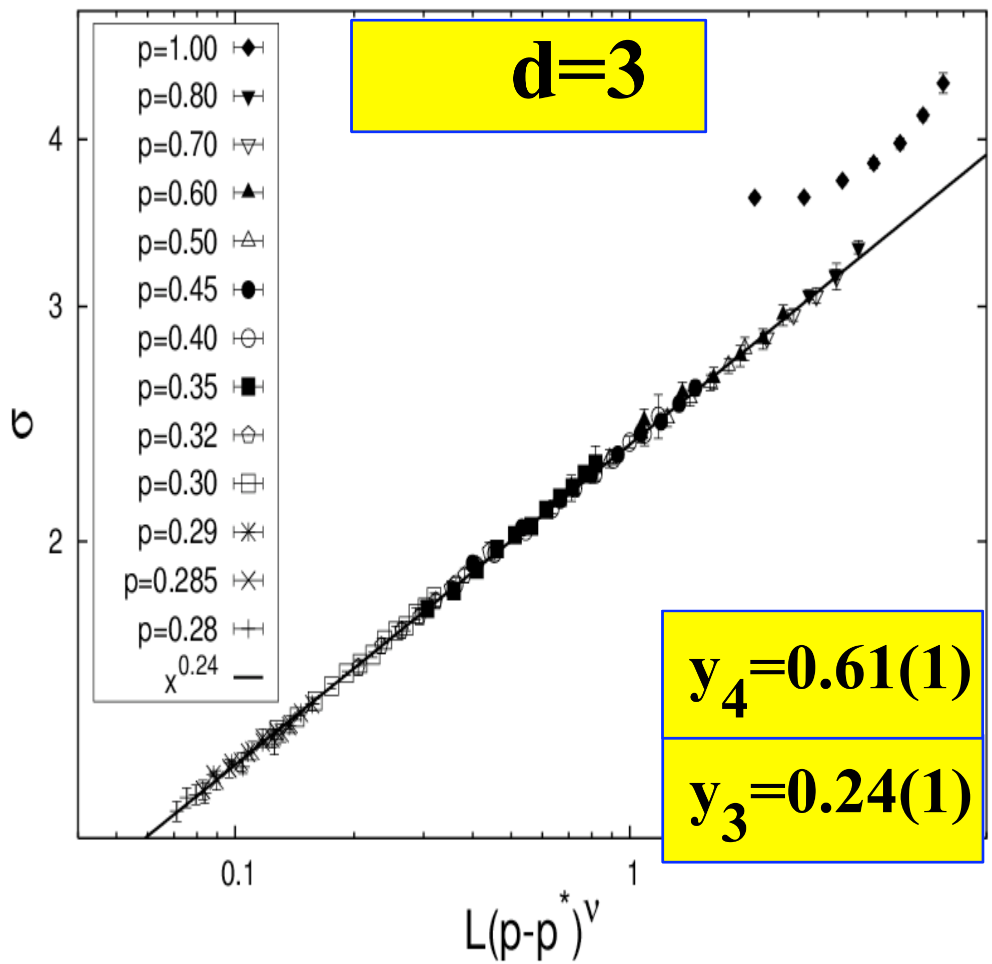

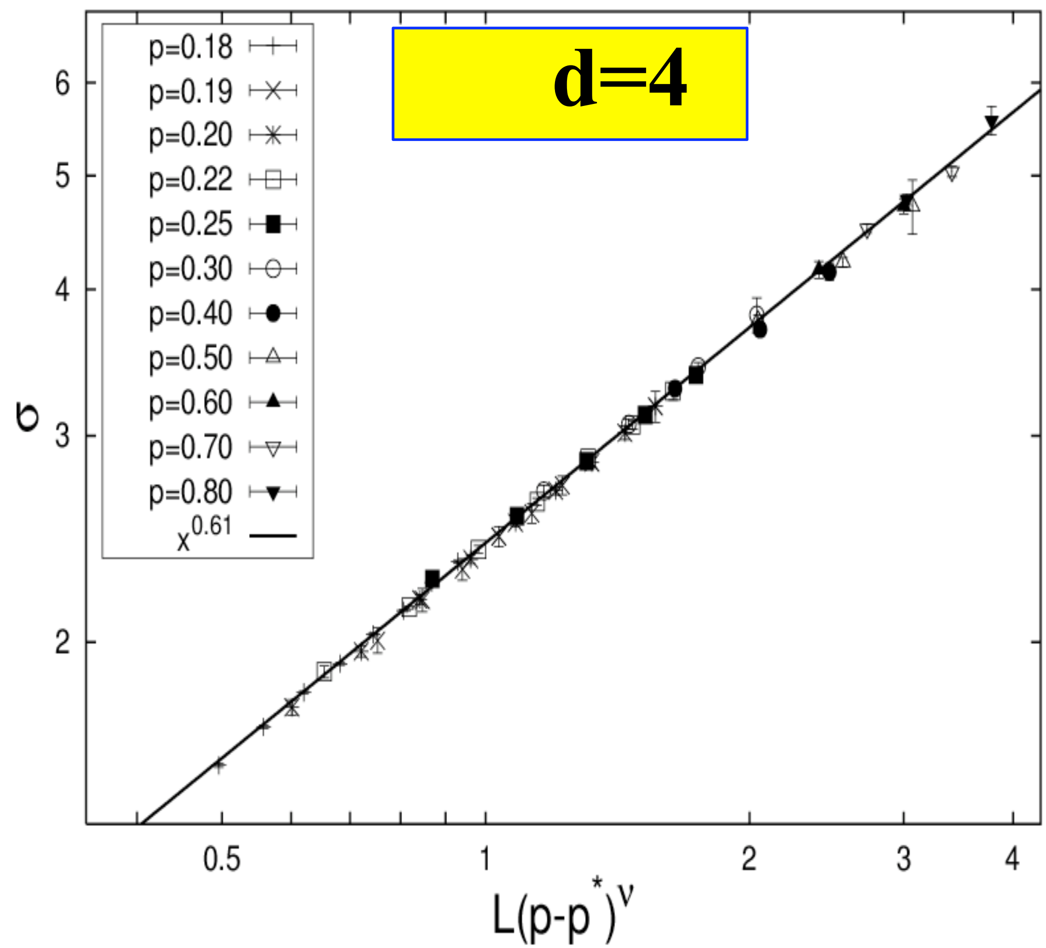

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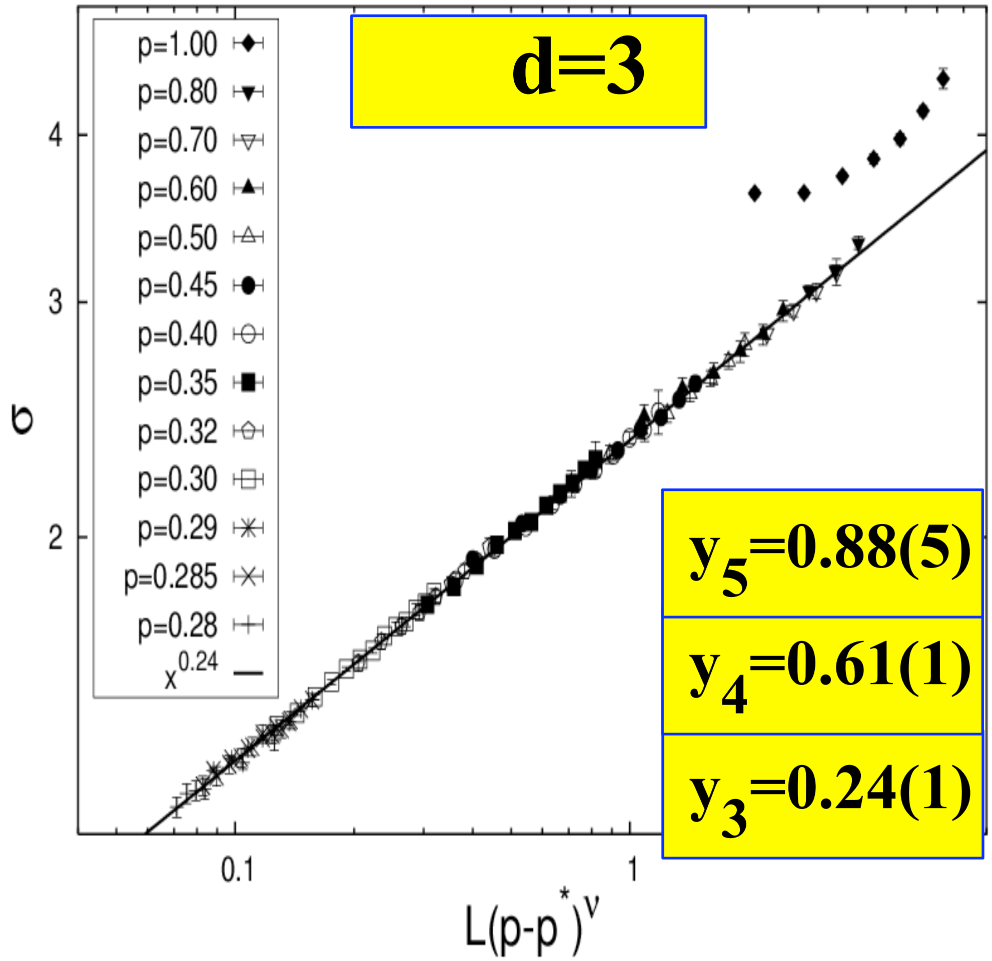

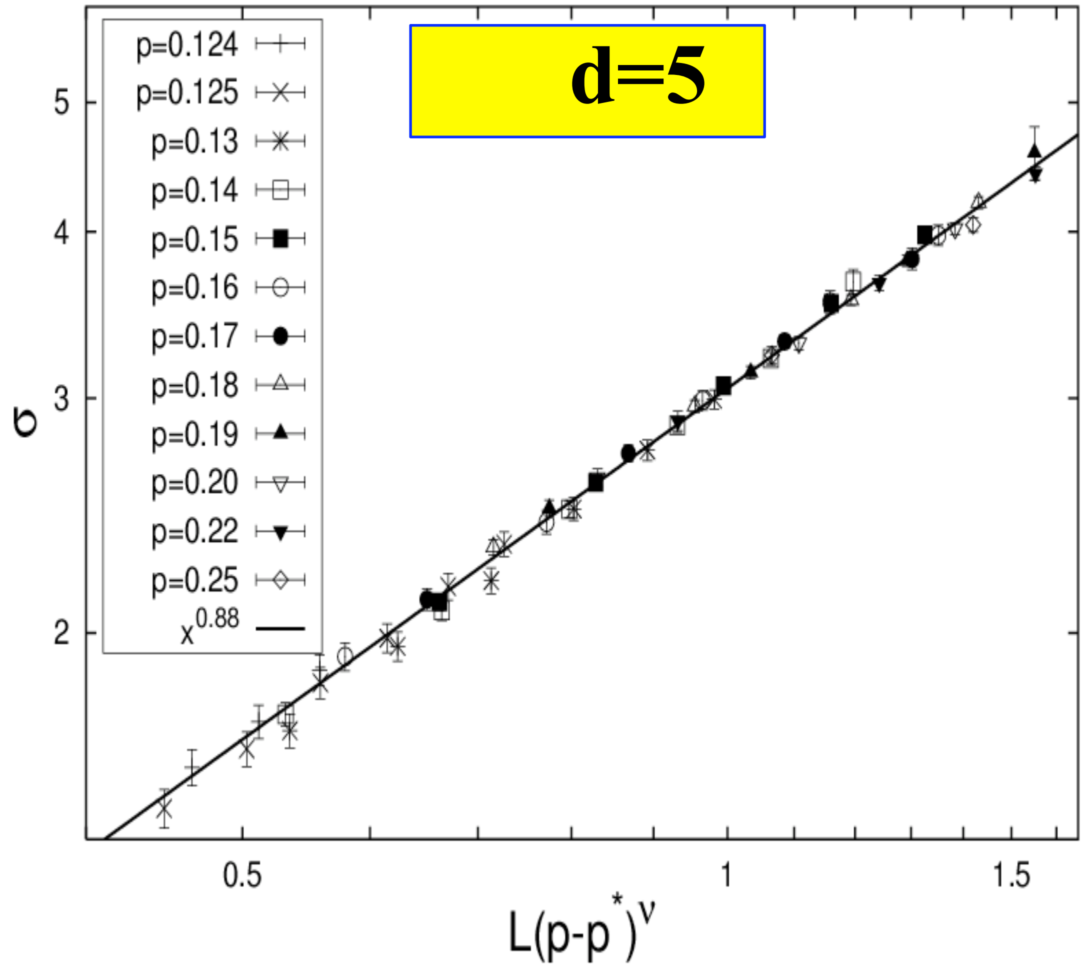

$y_5 = 0.88(5)$

$y_4 = 0.61(1)$

$y_3 = 0.24(1)$

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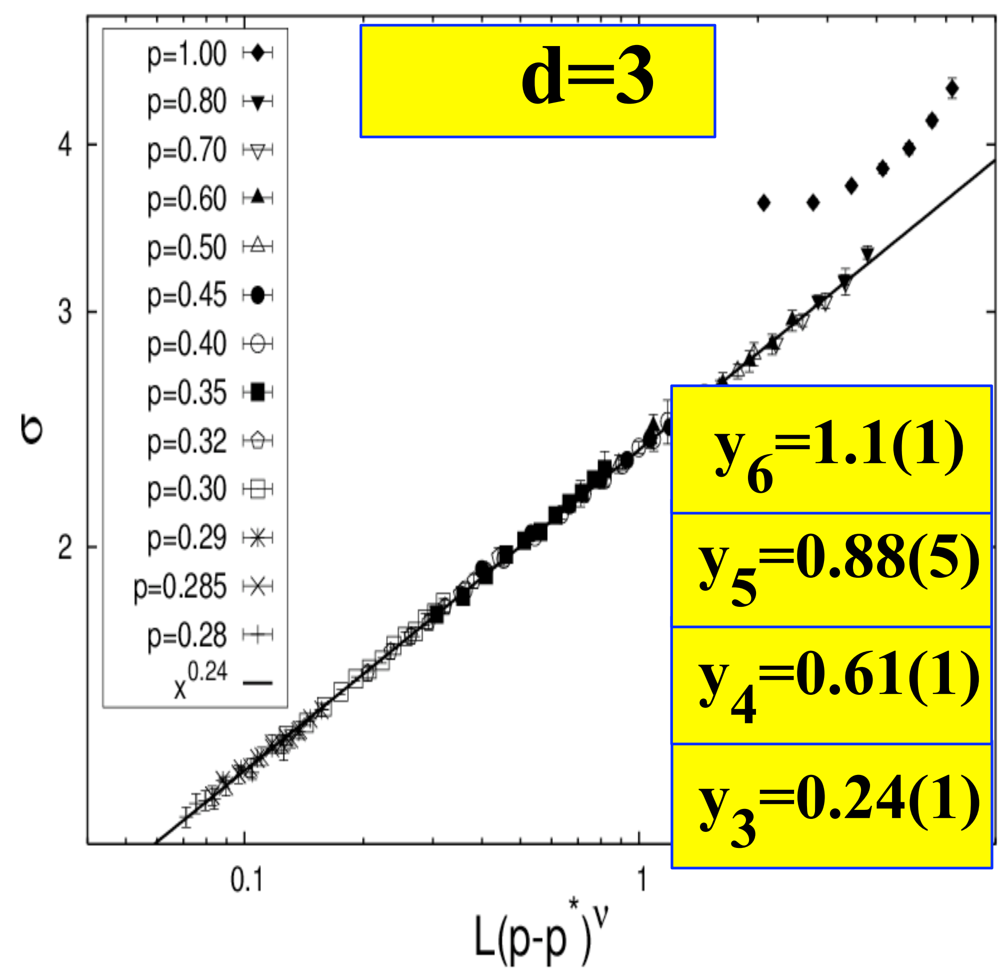

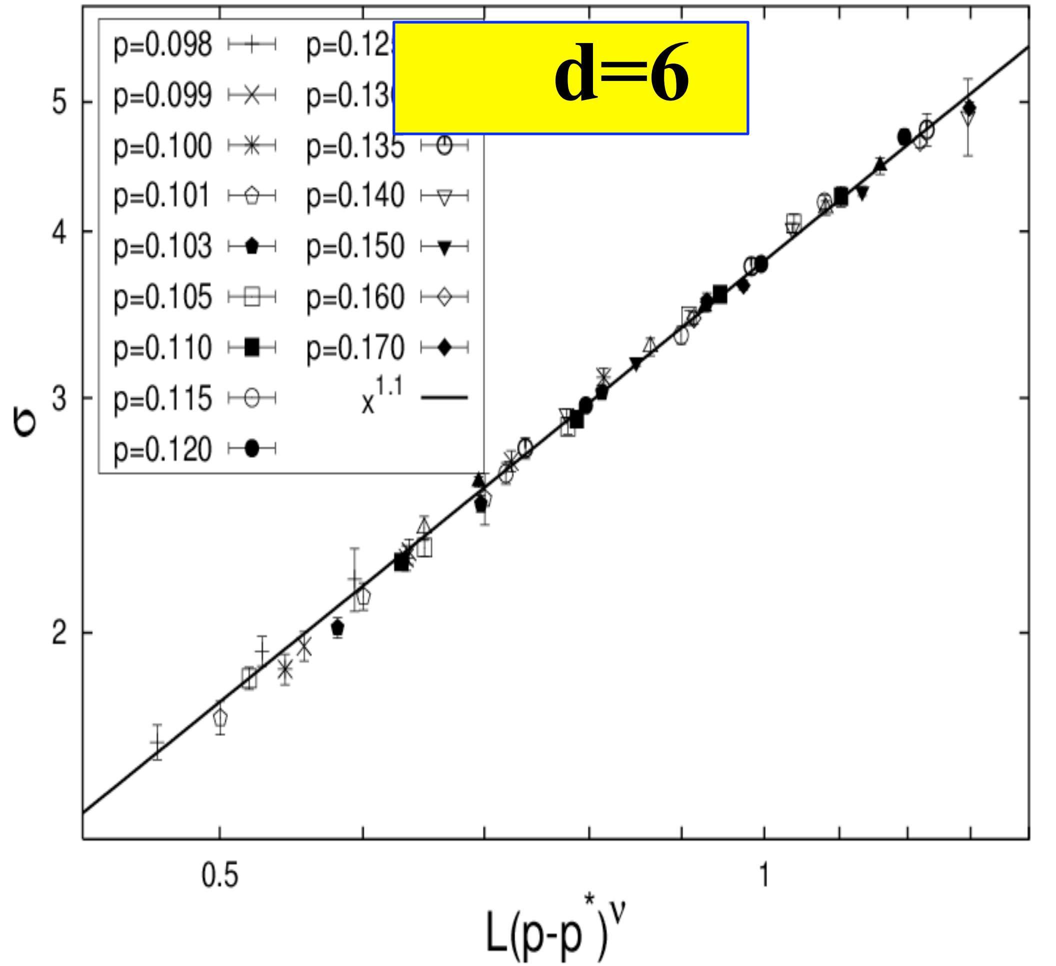

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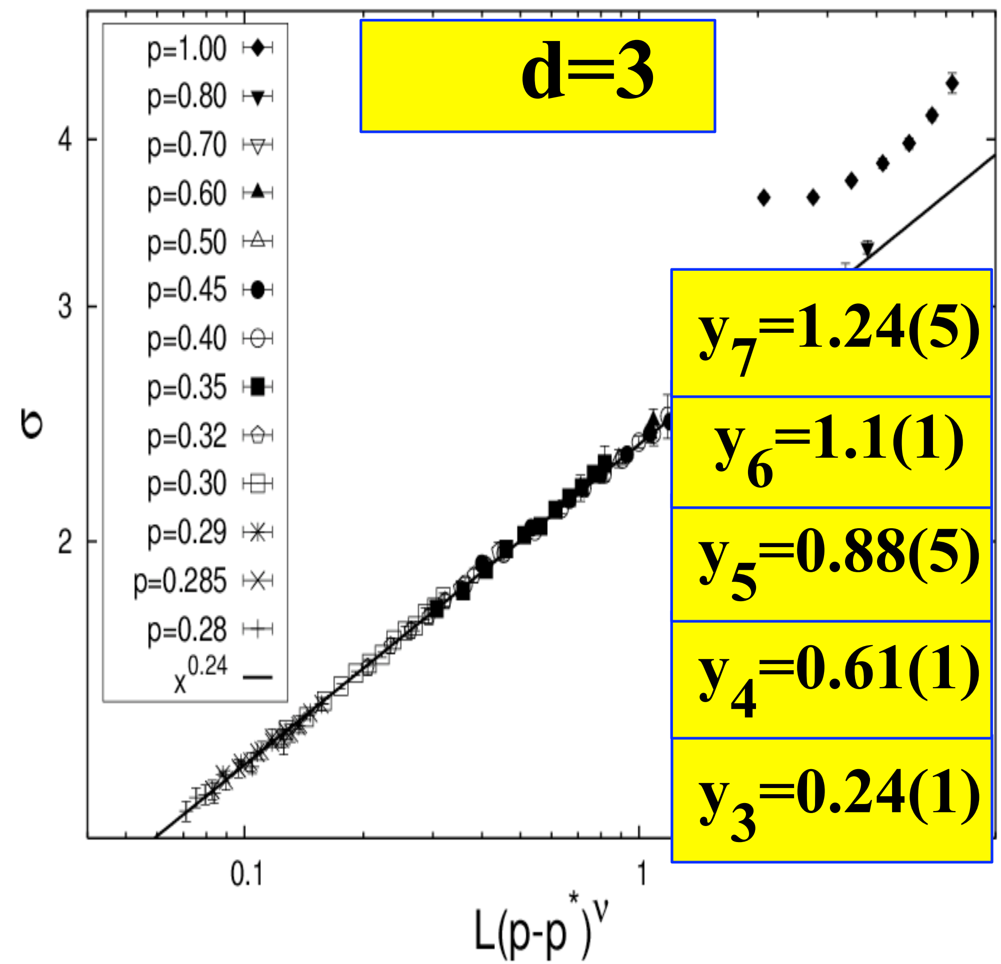


d=3

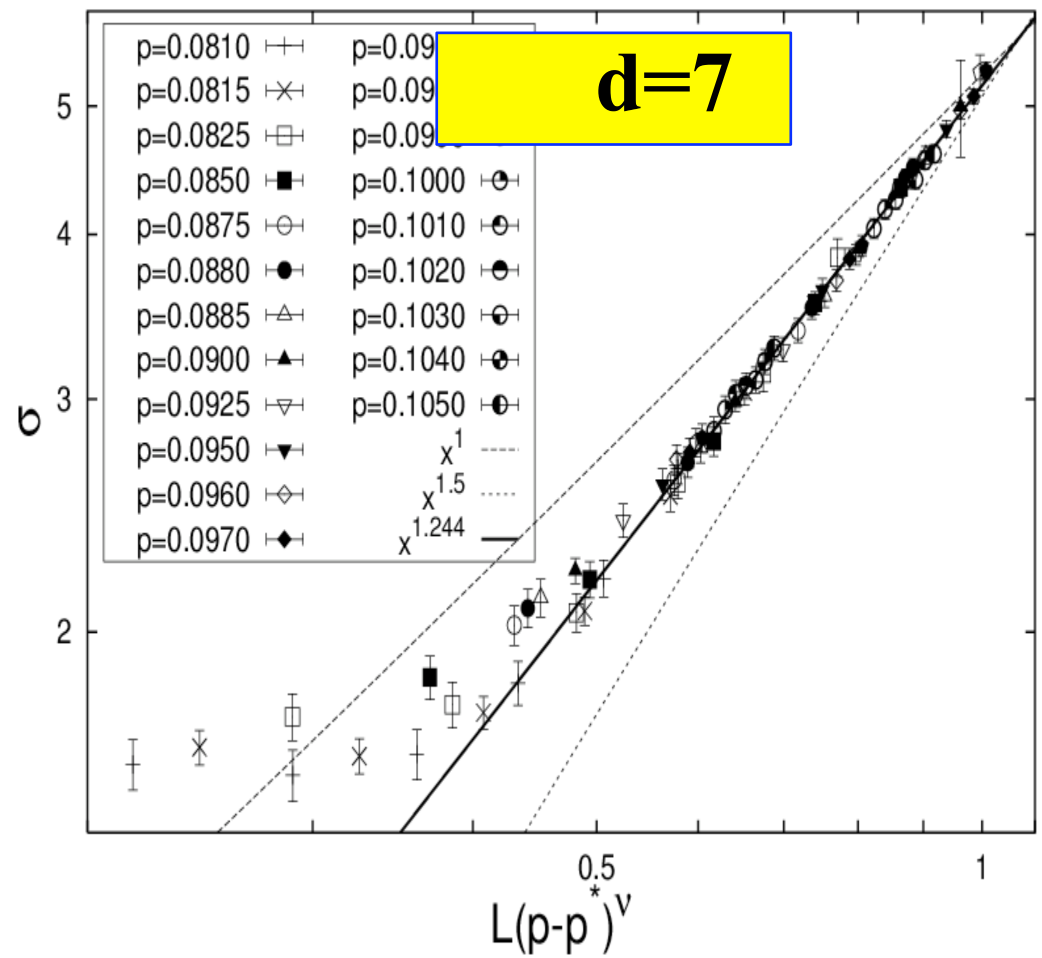
y_7=1.24(5)

y_6=1.1(1)

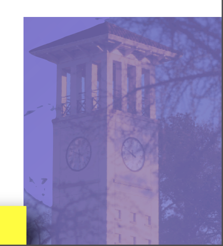
y_5=0.88(5)

y_4=0.61(1)

y_3=0.24(1)



d=7



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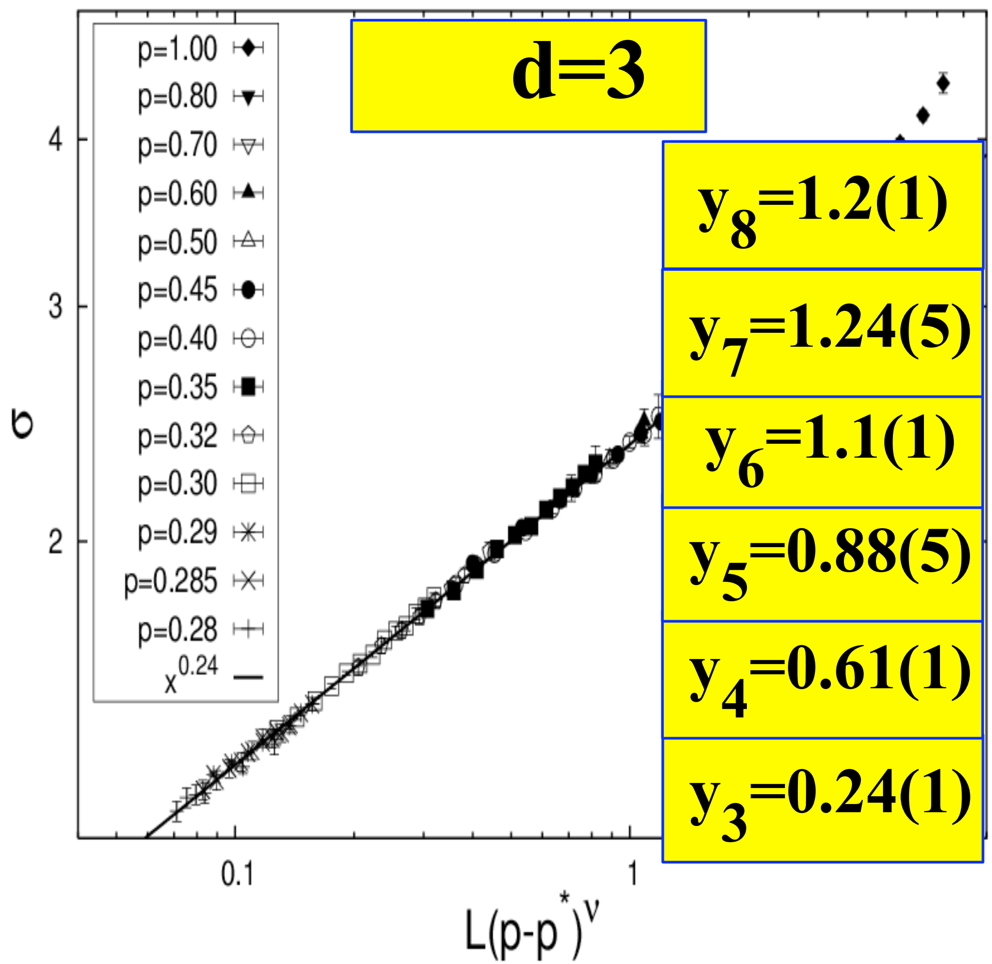


y_8=1.2(1)

y_7=1.24(5)

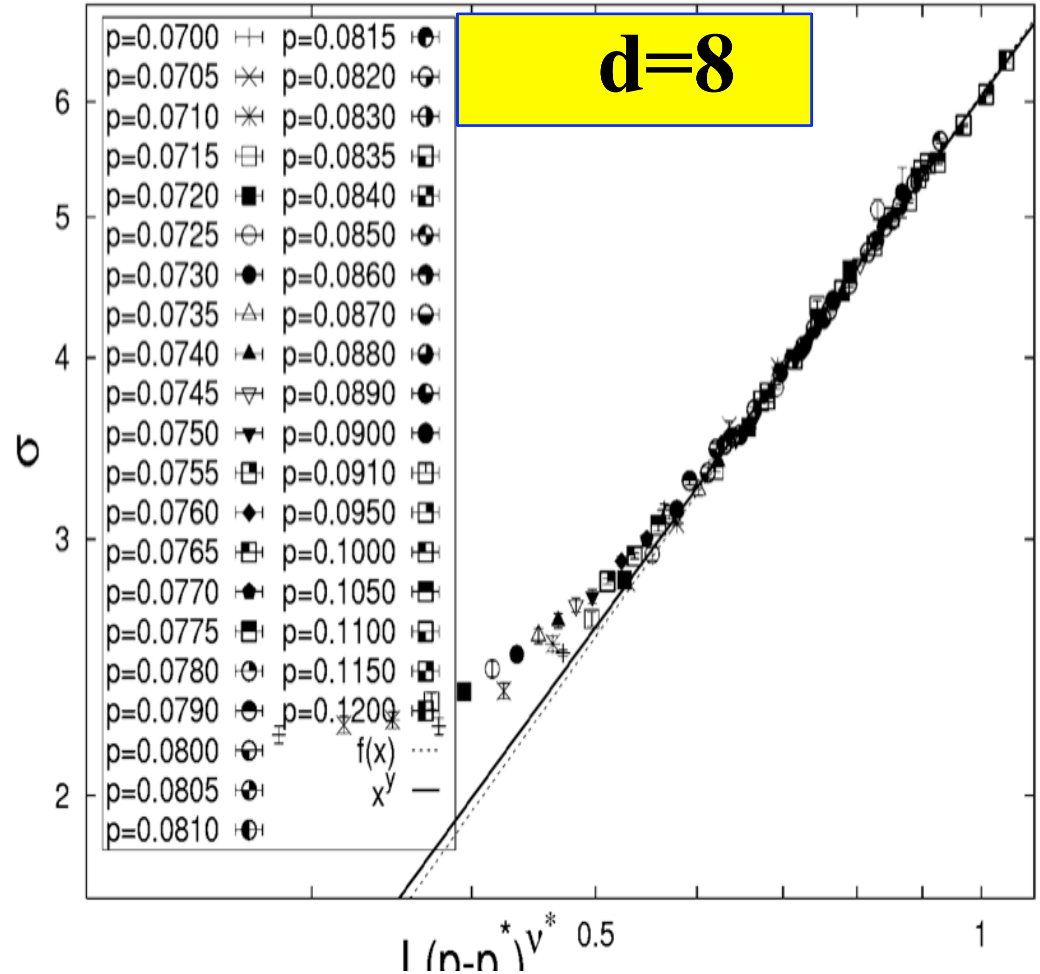
y_6=1.1(1)

y_5=0.88(5)

y_4=0.61(1)

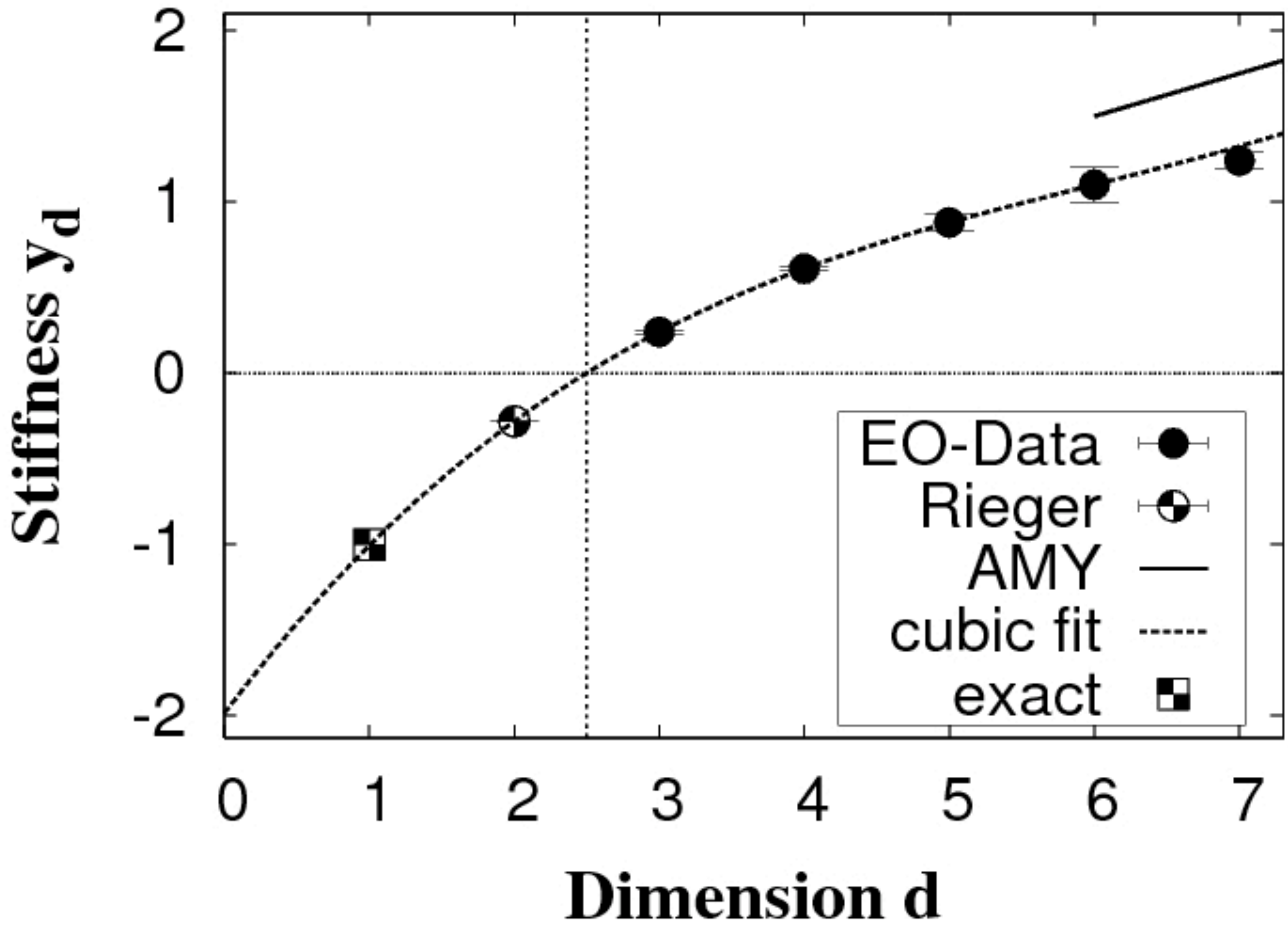
y_3=0.24(1)

d=8



Comparing with Theory:

"Stiffness": $\sigma(\Delta E) \sim L^{y}$



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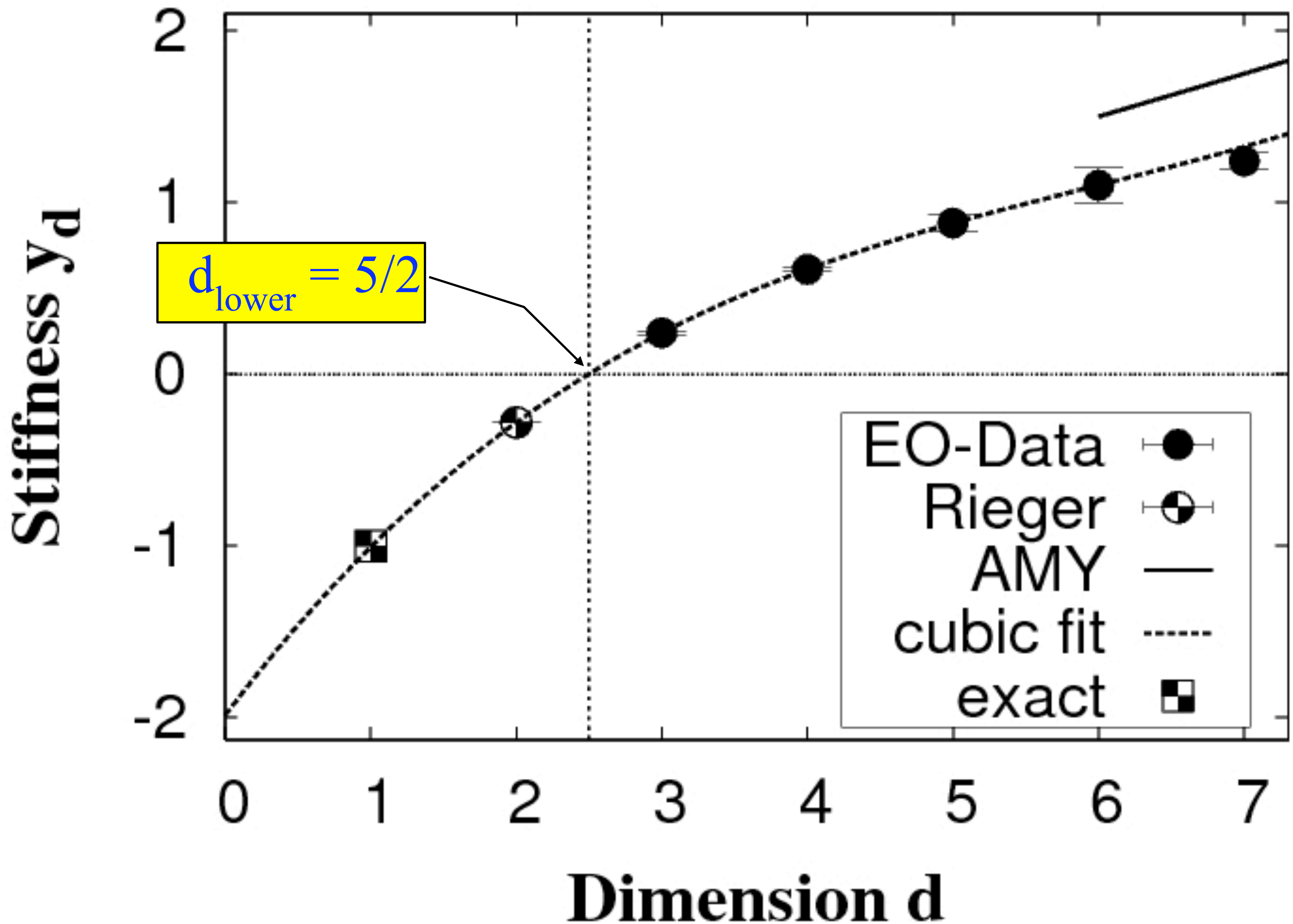

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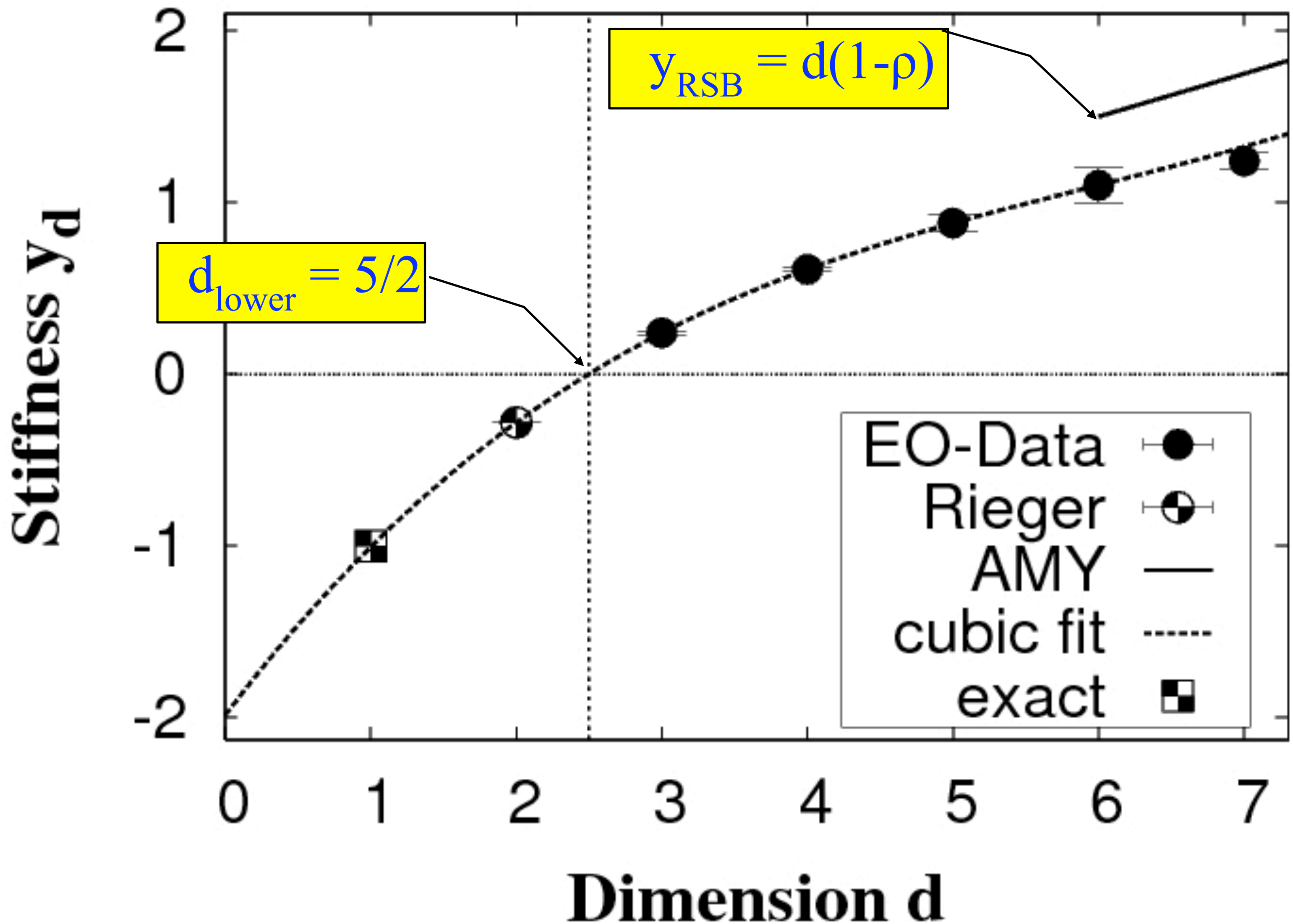

Other Evidence for $d_l=5/2$:

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$$T_g \approx \sqrt{2d} \qquad (d \to \infty)$$

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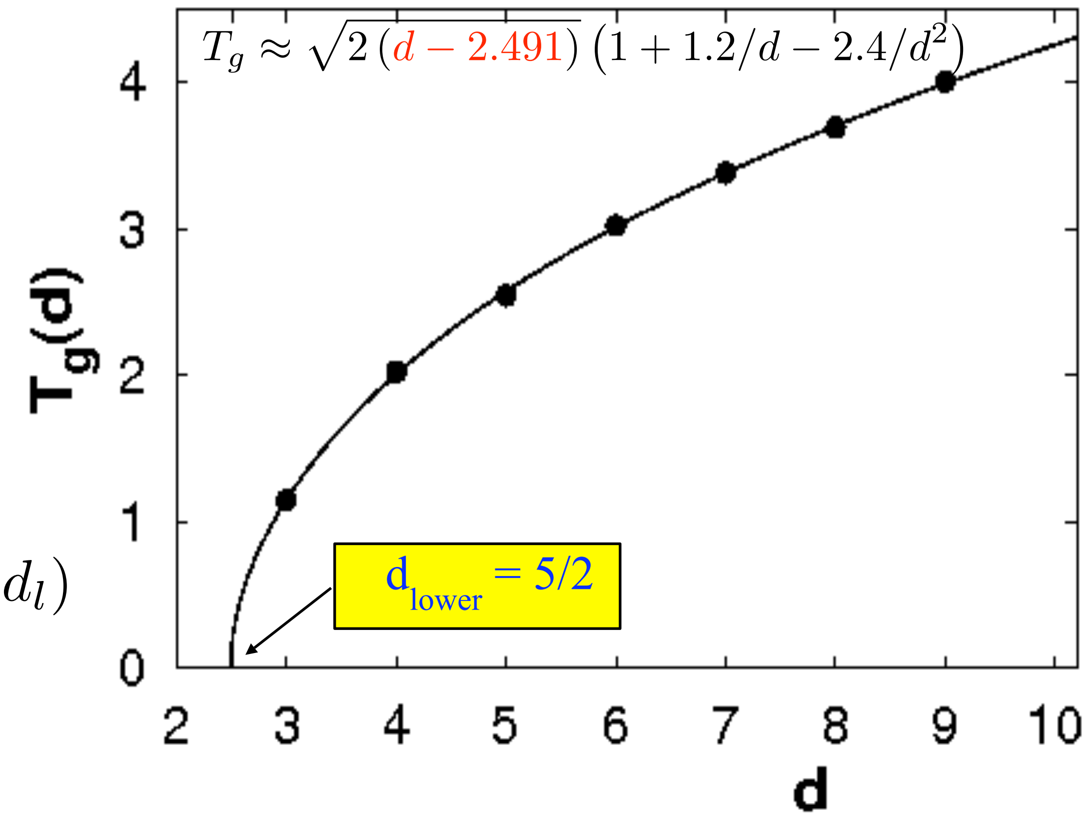

Data from:
▫MC (Ballesteros et al) for d=3,4
▫High-T Series (Klein et al) for d≥5

Comparing with Theory:

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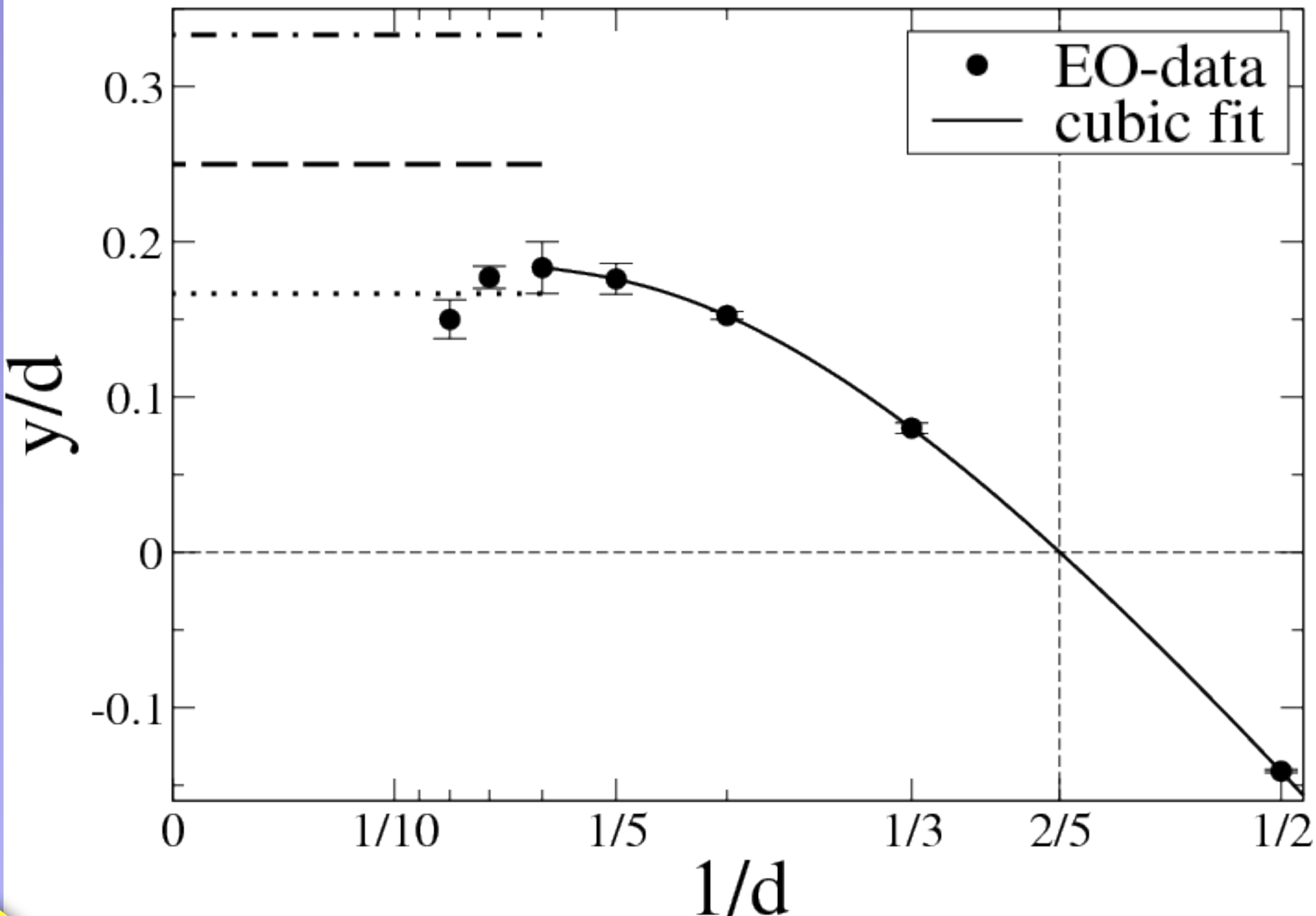

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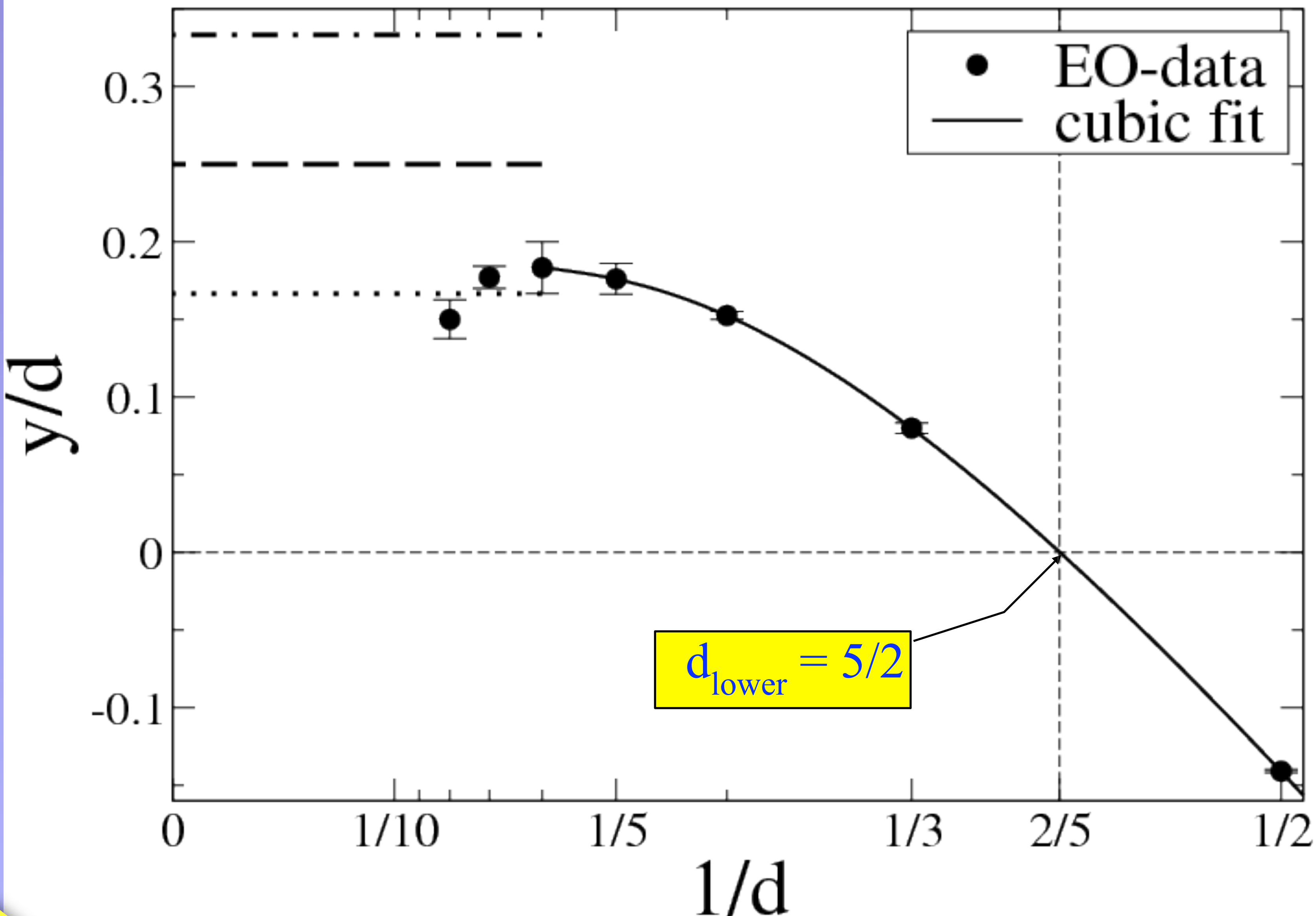

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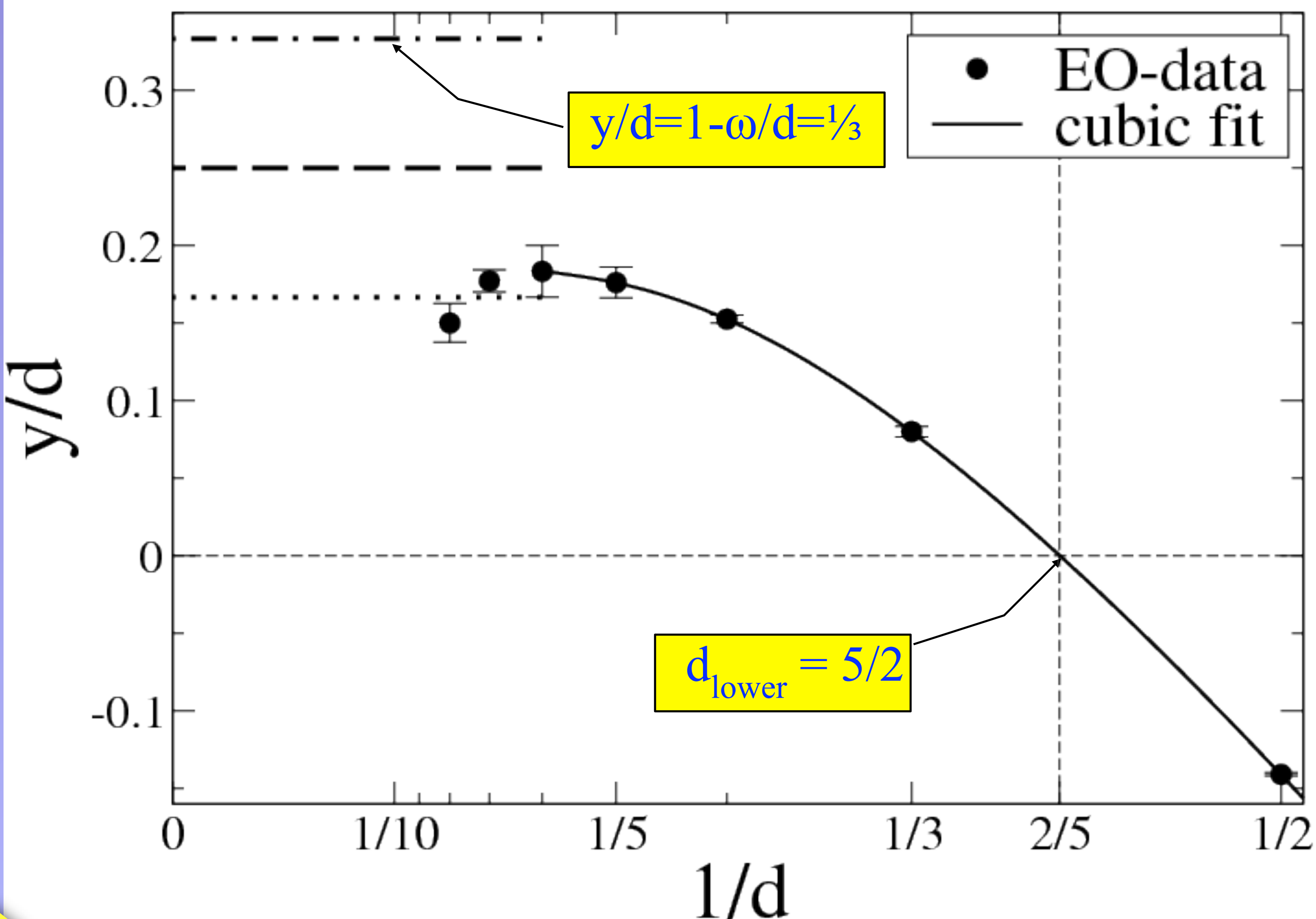

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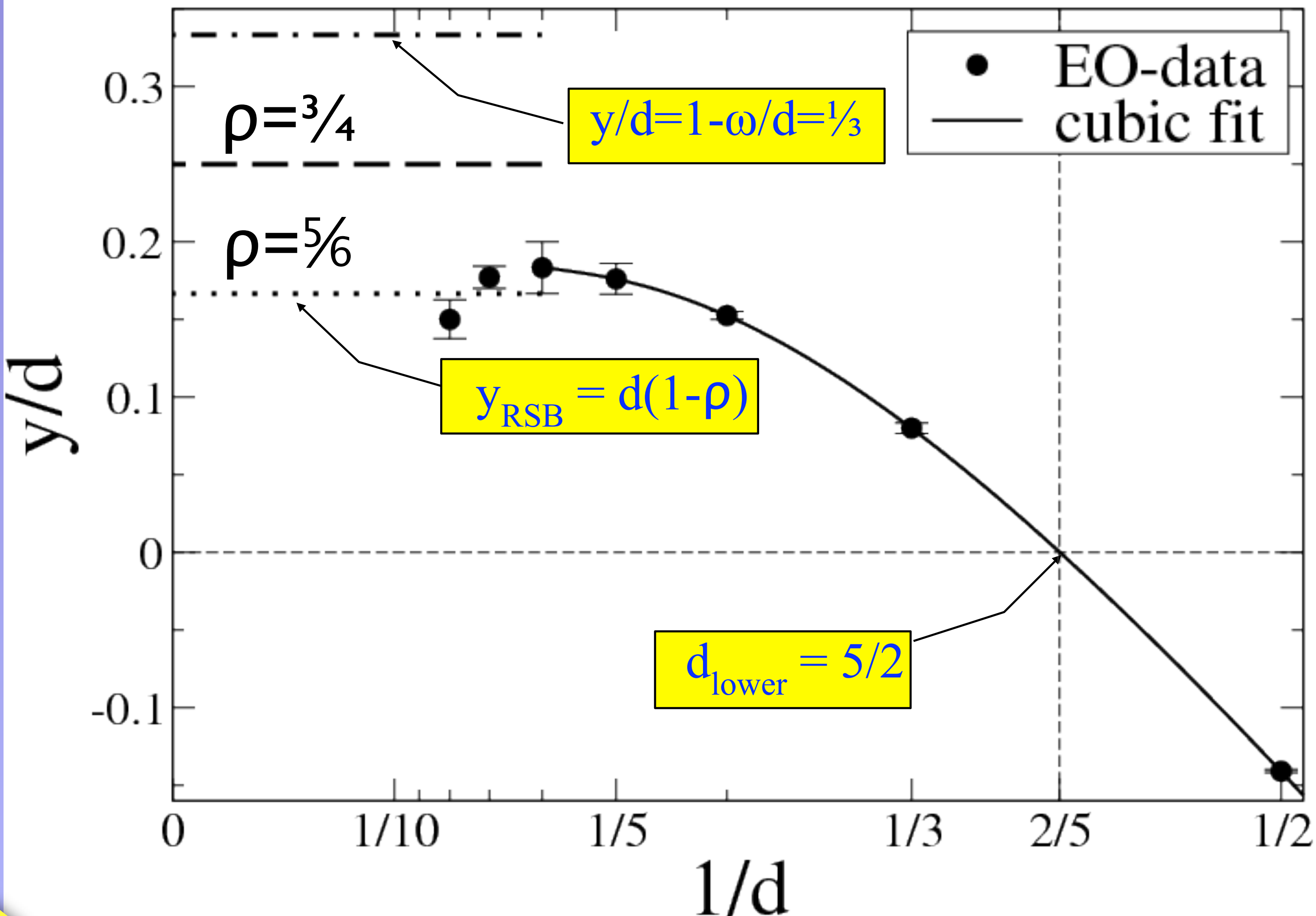

Corrections-to-Scaling in EA:

Ground State Energy: $E(L) \sim e_0 L^d + A L^y \qquad (L \to \infty)$

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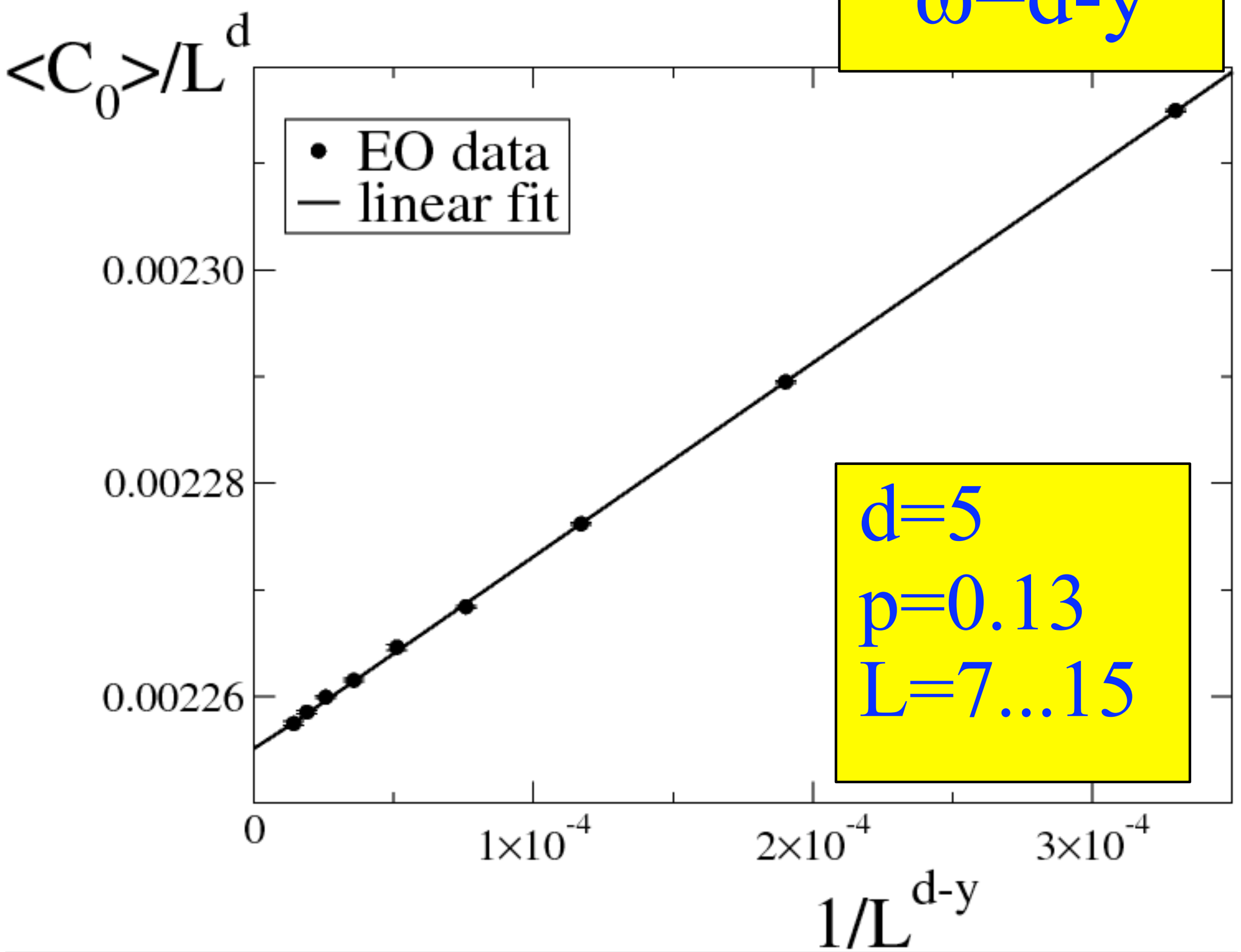
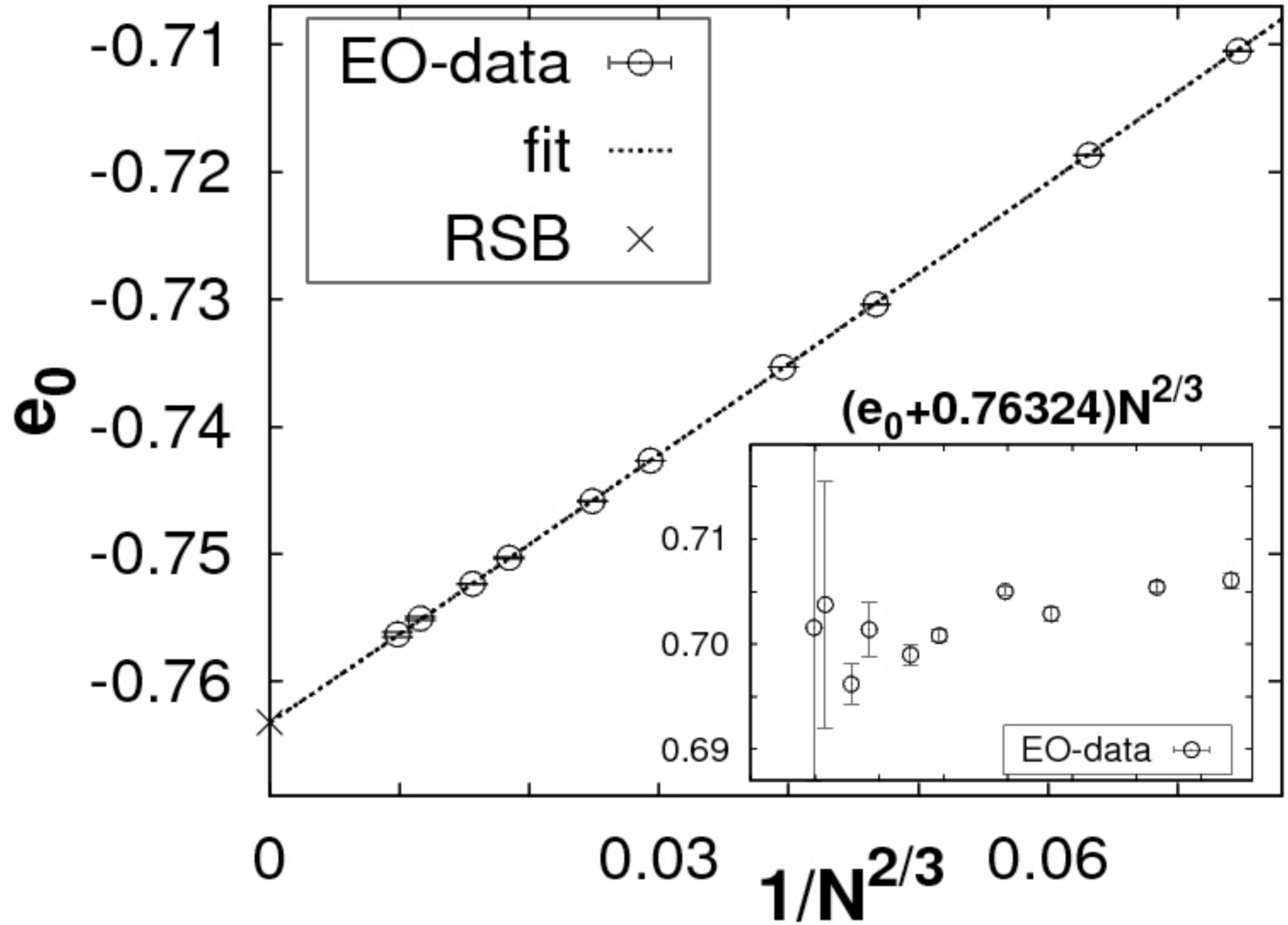


τ-EO for Sherrington-Kirkpatrick

- Mean-Field ($d \to \infty$) Spin Glasses:



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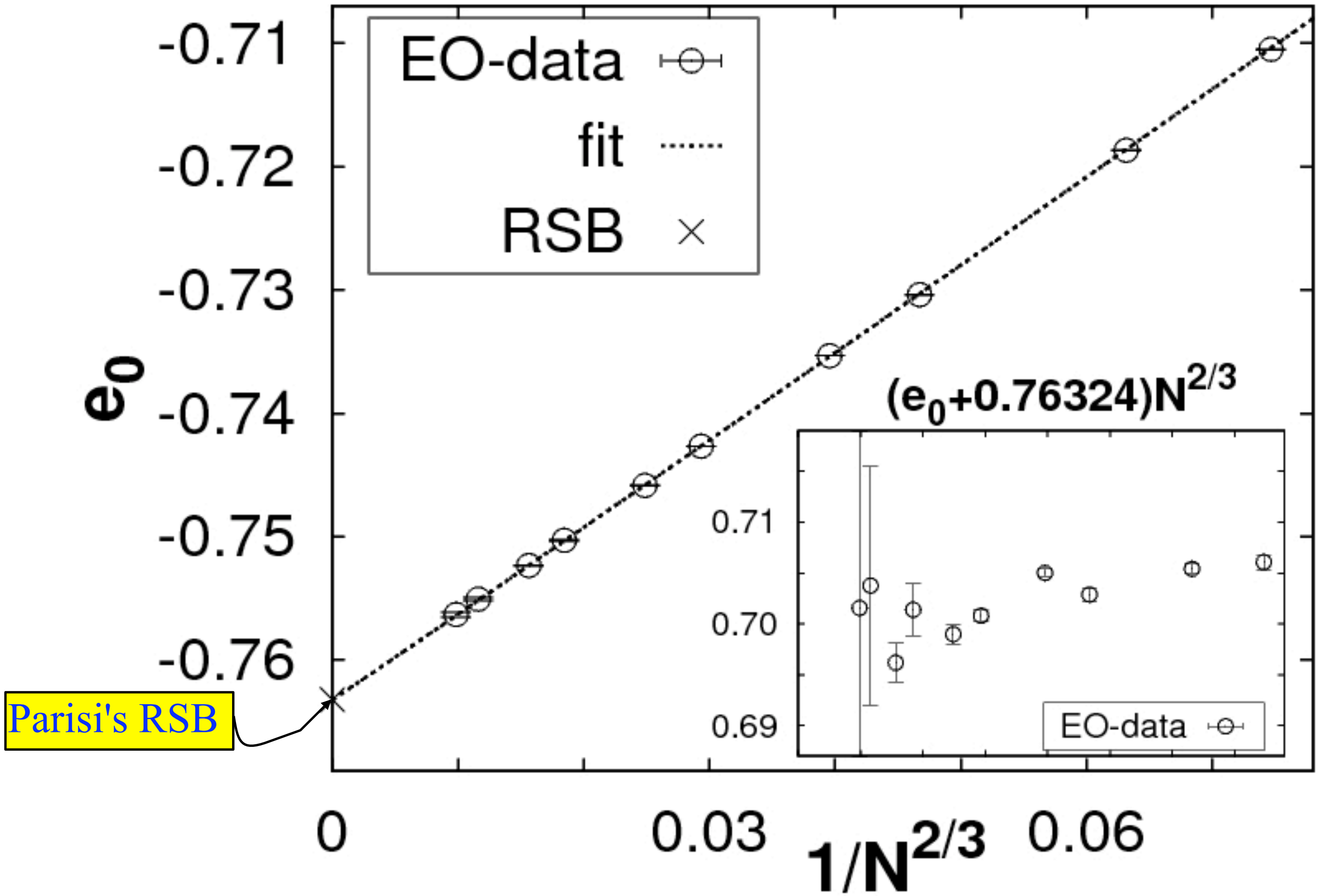

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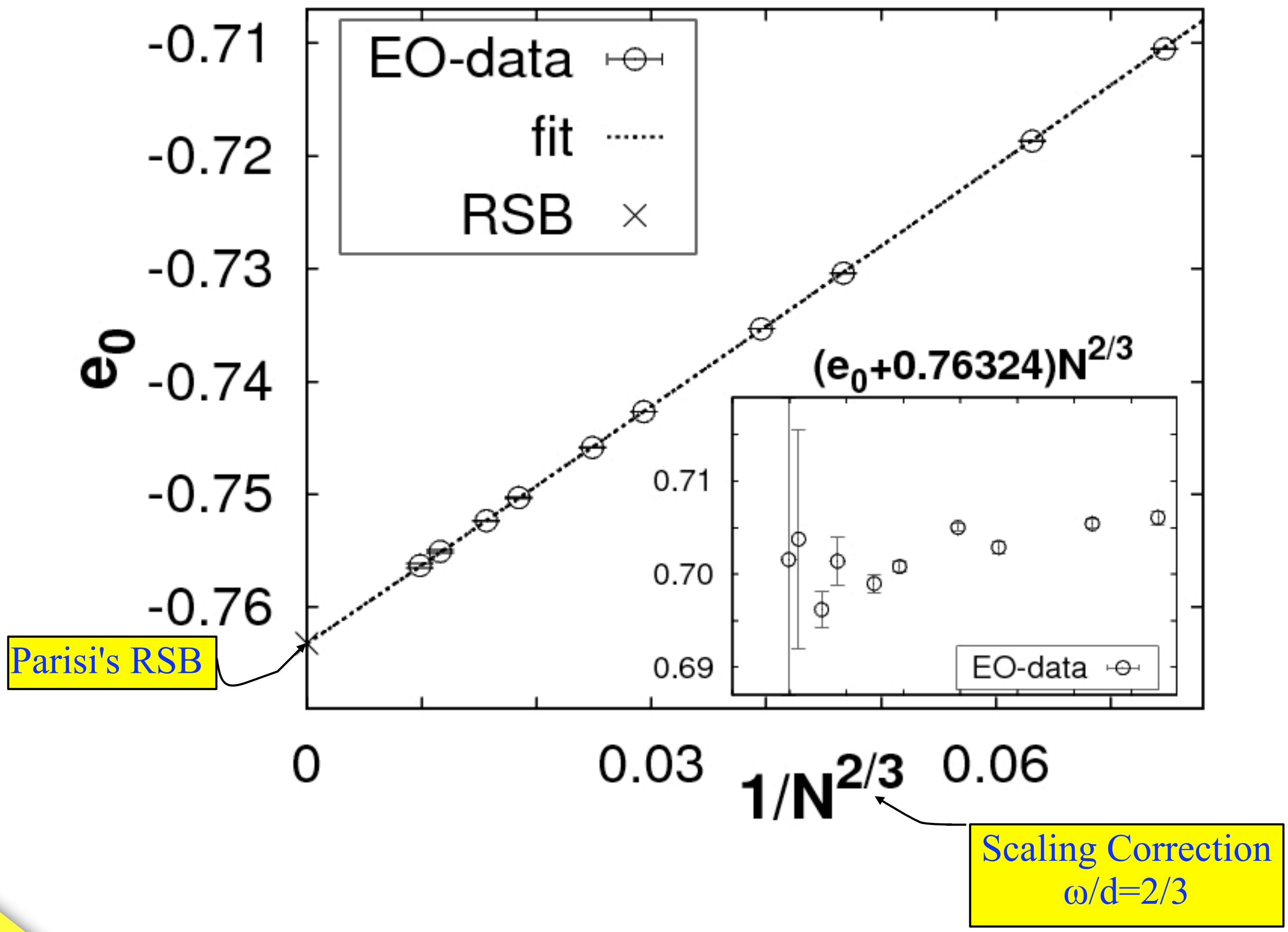

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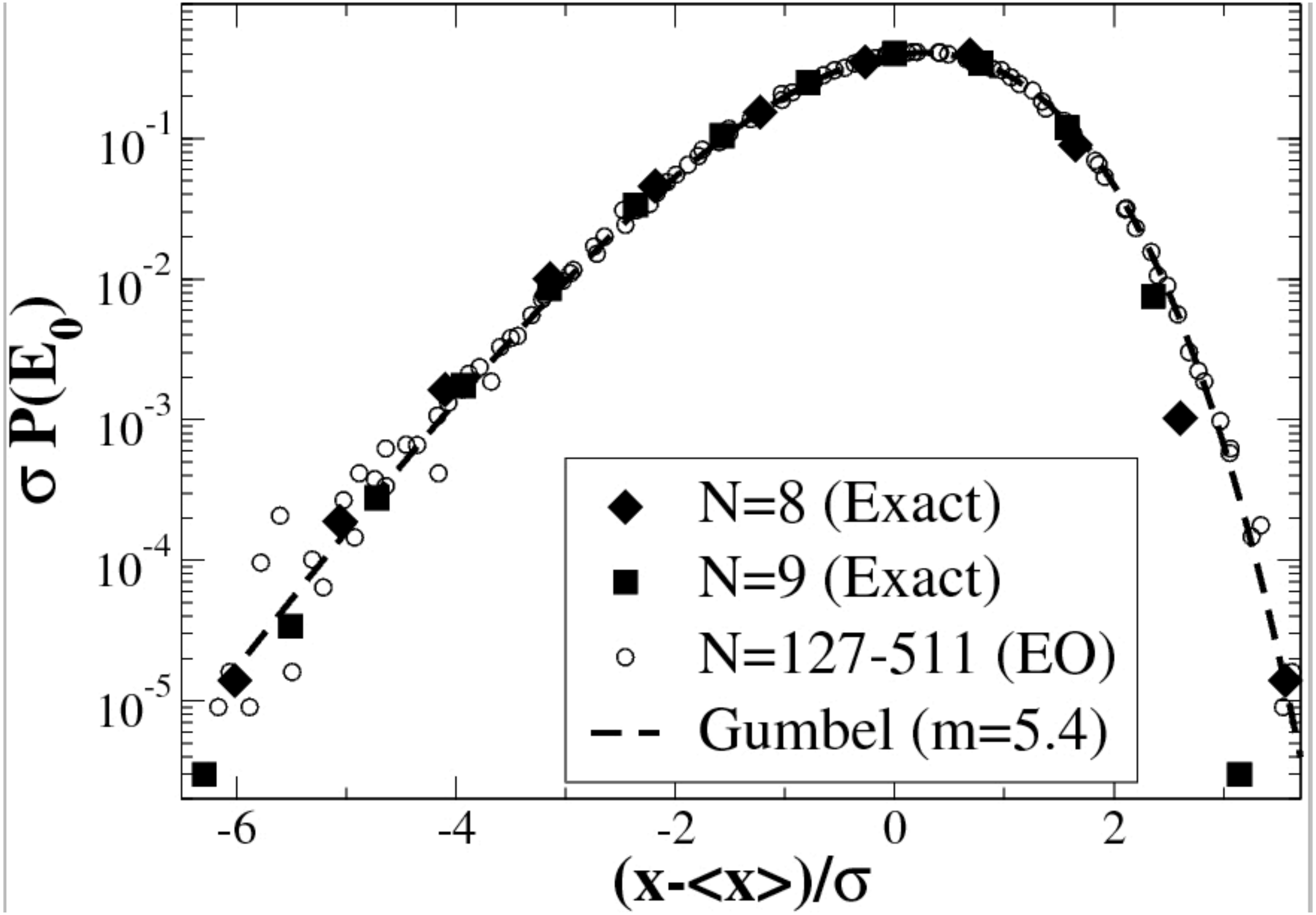

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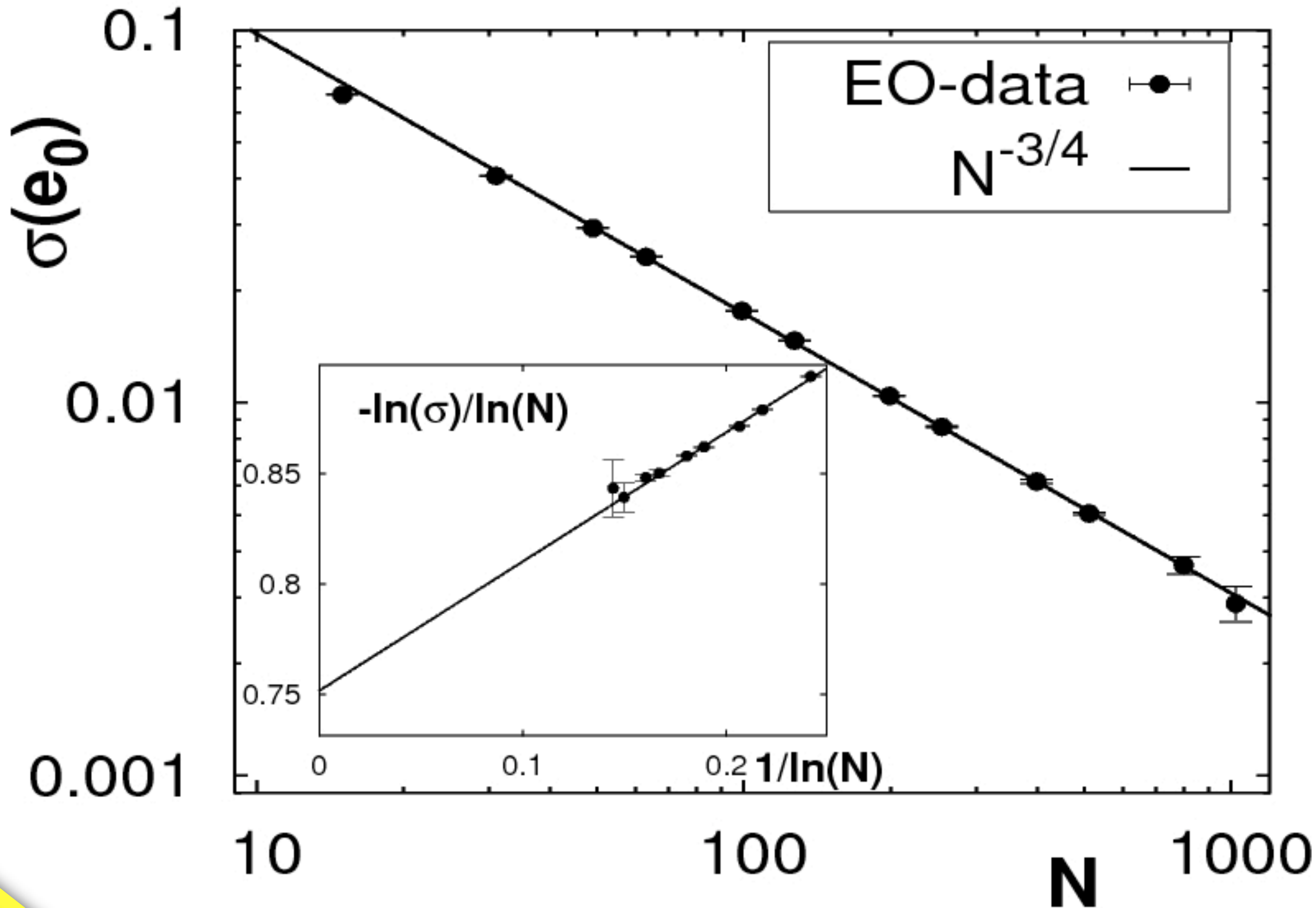

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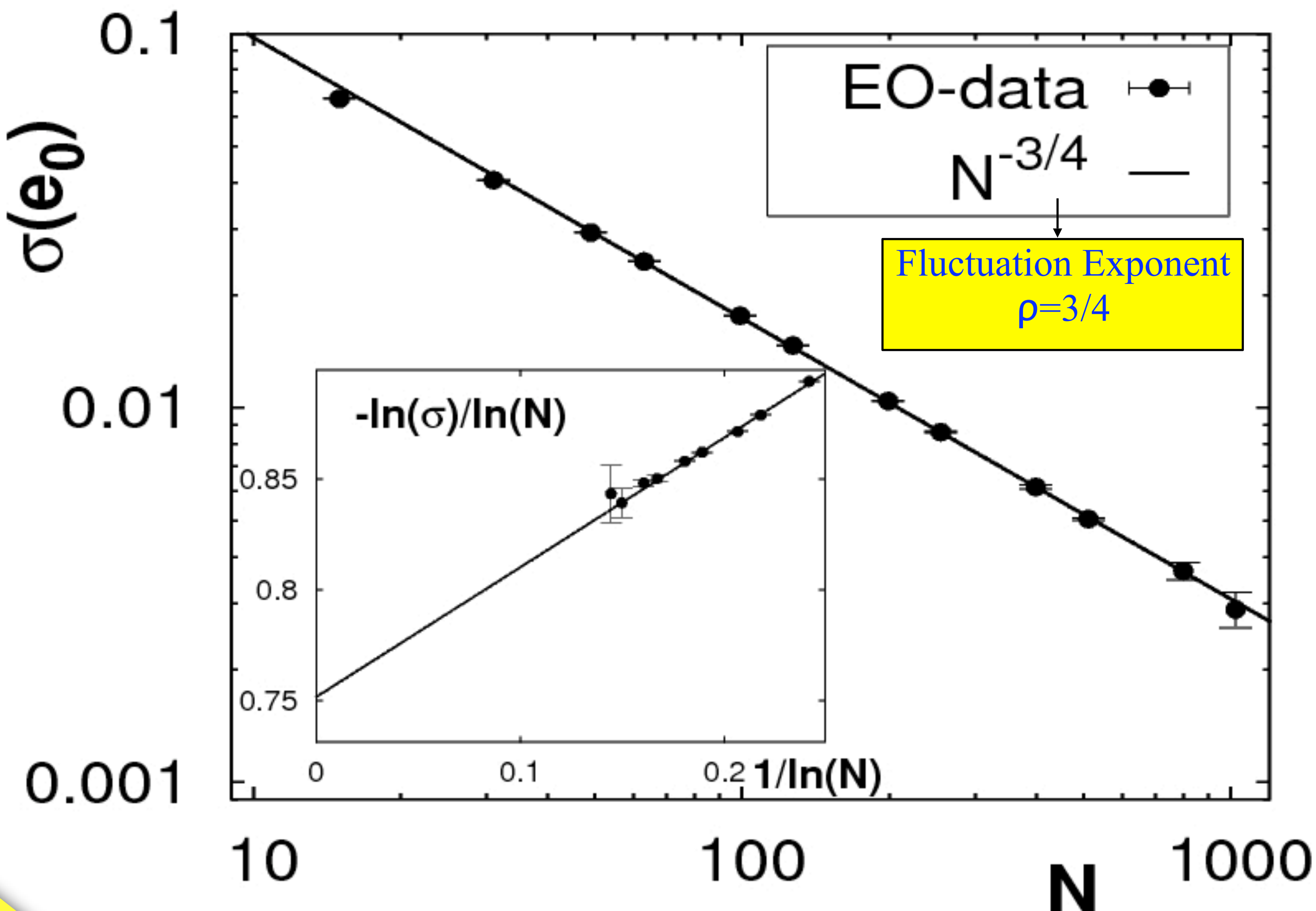
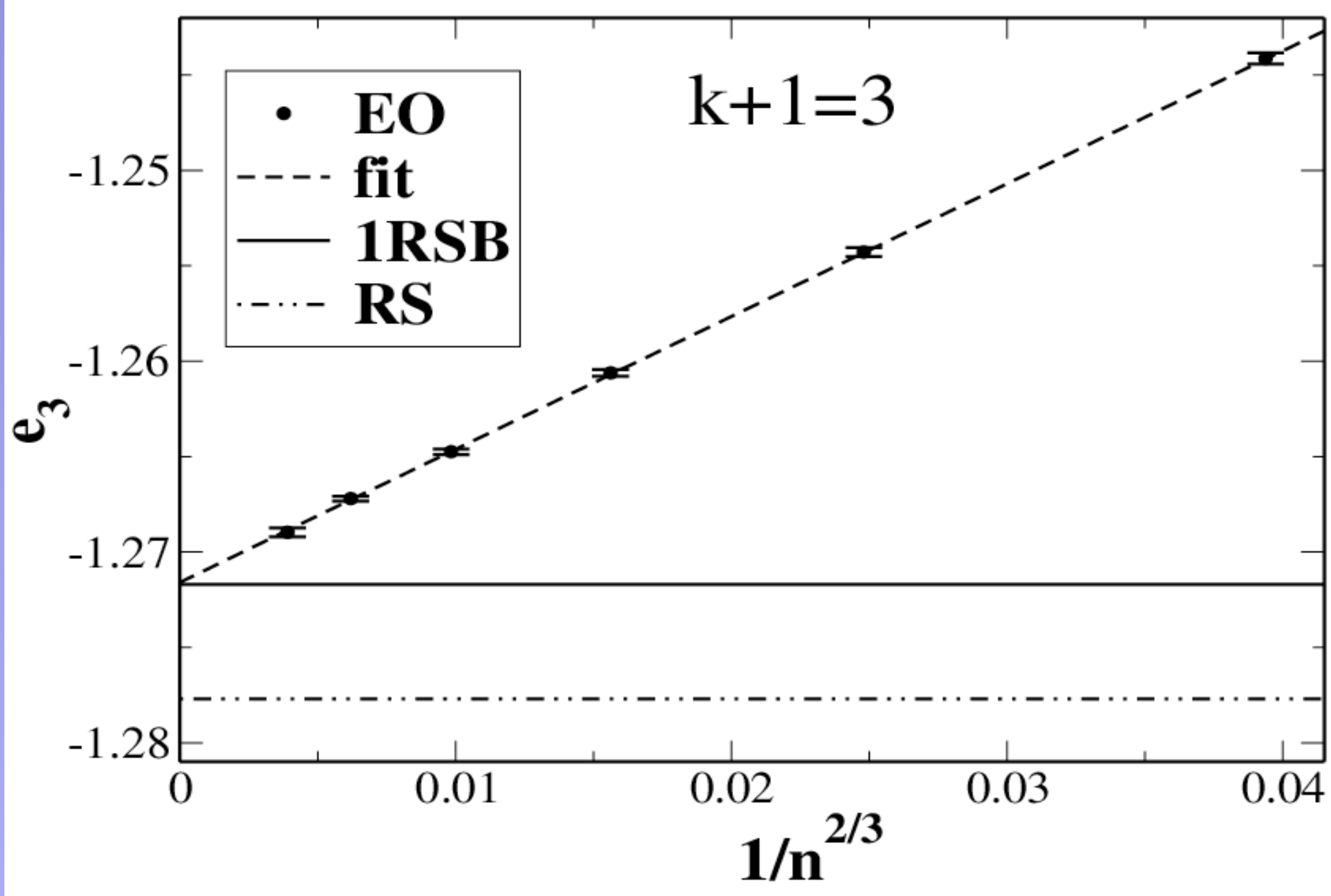


τ-EO for Bethe Lattices:

EO for 3-connected Bethe Lattice Glass w/ Replica Sym. Breaking:



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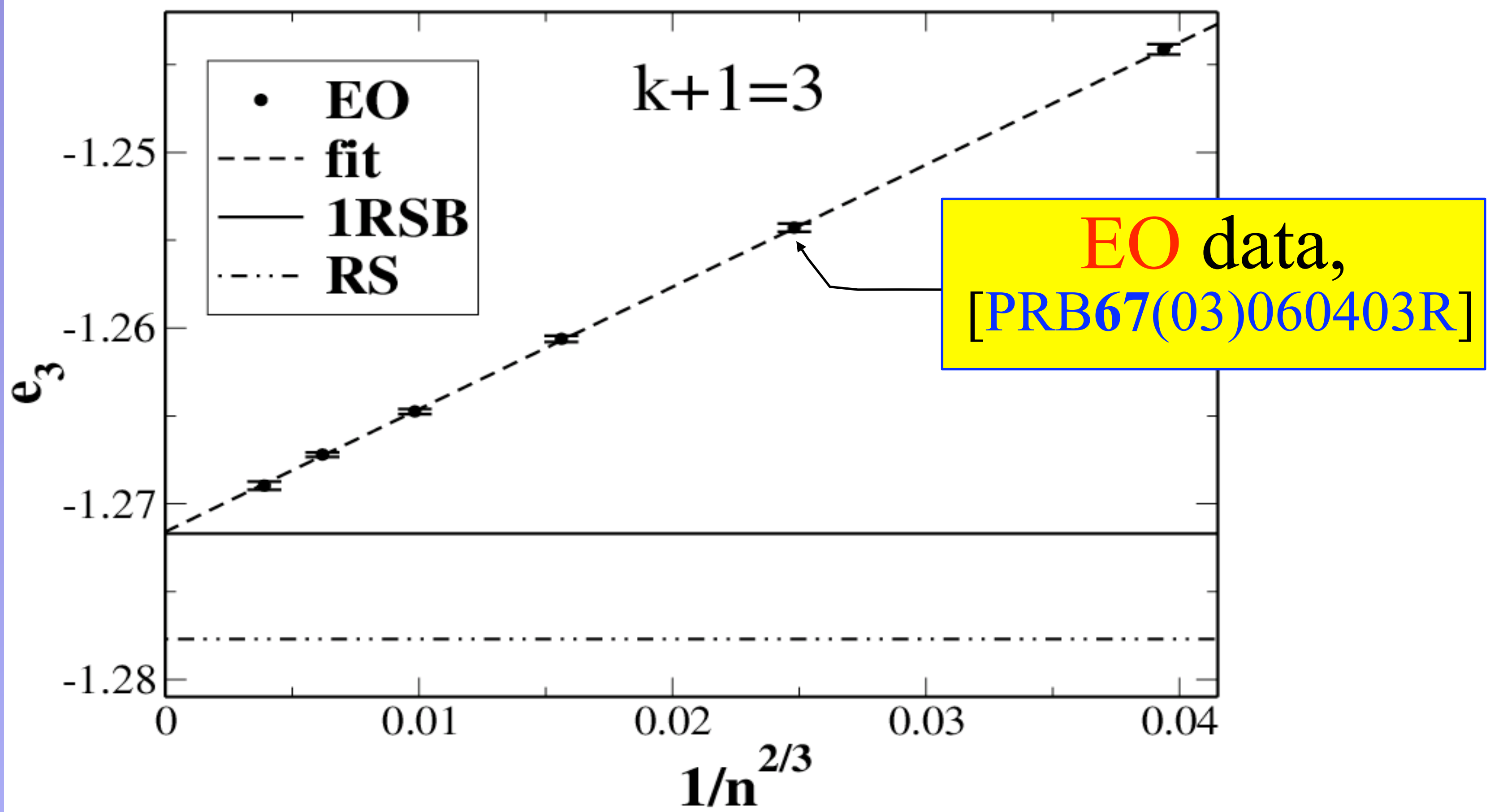

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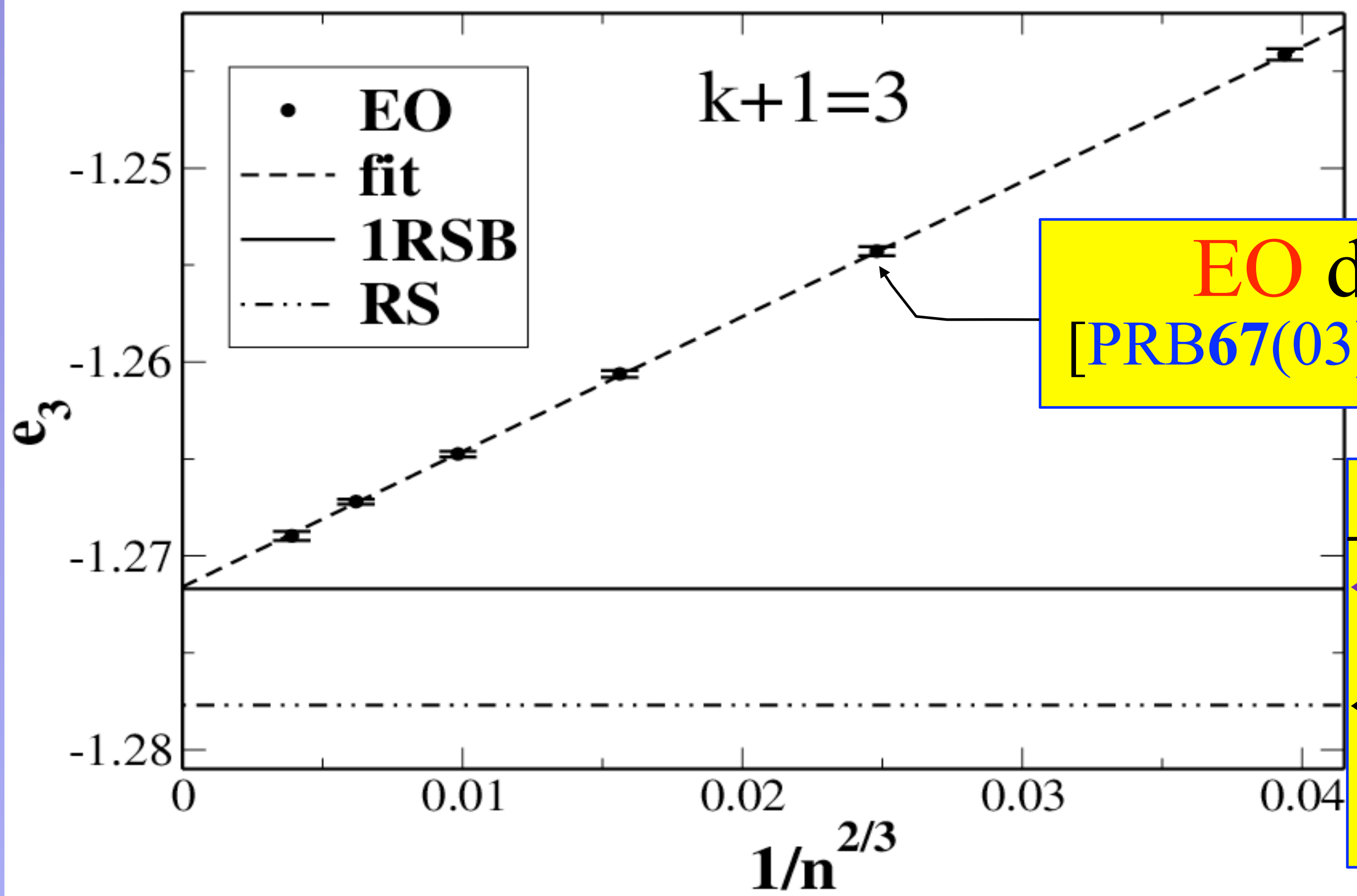

EO data, [PRB**67**(03)060403R]

Replica Theory:
⇐ 1RSB,
⇐ no RSB,
[J.Stat.Phys.**111**(03)1]

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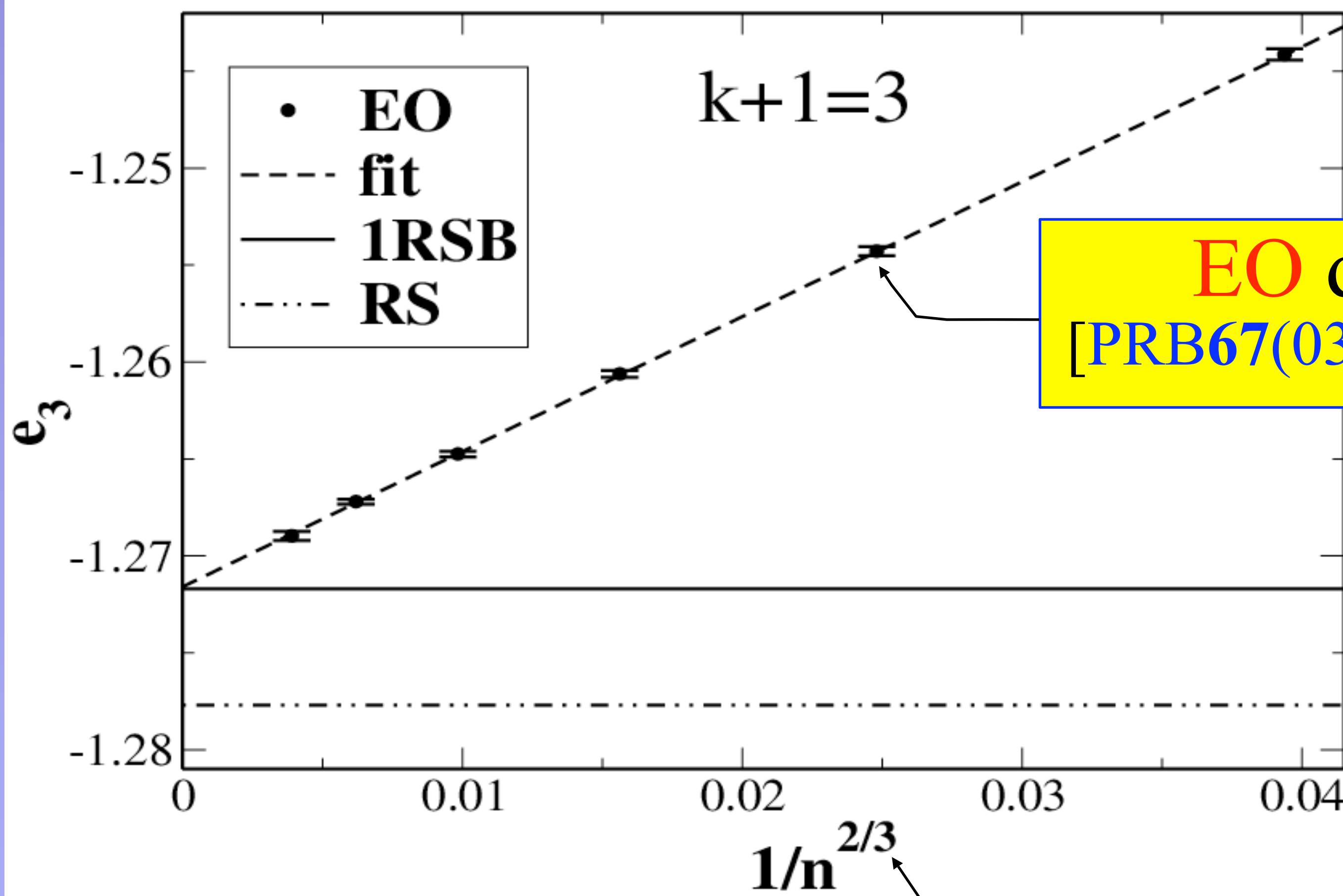

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Scaling corrections

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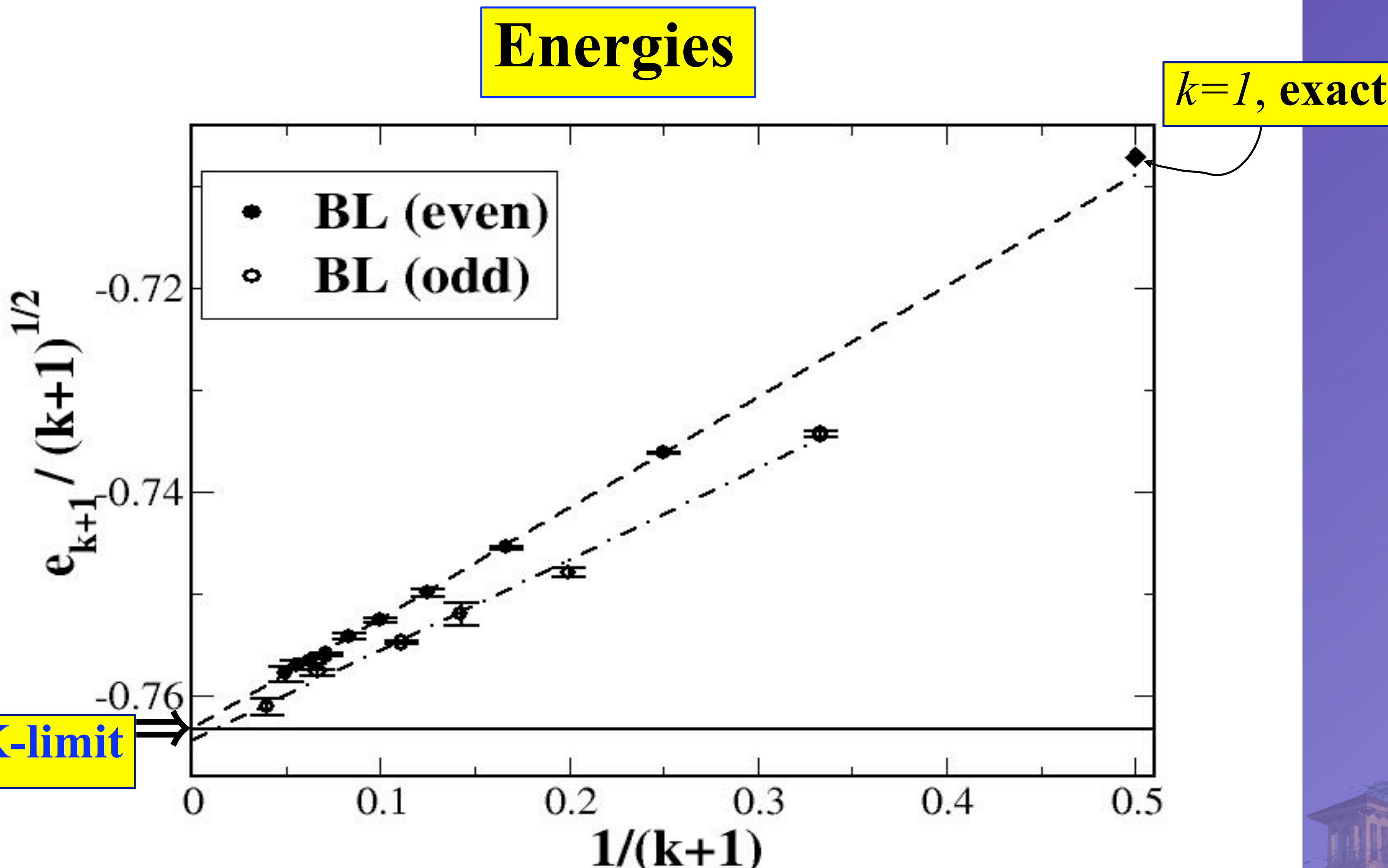
EO for $(k+1)$-connected Bethe Lattice Glasses for $(k+1)\rightarrow\infty$:



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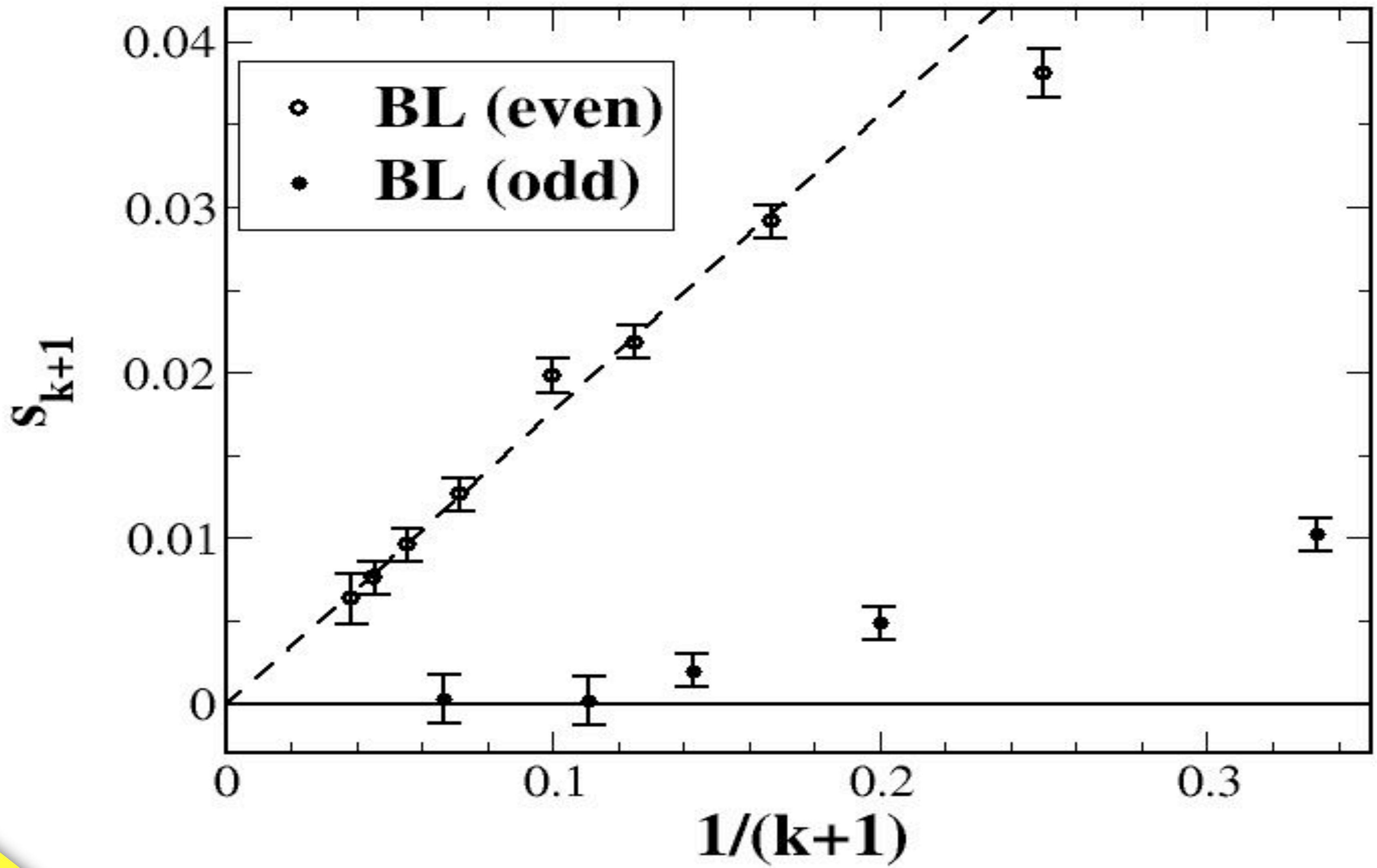
Entropies



Distribution of Ground State Energies:

Deviation $\sigma(e_0)$ of PDF for Bethe Lattices of Degree $z(=k+1)$:

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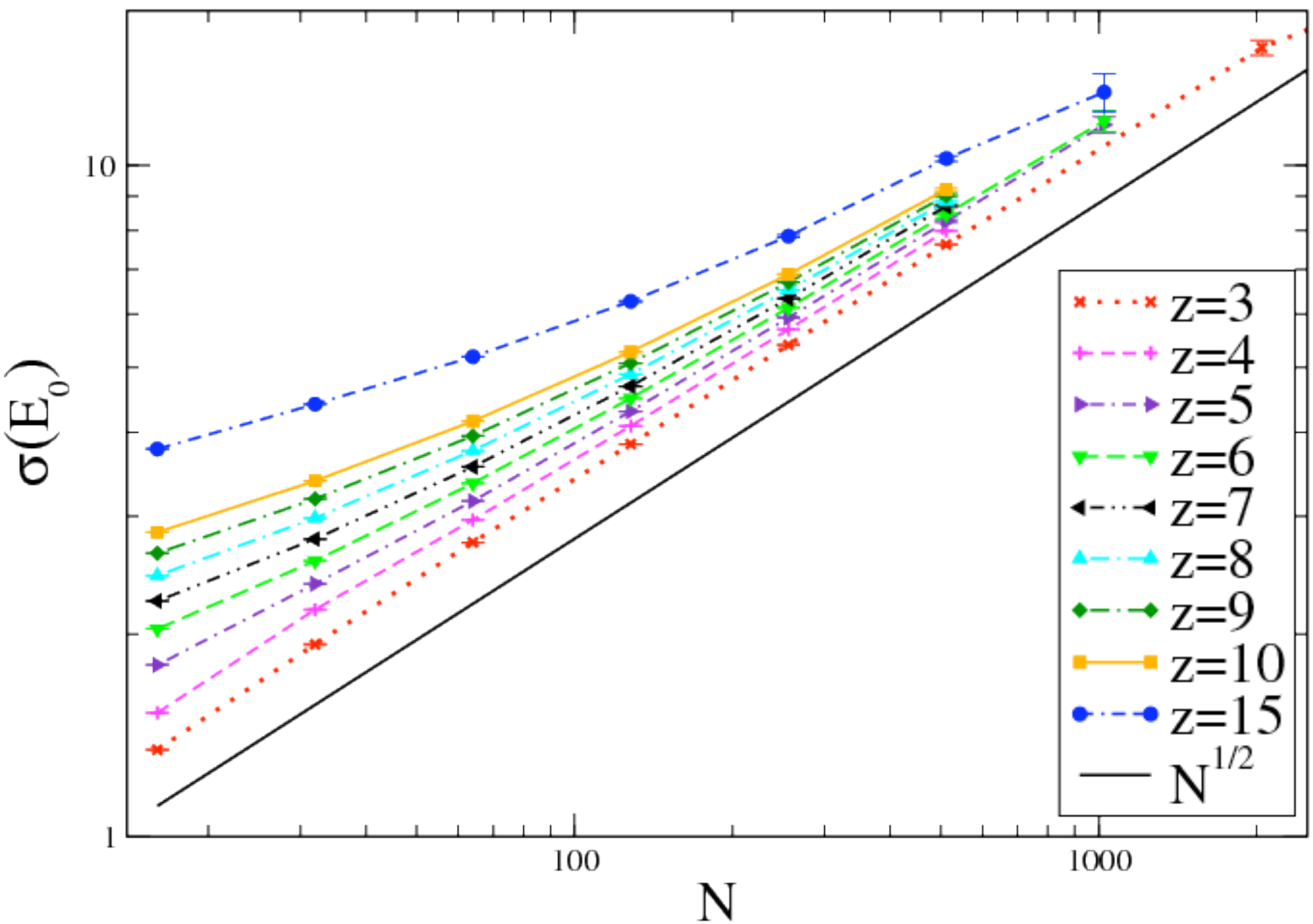

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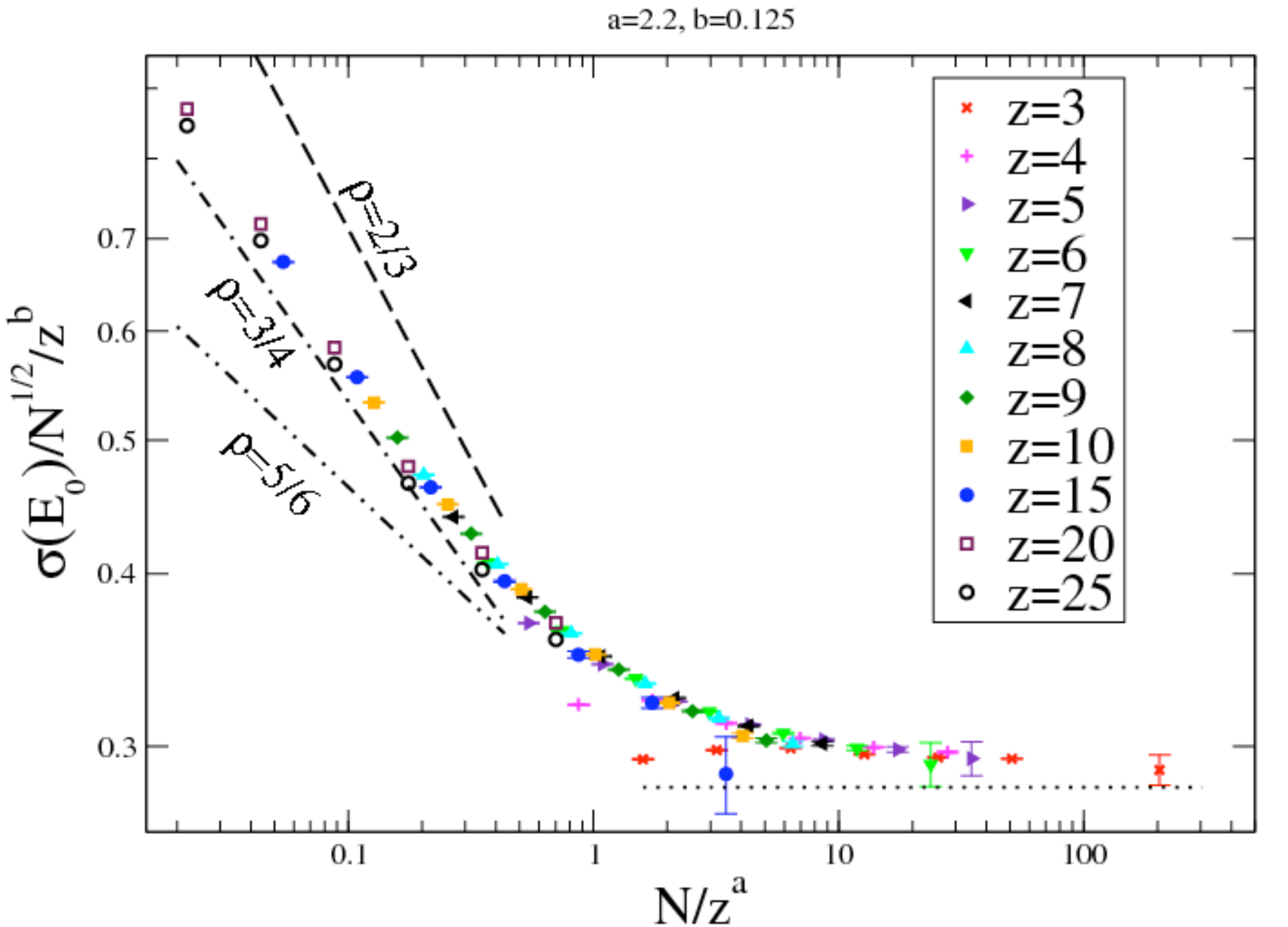

Comprehensive View on Spin Glasses:

A Set of Models:

d: dimension,
p: bond density,
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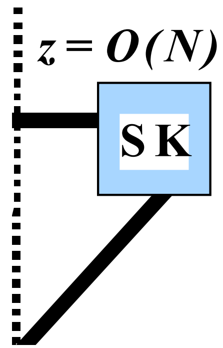

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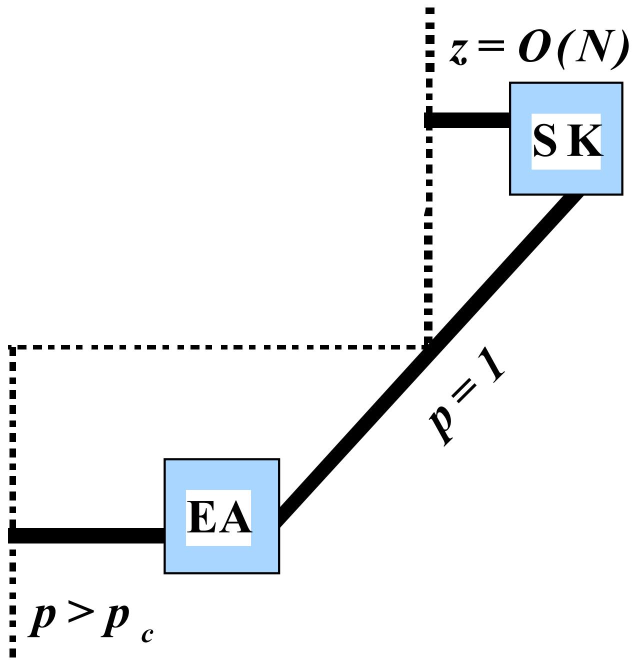

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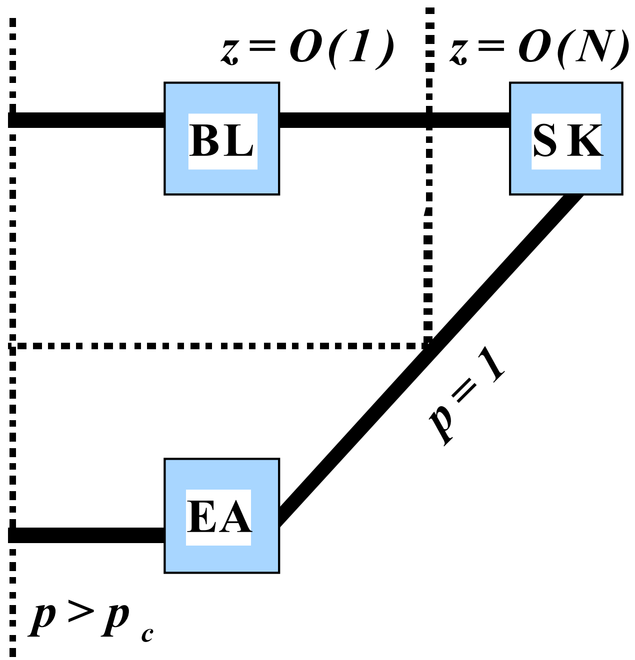

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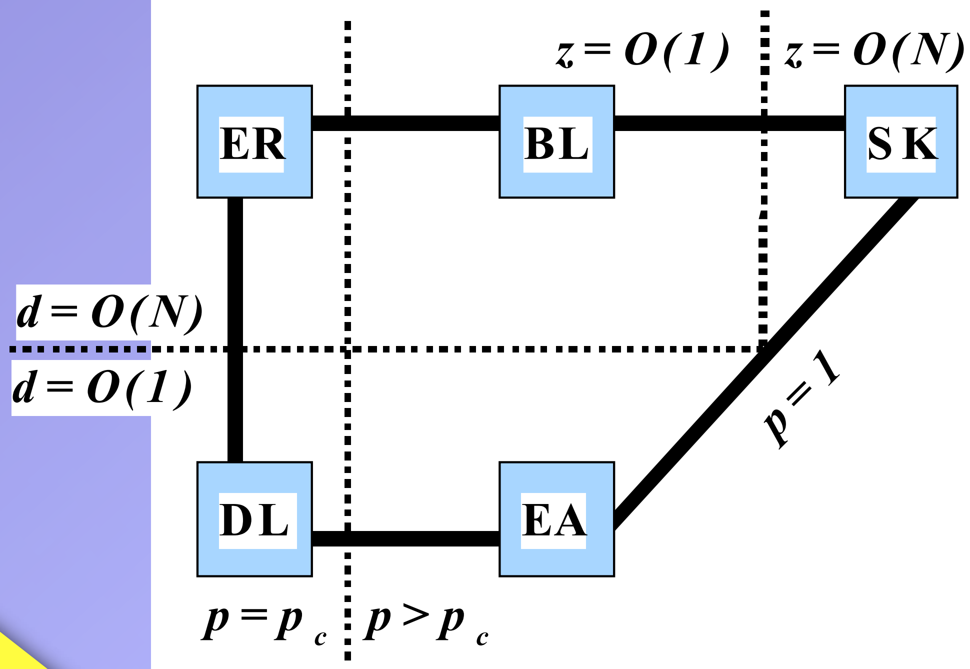

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- Dilute Lattice (**DL**)
  ↳**EA** at p_c
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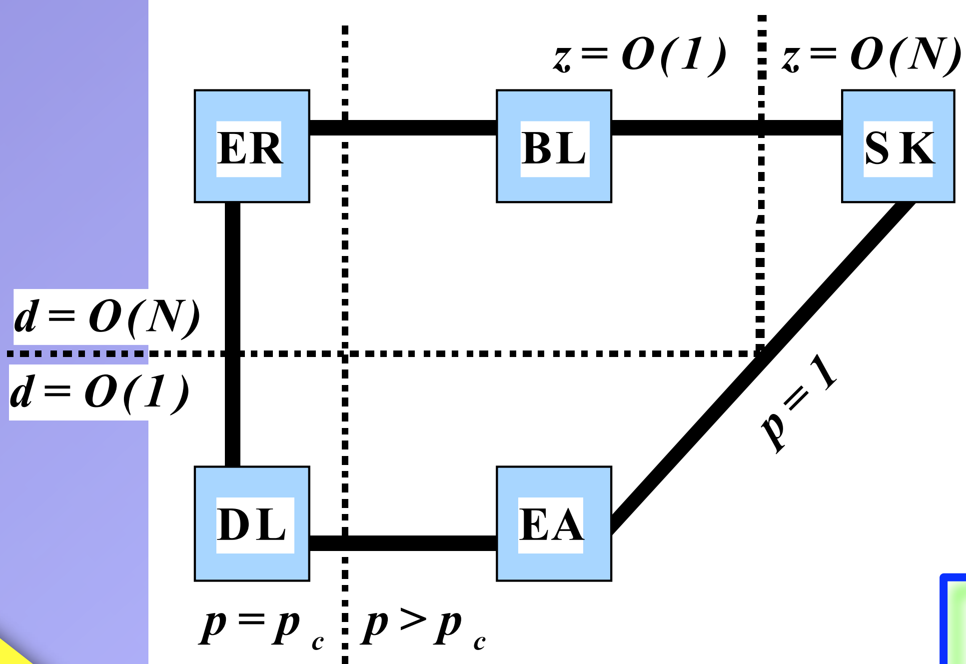

$$H = \frac{1}{\sqrt{2dp}} \sum_{<i,j>} J_{i,j} x_i x_j$$

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1) Distribution $P(e_0)$, width $\sigma(e_0) \sim N^{-\rho} = L^{-d\rho}$

- In **EA**: $\rho = ½$ (Wehr&Aizenman: ρ exact!)

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- In **EA**: $y \approx 0.24,\ldots,1.2$ for $d=3,\ldots,7$

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3) Corrections-to-Scaling : $e_0(N)-e_0(\infty) \sim N^{-\omega/d} = L^{-\omega}$

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- In **EA**: $y \approx 0.24,\ldots,1.2$ for $d=3,\ldots,7$
- In **SK**: $y/d = 1 - \rho \to 1/4$, too high for **EA** at $d \geq 6$

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A Set of Exponents:

1) Distribution $P(e_0)$, width $\sigma(e_0) \sim N^{-\rho} = L^{-d\rho}$

- In **EA**: $\rho = 1/2$ (Wehr&Aizenman ρ exact!)
- In **SK**: $\rho \approx 3/4$ (Highly Skewed)

2) Distribution $P(\Delta E_0)$, width $\sigma(\Delta E_0) \sim N^{y/d} = L^{y}$

- In **EA**: $y \approx 0.24,\ldots,1.2$ for $d=3,\ldots,7$
- In **SK**: $y/d = 1-\rho \to 1/4$, too high for **EA** at $d \geq 6$

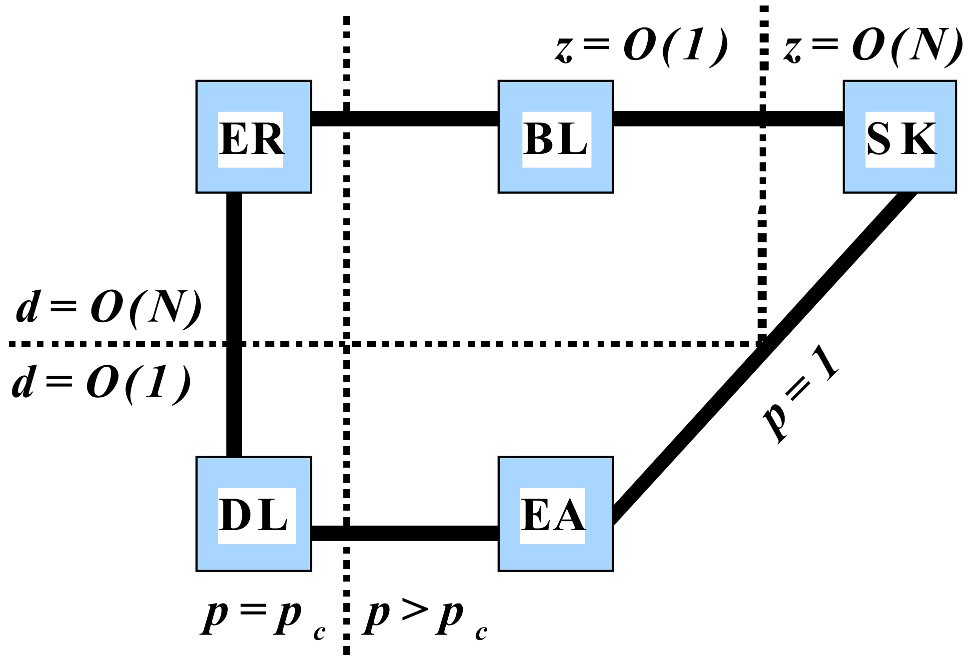
3) Corrections-to-Scaling : $e_0(N)-e_0(\infty) \sim N^{-\omega/d} = L^{-\omega}$

- In **EA**: $\omega/d = 1 - y/d$
- In **SK**: $\omega/d \approx 2/3 \neq 1 - y/d$

Comprehensive View on Spin Glasses:

A Set of Models:

d: dimension,
p: bond density,
z: bond degree



Stefan Boettcher
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Comprehensive View on Spin Glasses:

A Set of Models:

d: dimension,
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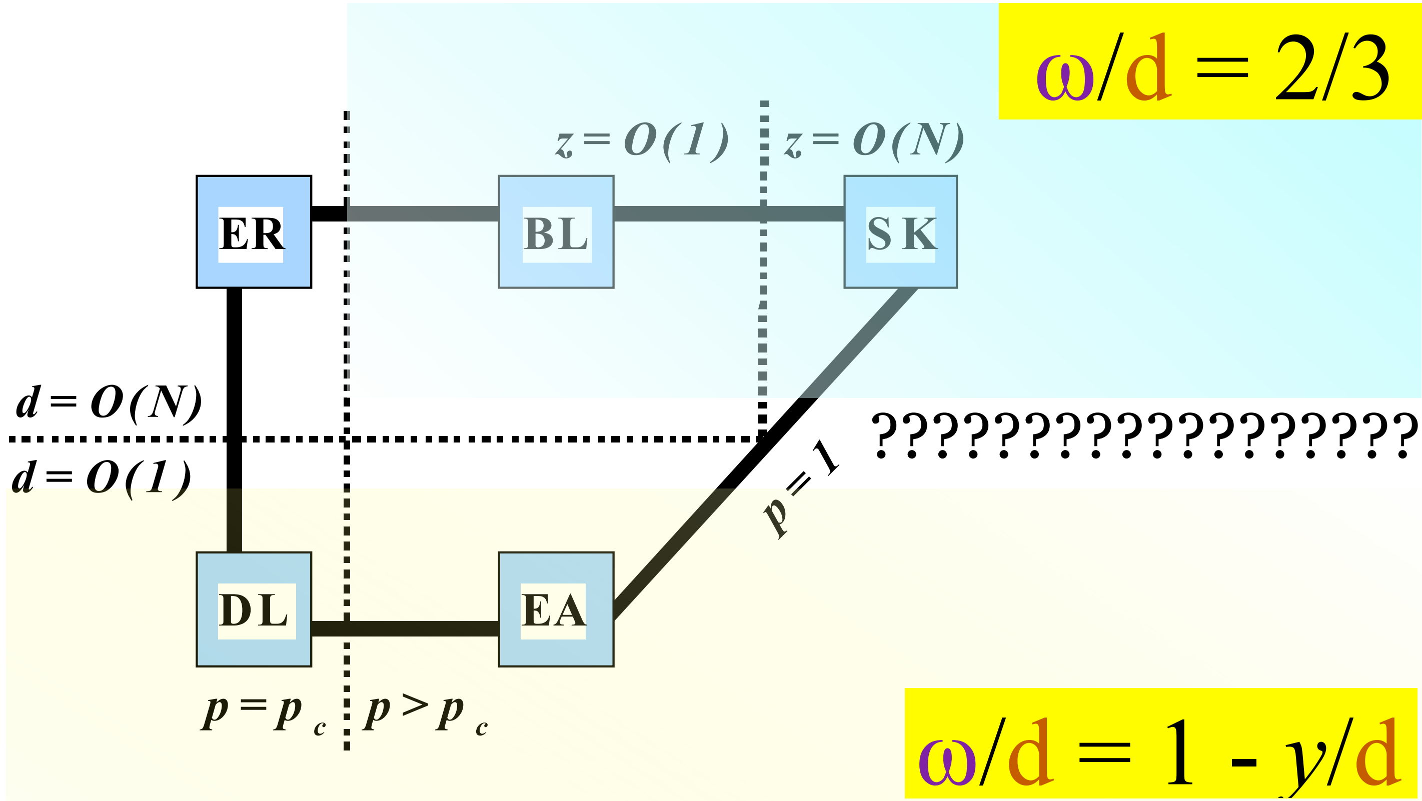
z: bond degree

- Corrections-to-Scaling: ω

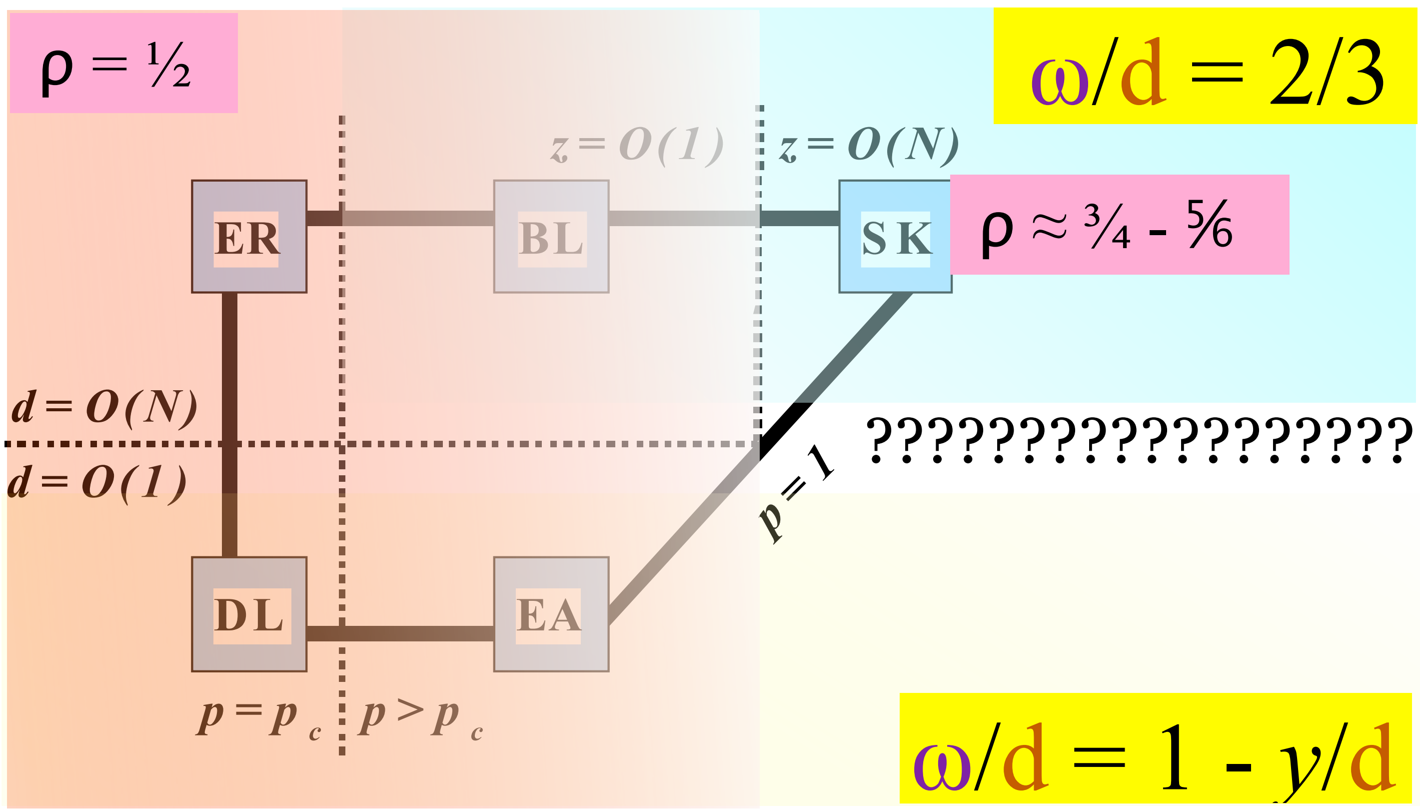


Comprehensive View on Spin Glasses:

A Set of Models:

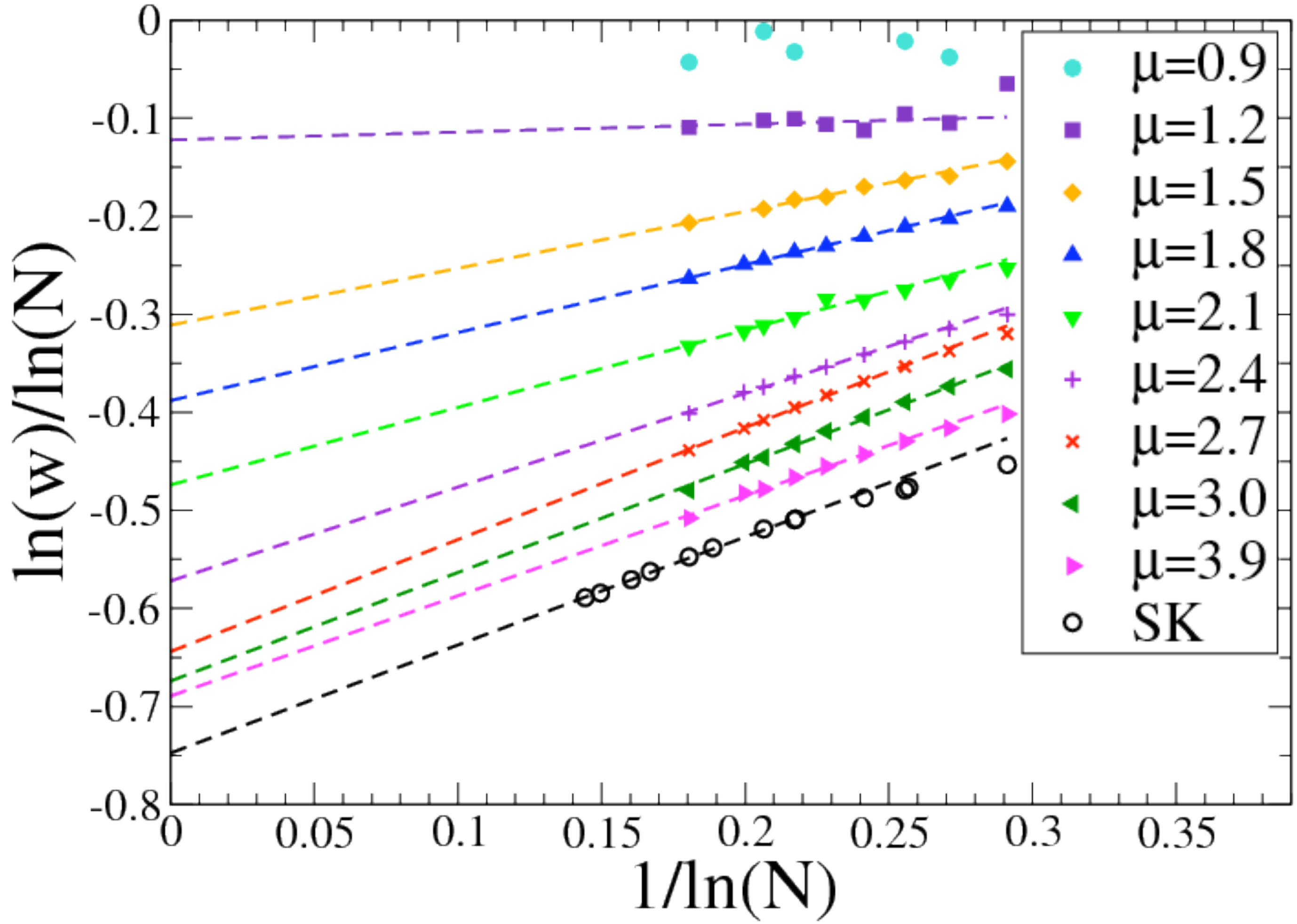
d: dimension,
p: bond density,
z: bond degree

- Corrections-to-Scaling: ω
- Energy Fluctuations: ρ



SK with Power-Law Bonds:

Power-Law Bonds: $P(J) \sim 1/|J|^{1+\mu}$ $(|J|>1)$



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