POTSDAM INSTITUTE FOR CLIMATE IMPACT RESEARCH

BASIN STABILITY:
LARGE PERTURBATIONS IN POWER GRIDS

Paul Schultz

PIK Research Domain IV, Transdisciplinary Concepts & Methods

April 21, 2015

Leibniz Association

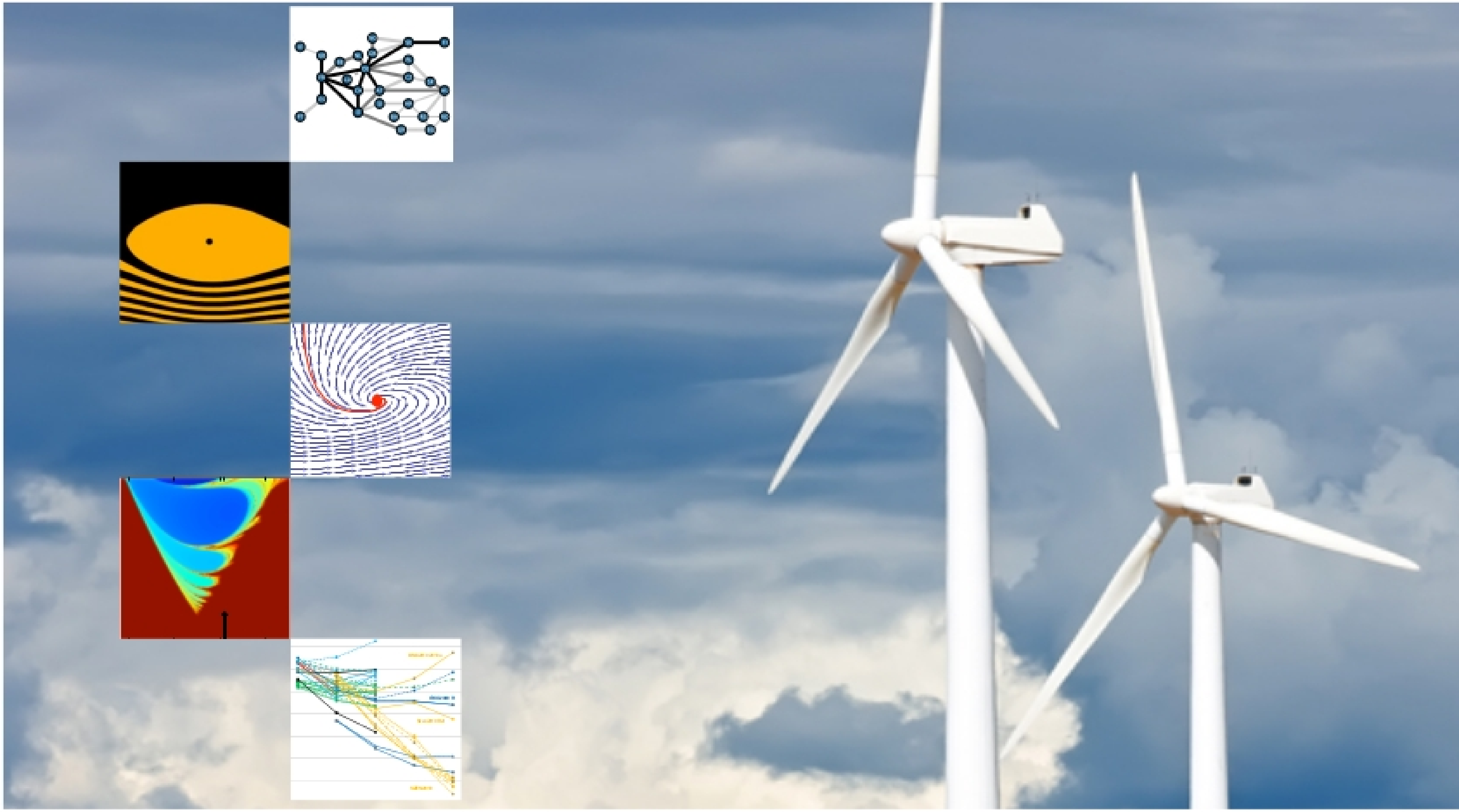
Humboldt-Universität zu Berlin

Bundesministerium für Bildung und Forschung

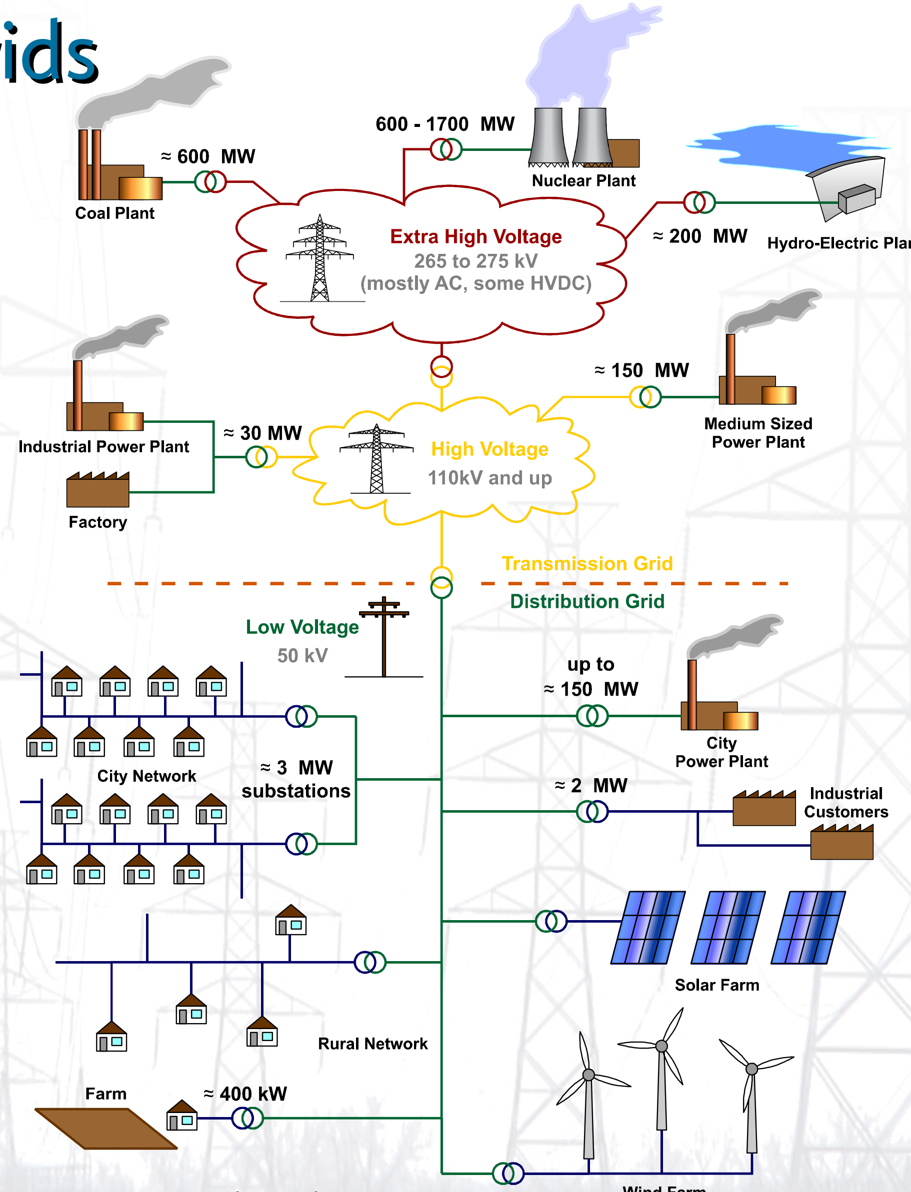
CoNDyNet

STROMNETZE Forschungsinitiative der Bundesregierung

Collective Nonlinear Dynamics of Power Grid Networks: Stability, Efficiency and Risks



Structure of Power Grids



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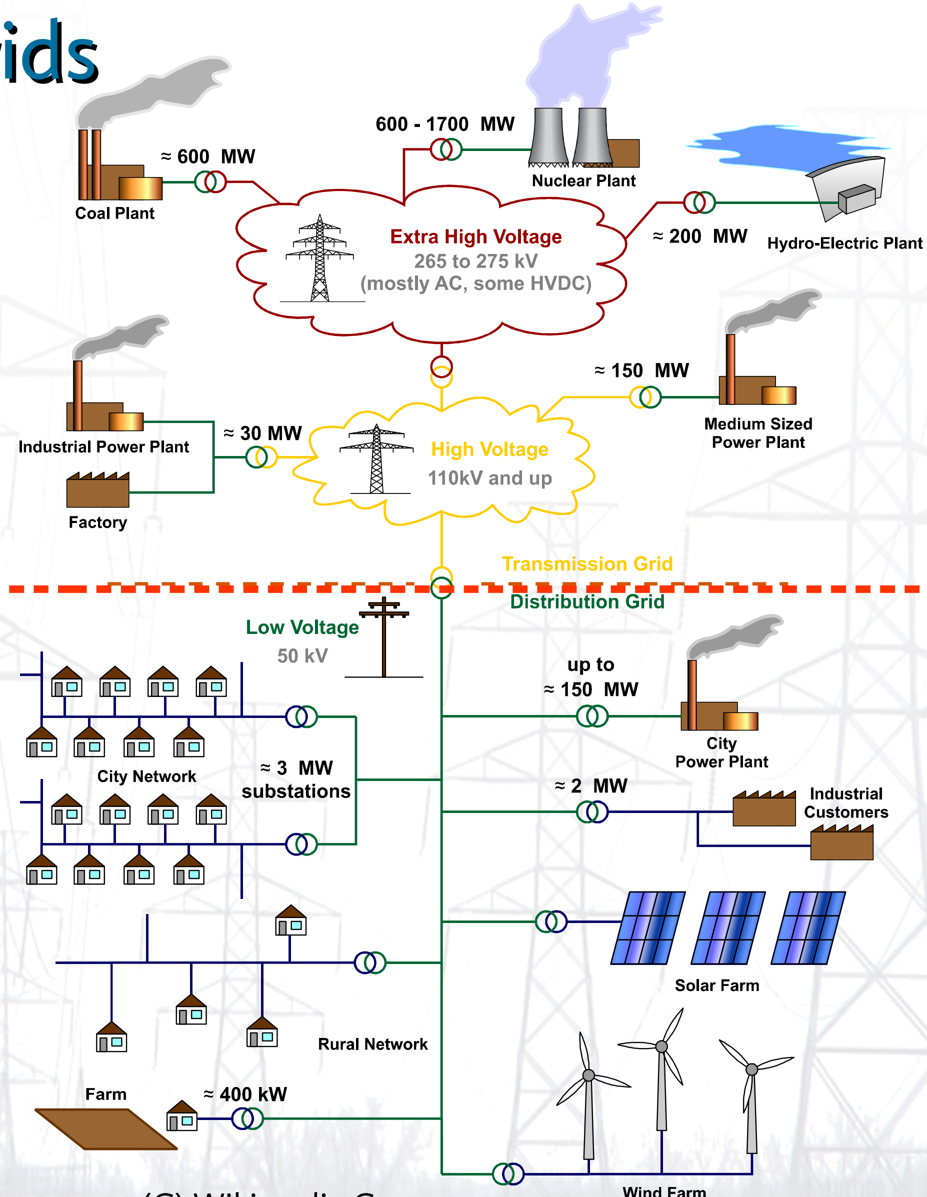

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A real world network

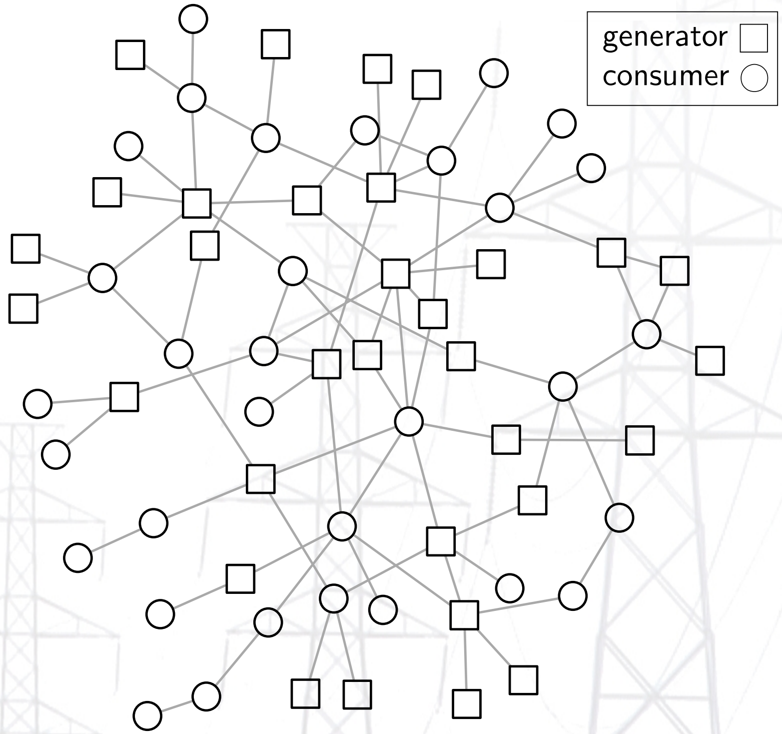
1. How to model Power Grids?

Power Grid Dynamics

$$\ddot{\phi}_i = P_i - \alpha_i \dot{\phi}_i - \sum_{j=1}^{N} K_{ij} \sin\left(\phi_i - \phi_j + \phi_{ij}\right) \quad \phi_{ij} := \arctan \frac{R_{ij}}{X_{ij}}$$

- physical model of generator dynamics
- effective network model

- simplifications:
  - constant couplings
  - dynamics close to Ω_{sync}



synchronised state:

$$\forall i : \dot{\phi}_i = 0$$

$$\forall (i,j) : \phi_i - \phi_j = \text{const.} \neq 0$$

Ref.: Filatrella et al., *EPJ B* 61(4), 2008

Ref.: Machowski et al, Wiley, *New York*, 2008

Ref.: Nishikawa & Motter, NJP (17) 15012, 2015

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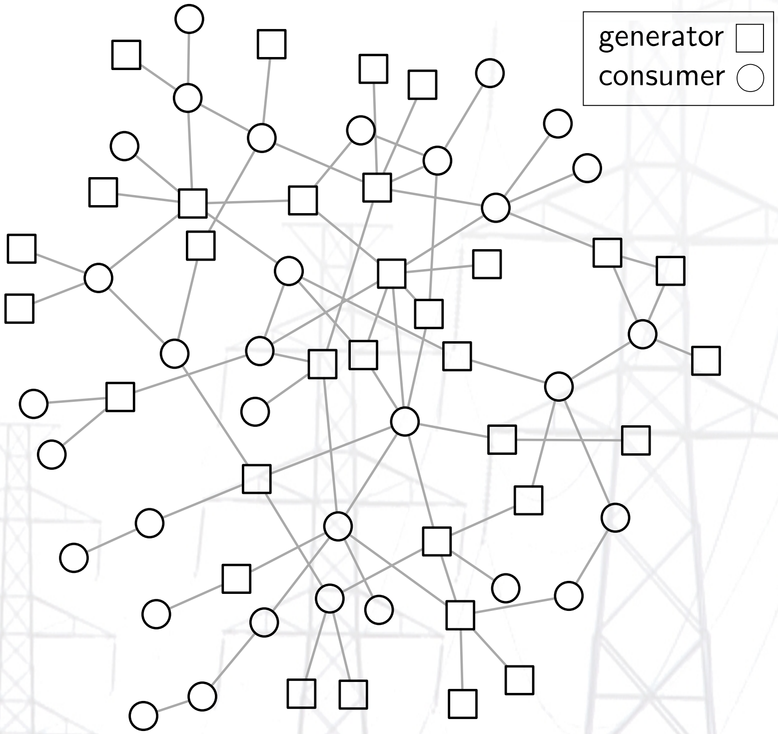

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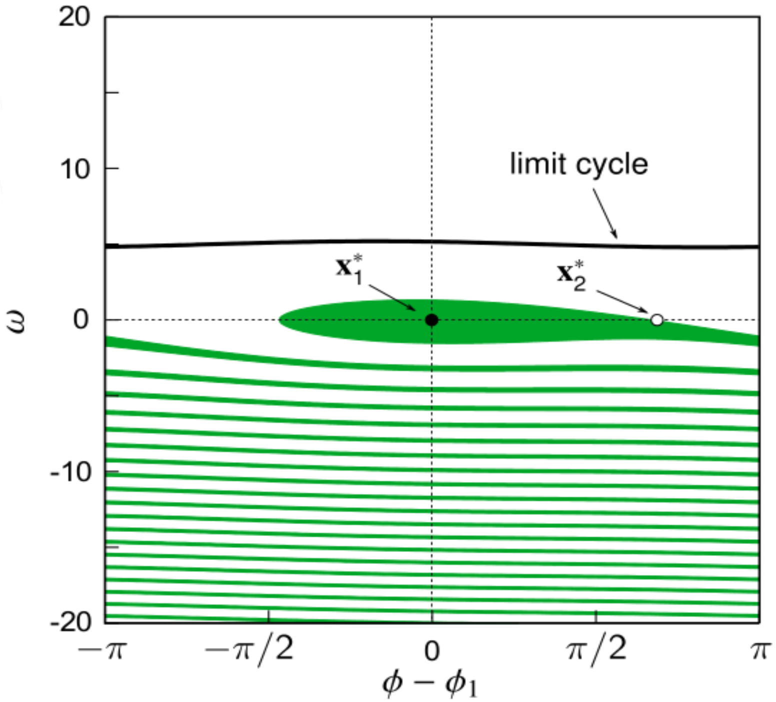
Estimation of Basin Stability



bistable **system:**

final state depends on IC

$$\mathbf{B} = \{x \in S | \Phi_t(x) \to \mathbf{I}\}$$

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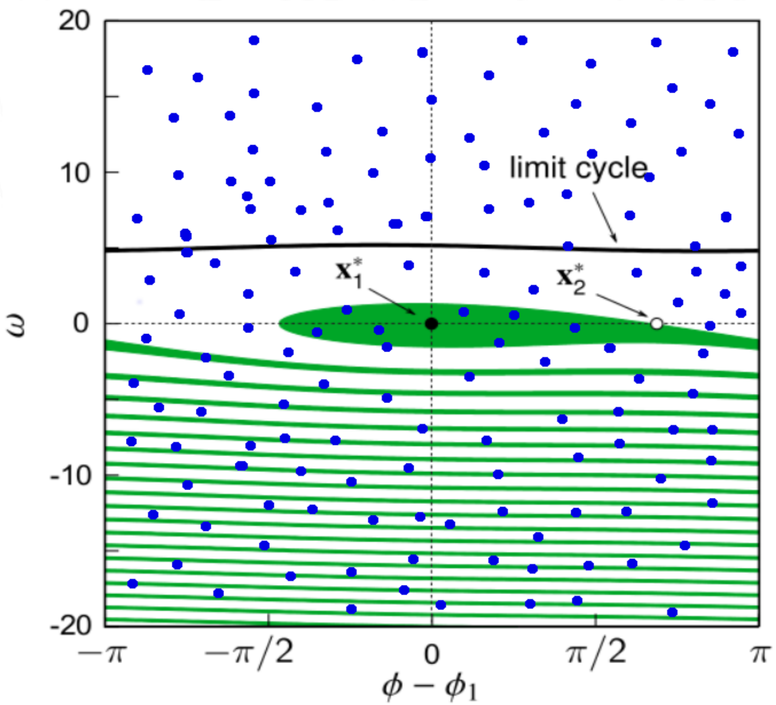

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final state depends on IC

$$\mathbf{B} = \{x \in S | \Phi_t(x) \to \mathbf{I}\}$$

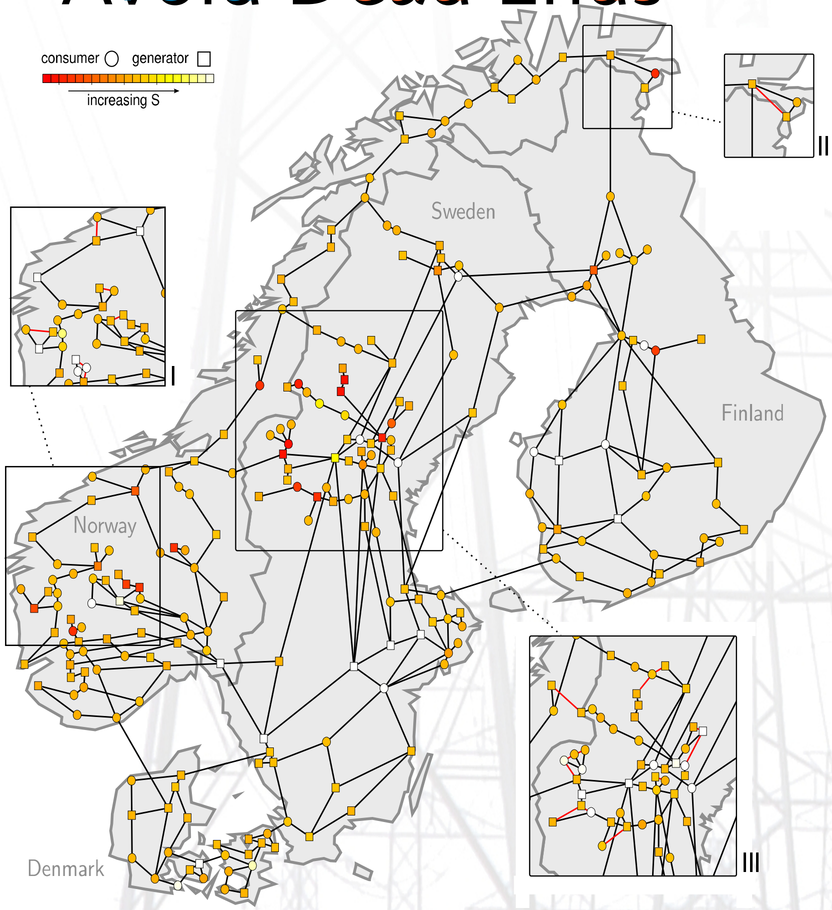
$$BS = \frac{\#\bullet \text{ in } \bullet}{\# \text{ all } \bullet}$$

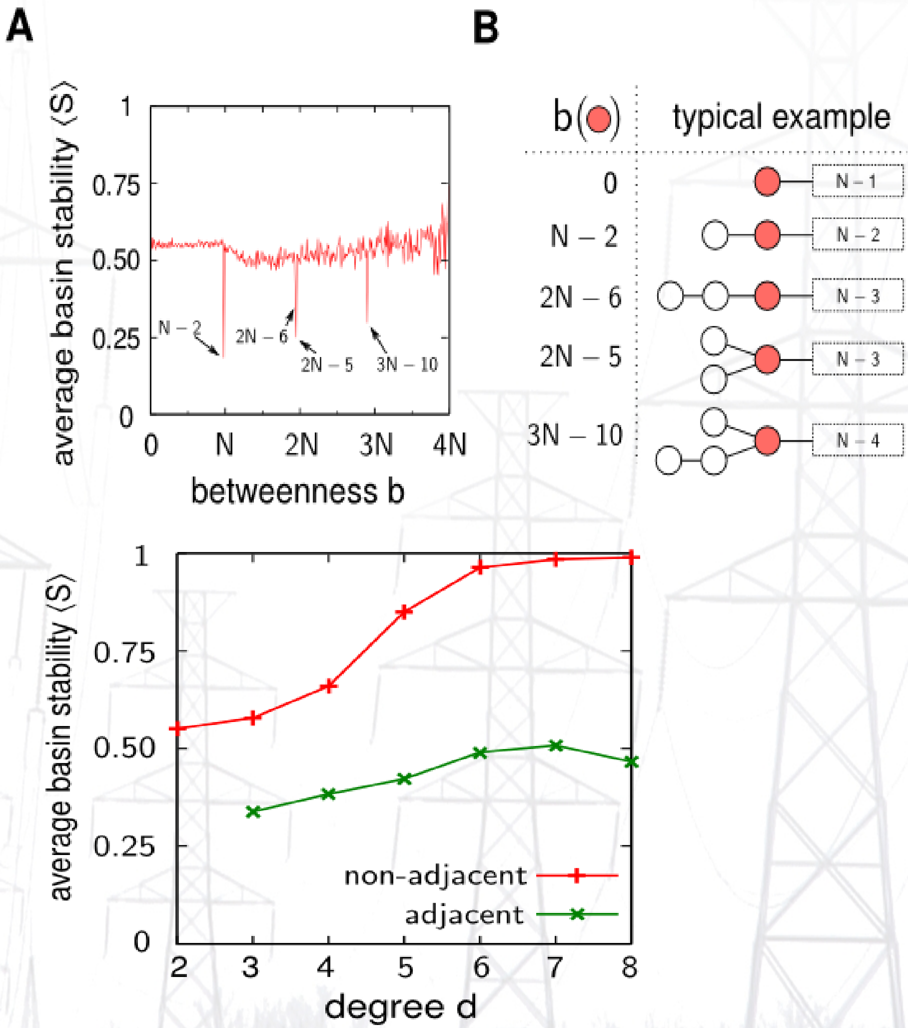
$$S_{\mathbf{B}} = \int_S \chi_{\mathbf{B}} \rho(x) dx$$

single node basin stability

$$\mathbf{B_i} := \{(\phi_i, \omega_i) \in \mathbf{B} | \forall_{j \neq i} (\phi_j, \omega_j) = (\phi_j^*, 0)\}$$

Avoid Dead Ends





Ref.: Menck et. al., Nature Com. (5), 2014

2. Random Network Model

Random Network Model: Key Ideas

- Model wide range of observed network properties
  → degree distribution, sparsety, connectivity, aspl, …

- Create realisticly appearing power grids with spatial embedding
  → supports random as well as given node locations

- Low computational complexity allows for extensive simulations

- Plausible construction mechanism using only few assumptions

- Two stages: initialisation and growth
  → tunable trade-off: cost minimisation vs. redundancy

Ref.: Rosas-Casals, Topological Complexity of the Electricity Transmission Network., UPC (2009)

Model Implementation

- Initialisation with minimum spanning trees
  → minimise overall edge weight (i.e. length)

Ref.: Borůvka 1926, Kruskal 1956, Prim 1957

- Transmission lines might be split if a new power plant appears closeby

- Trade-off between redundancy and costs

$$\text{maximise } f = \frac{(\text{new redundant lines})^r}{\text{spatial distance}}$$

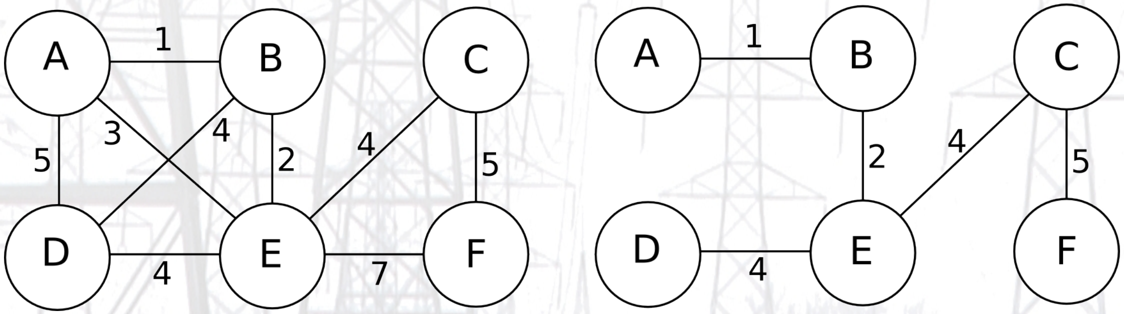
Minimum Spanning Tree MST

- Invented to design Moravia (Mähren) Power Grid

Ref.: Borůvka (1926), Kruskal (1956), Prim (1957)

- Draws connected graph (i.e. a tree) with N-1 edges between a set of nodes that minimises a given edge weight → spatial distance, resistance, general cost function

- Unique if all edge weights are mutually distinct

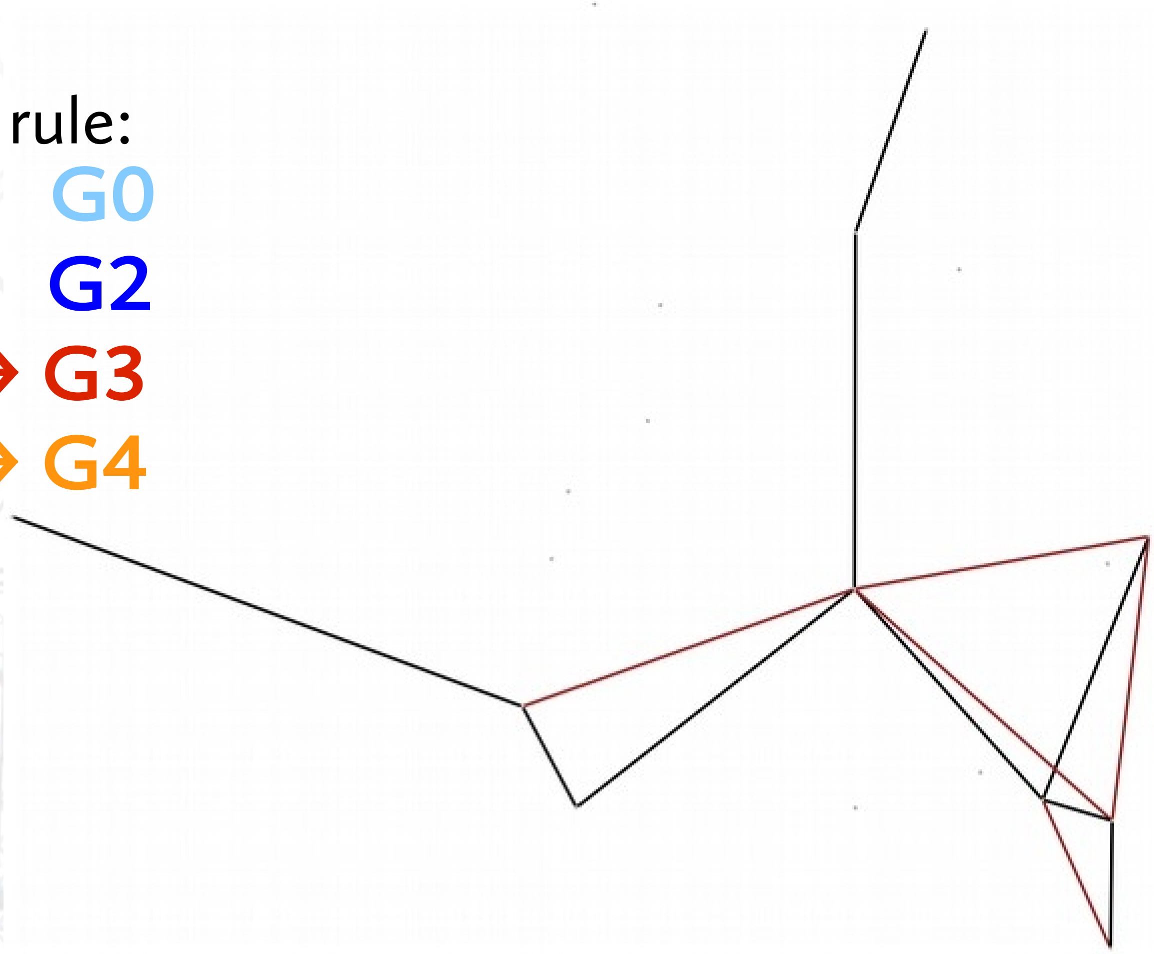


Growth Algorithm

attachment rule:

s → G0

1-s → G2
- p → G3
- q → G4



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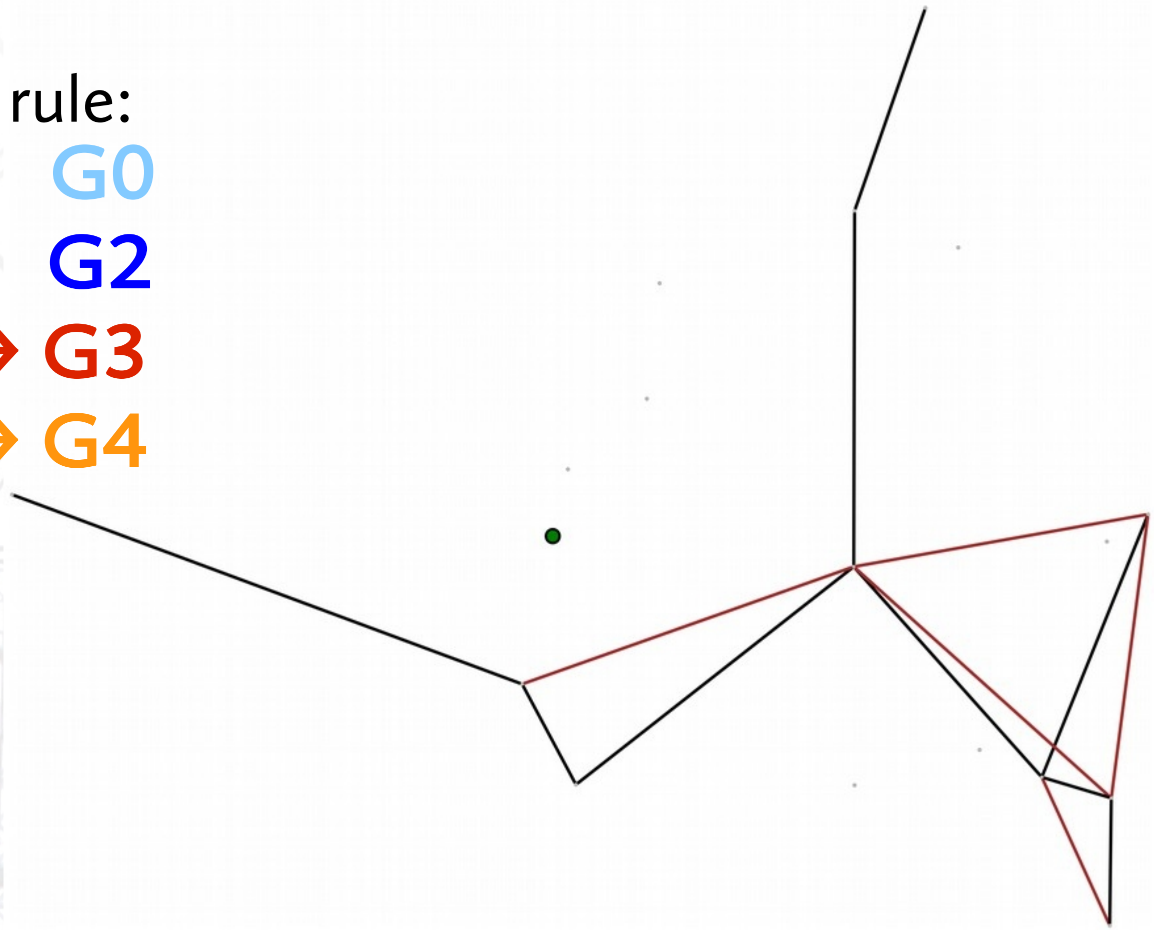

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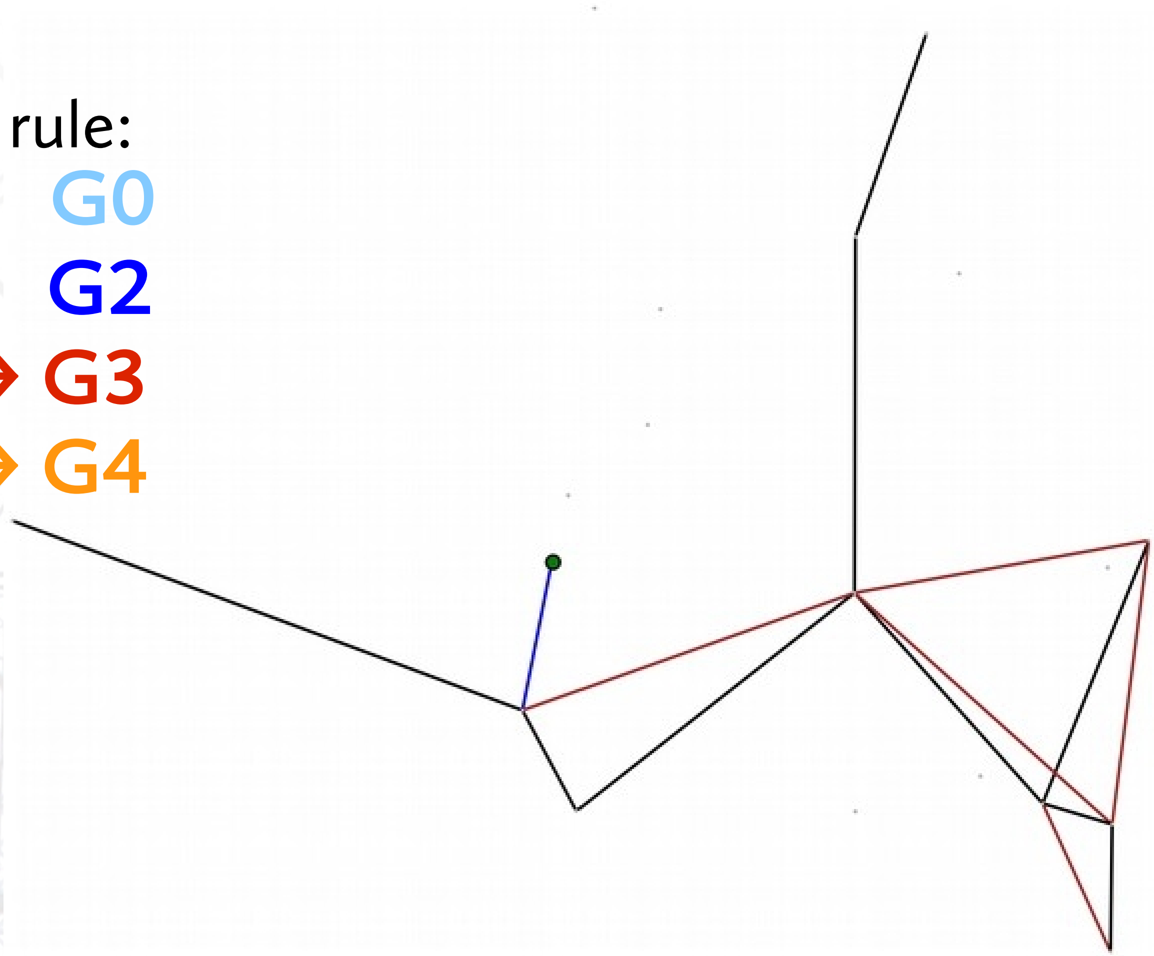

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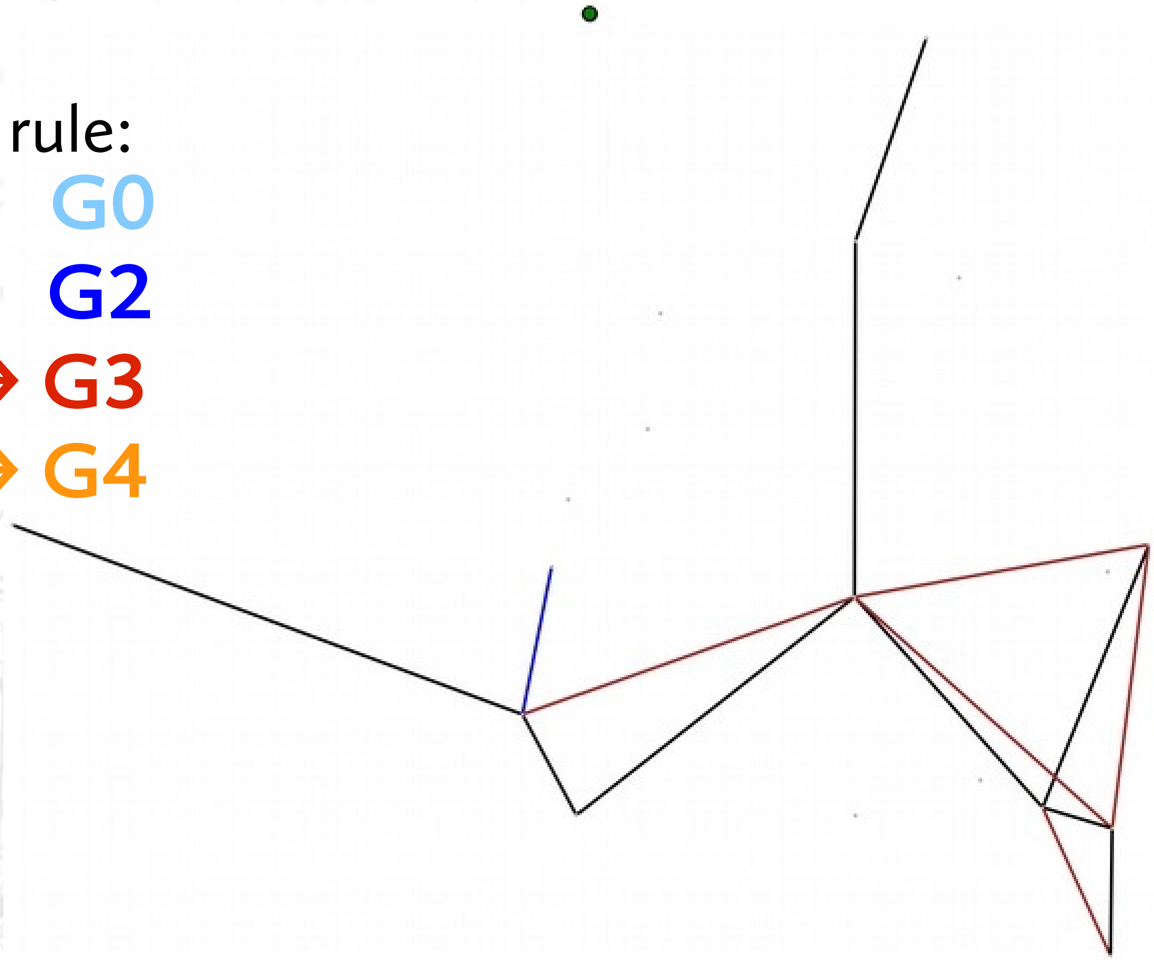

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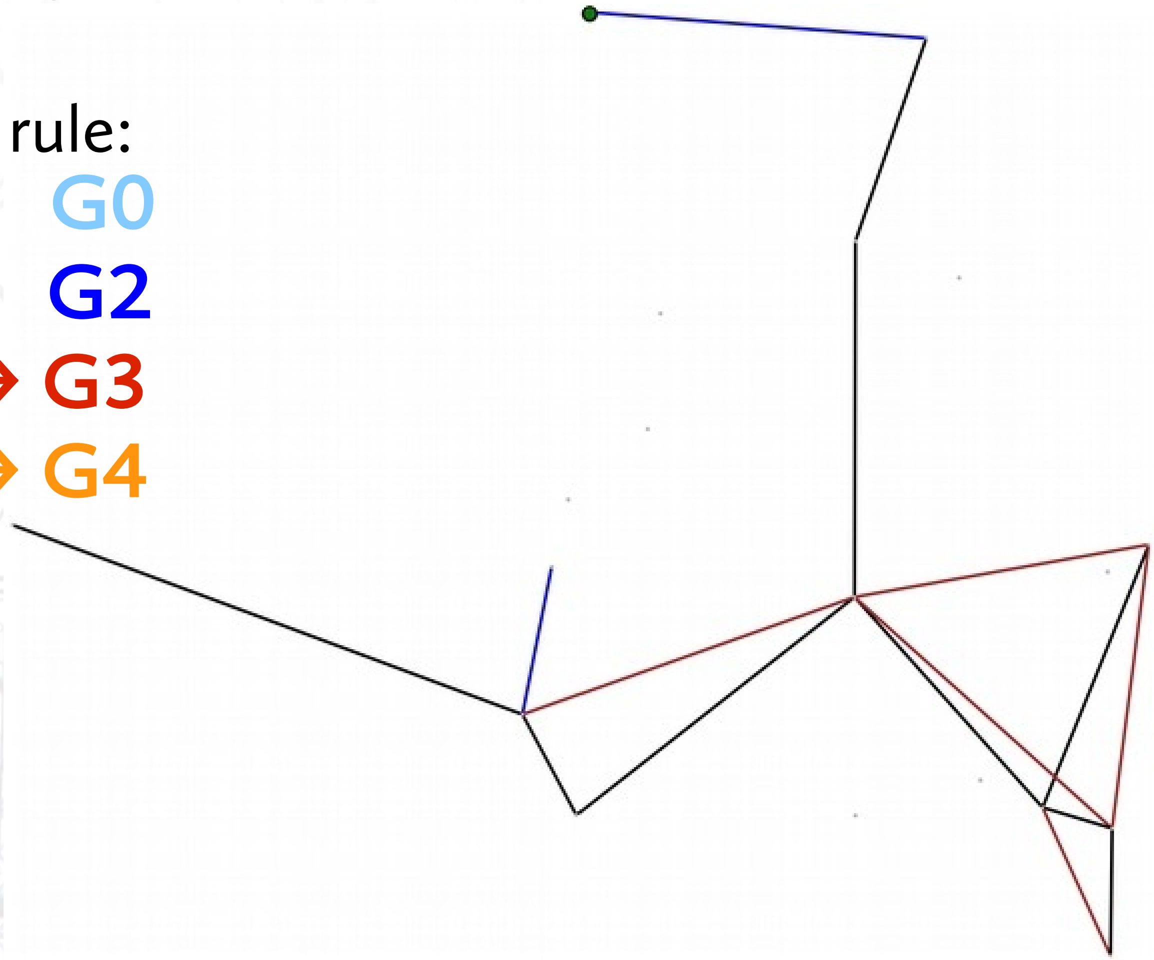

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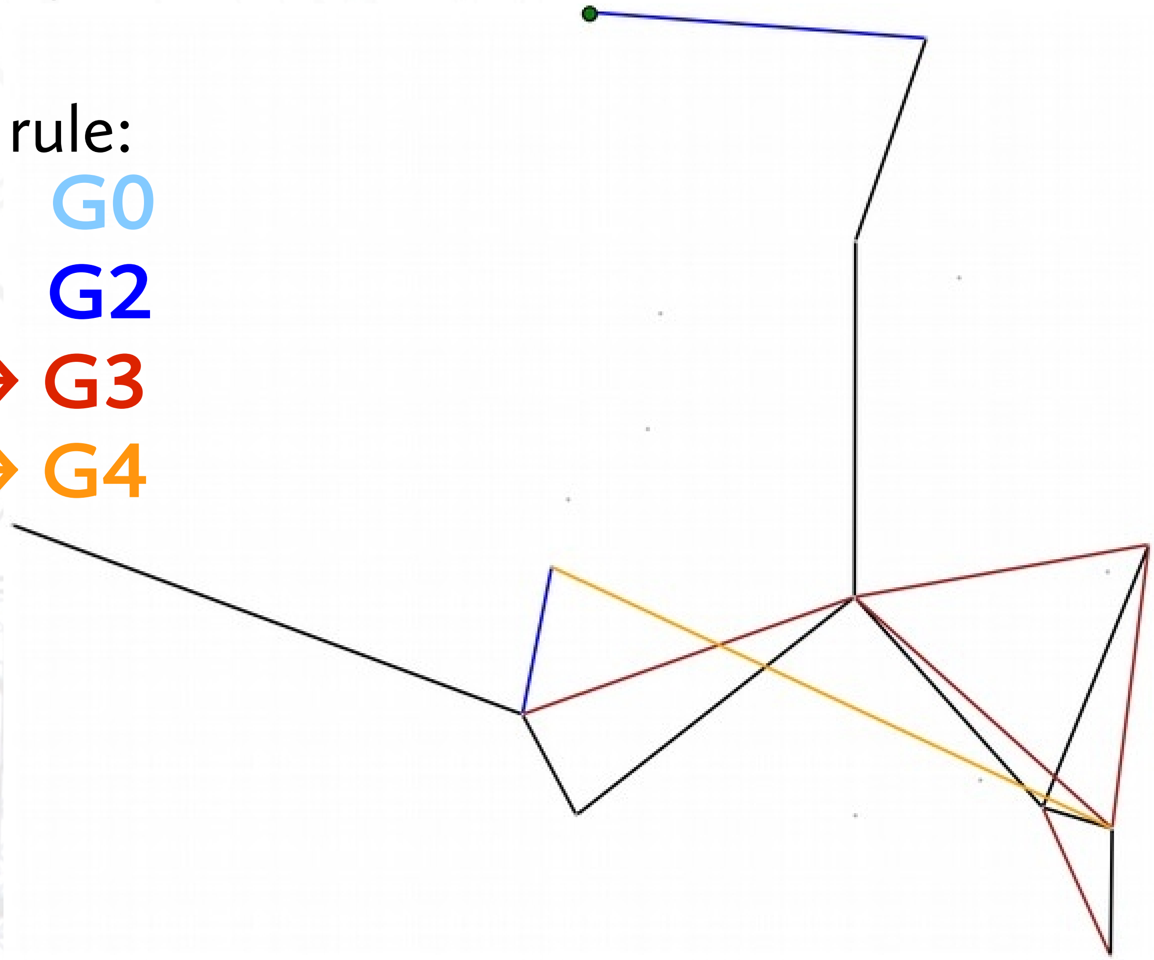

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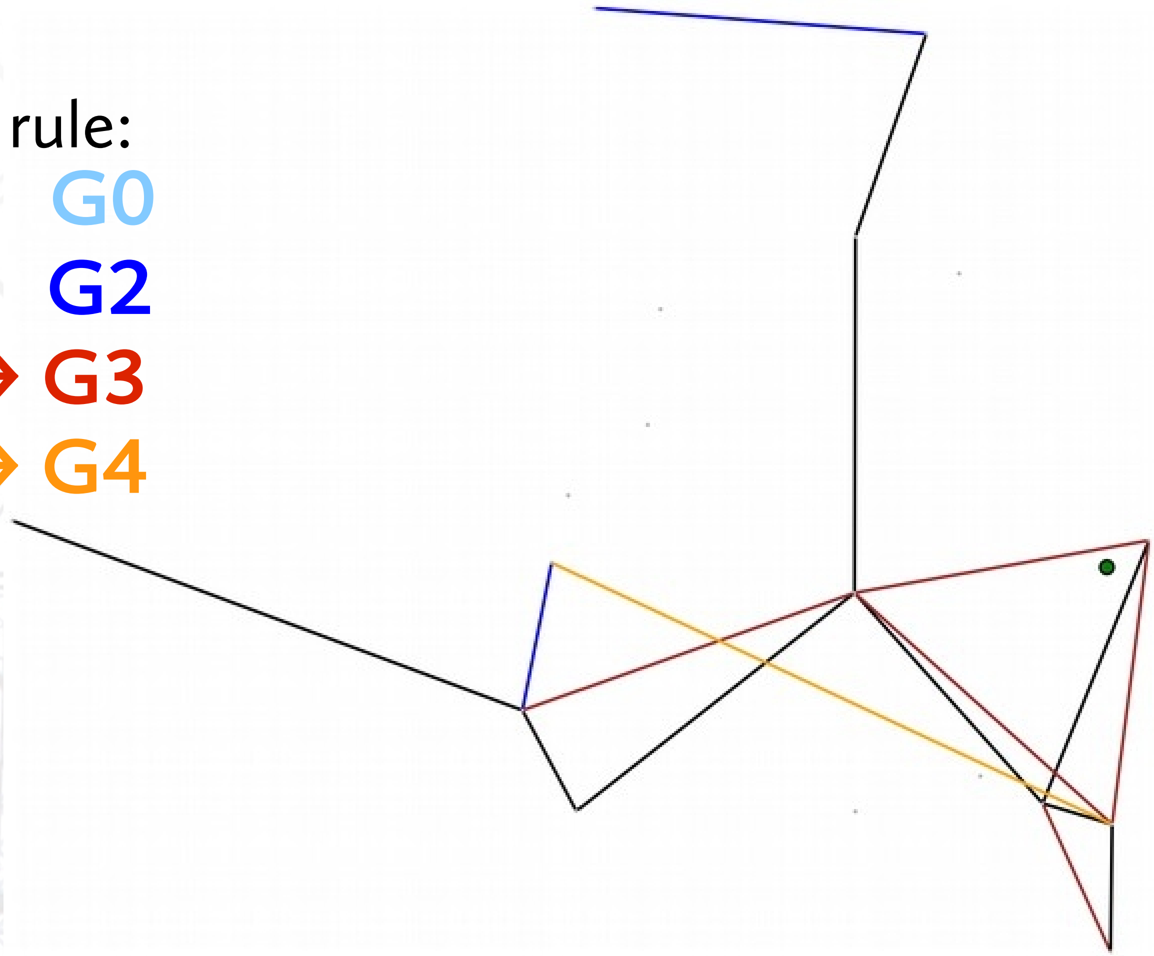

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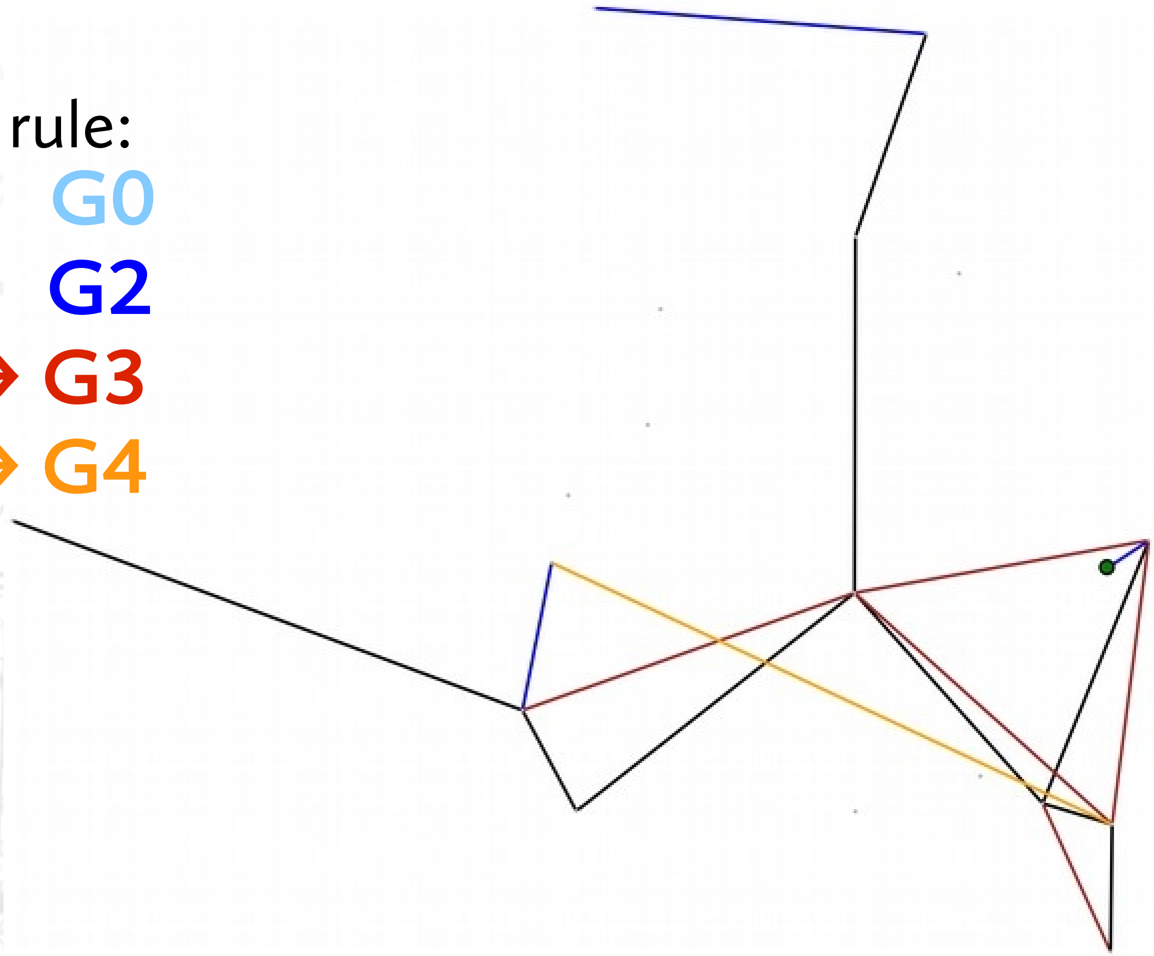

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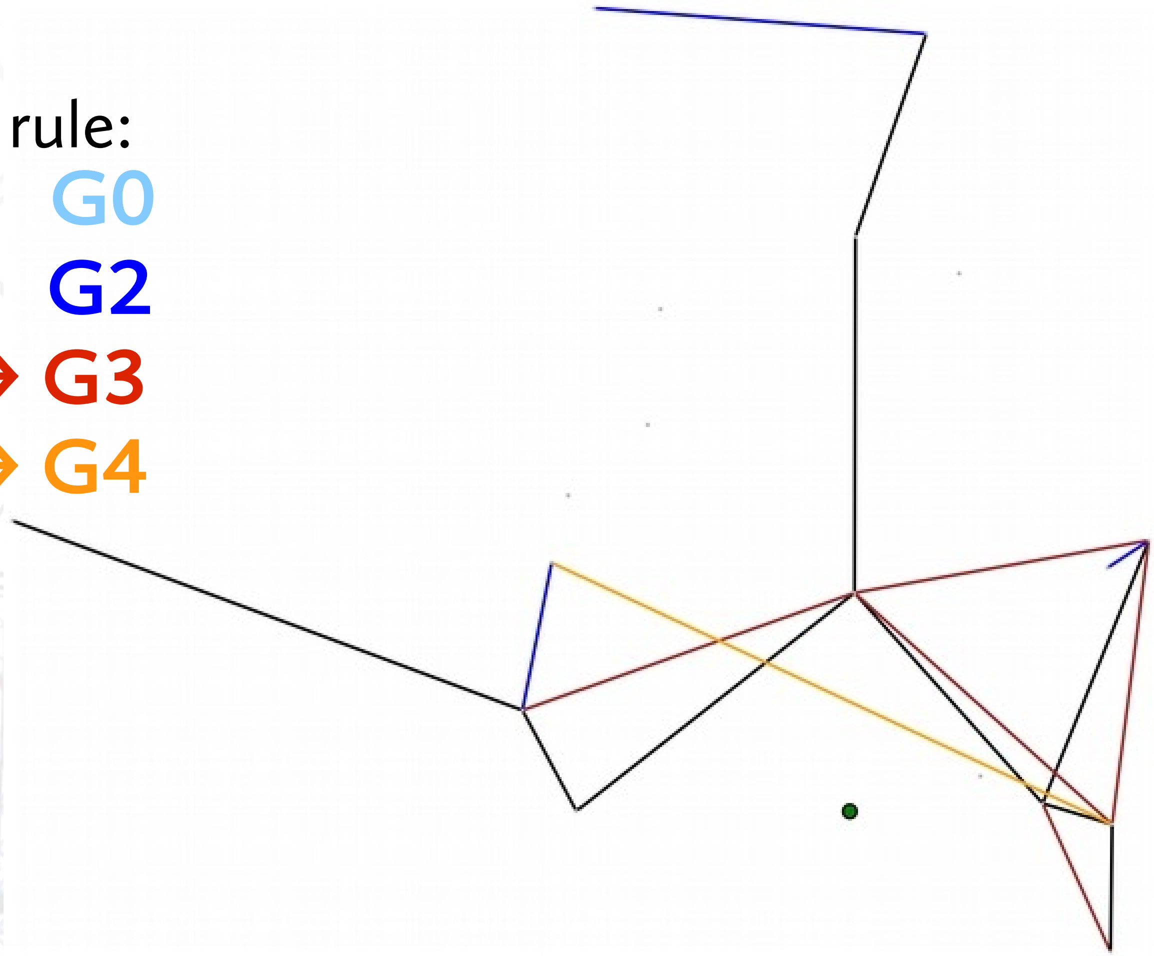

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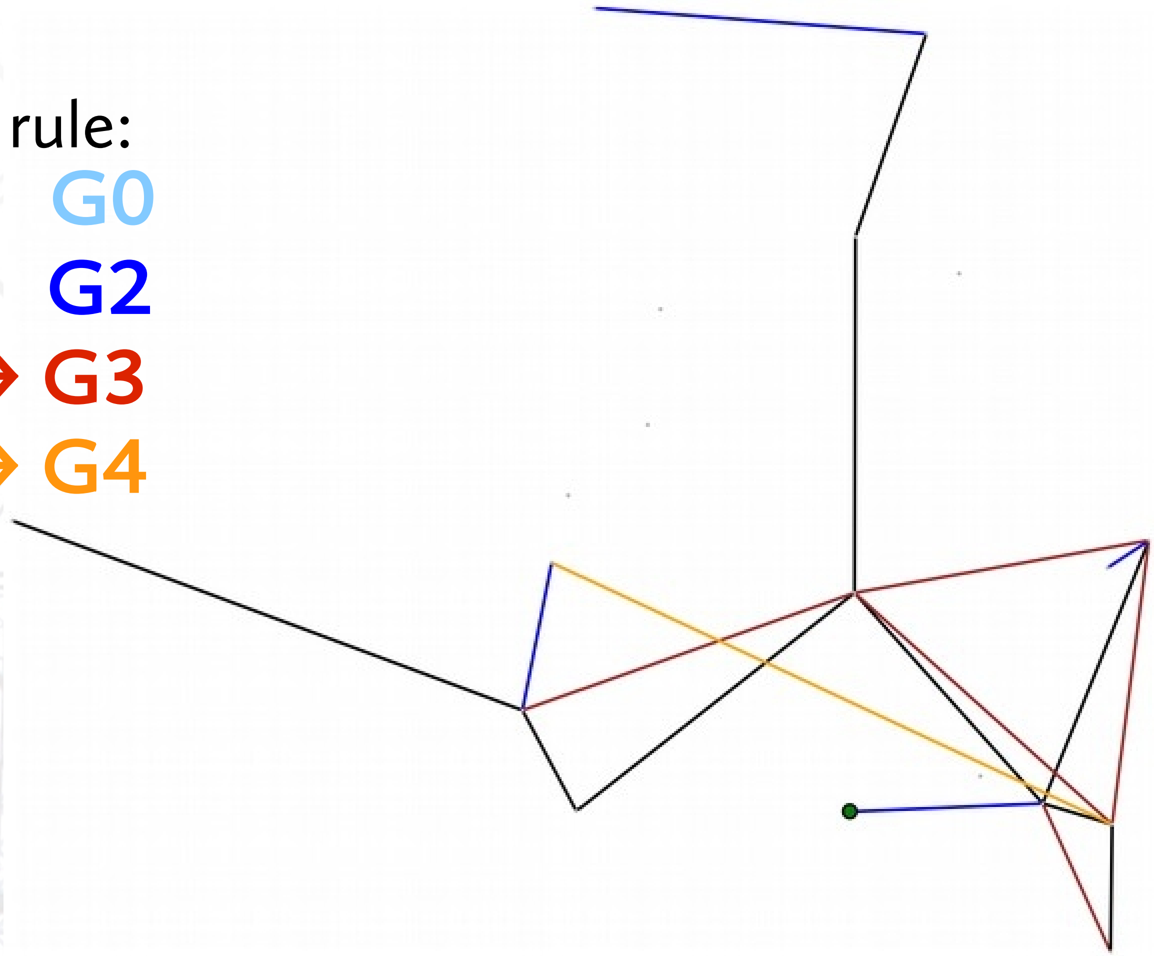

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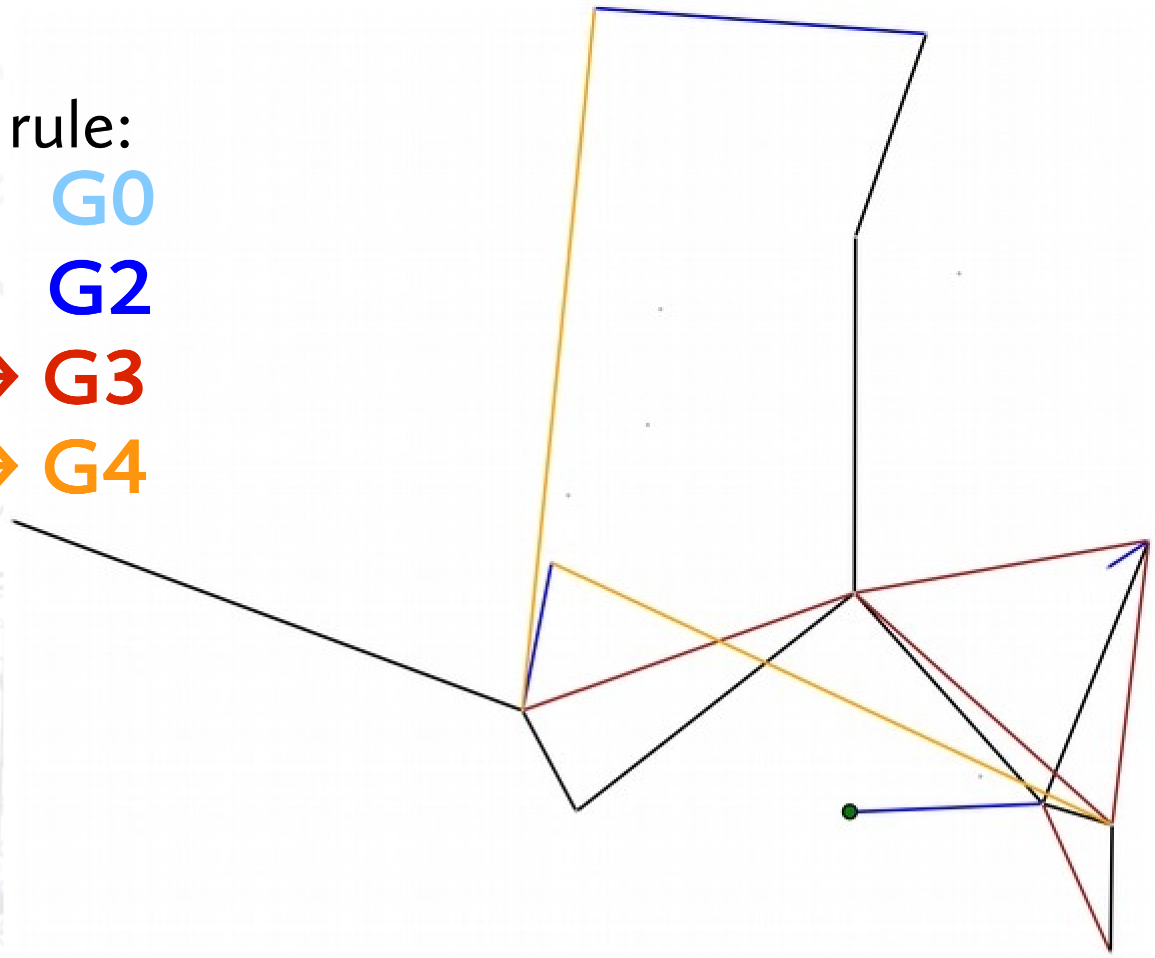

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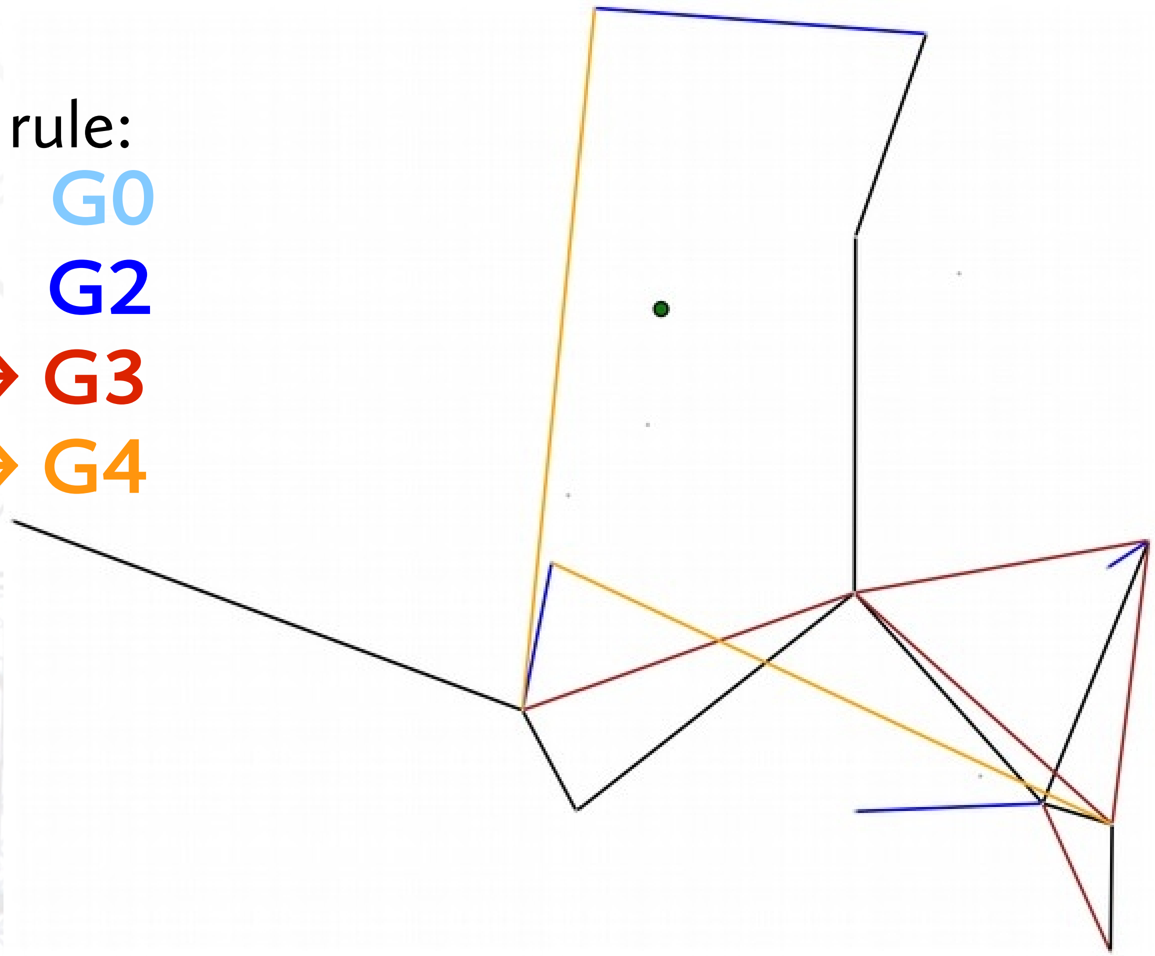

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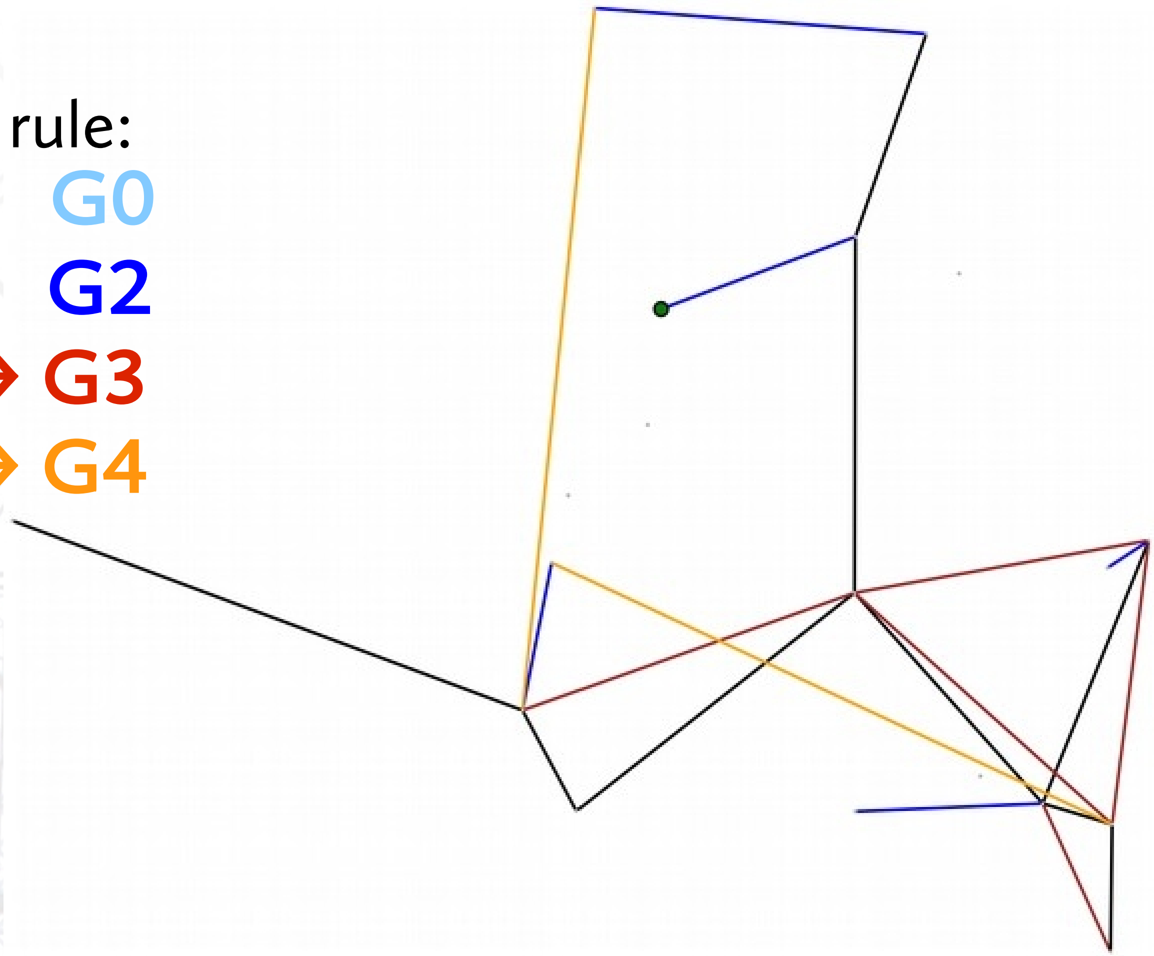

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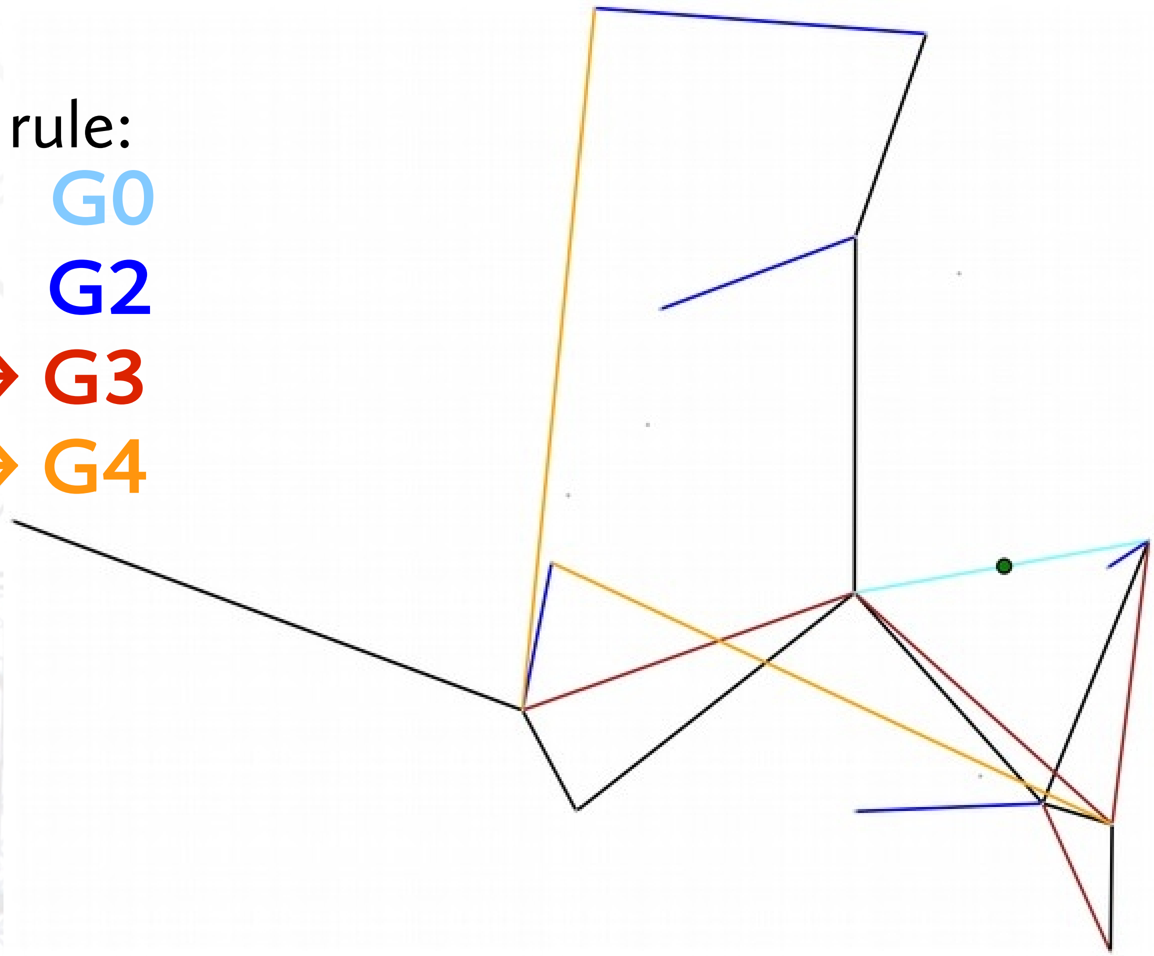

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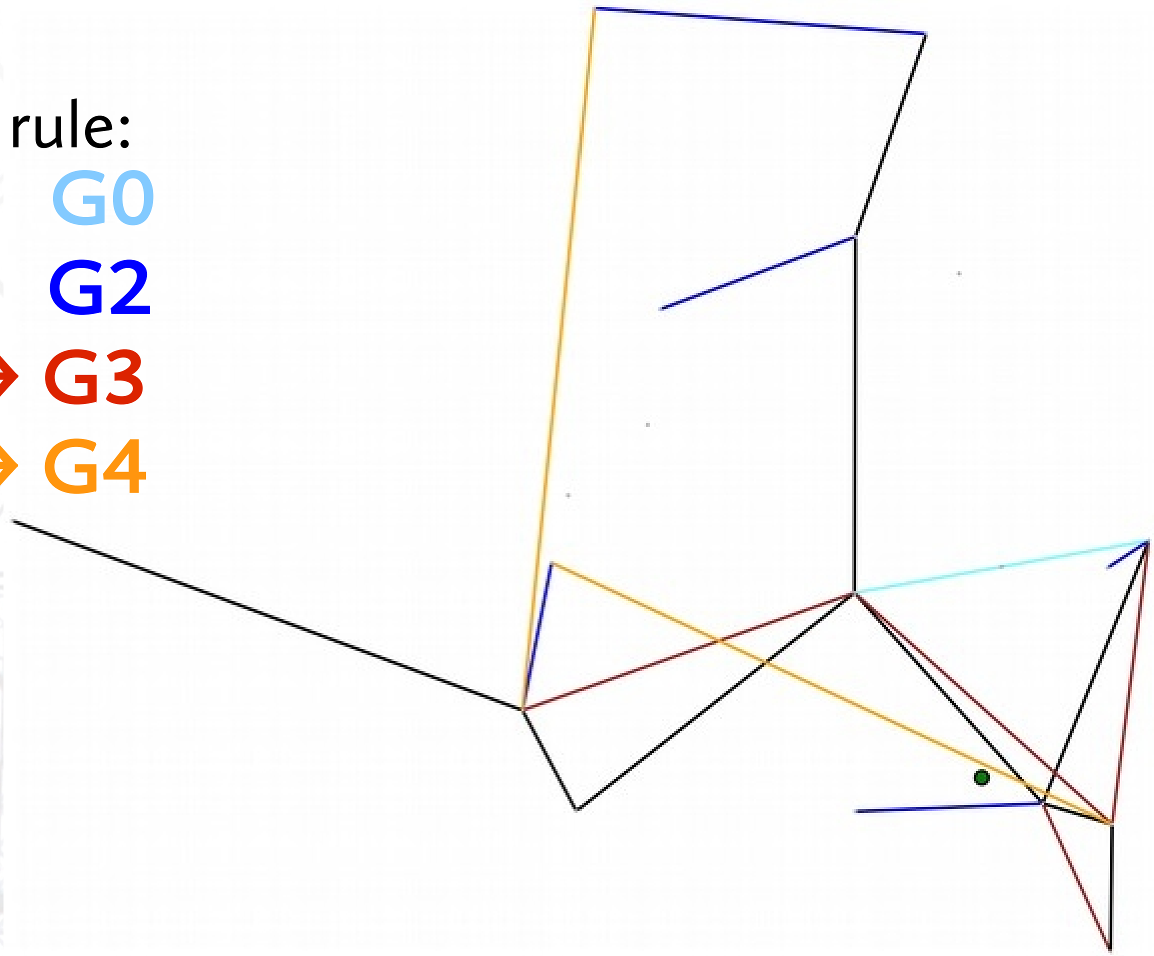

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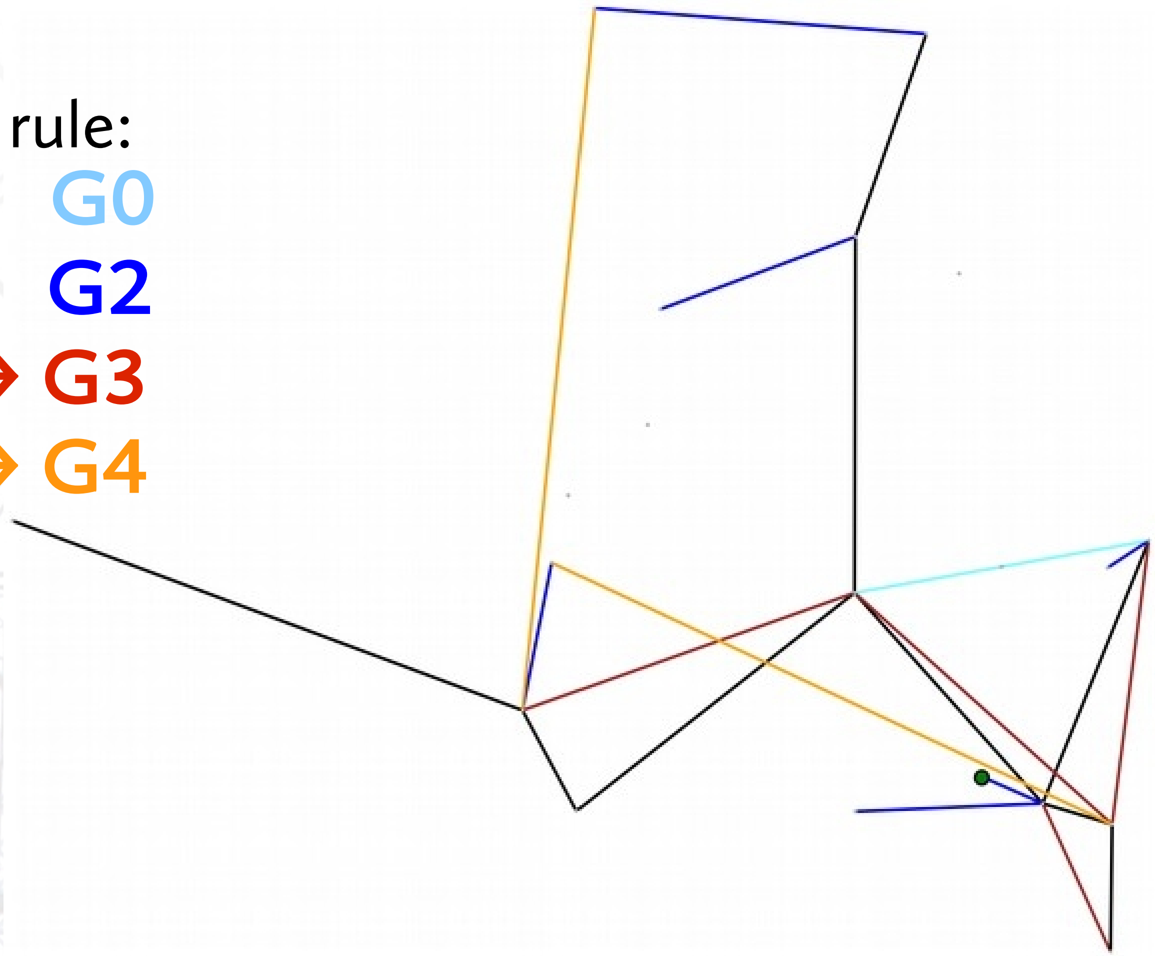

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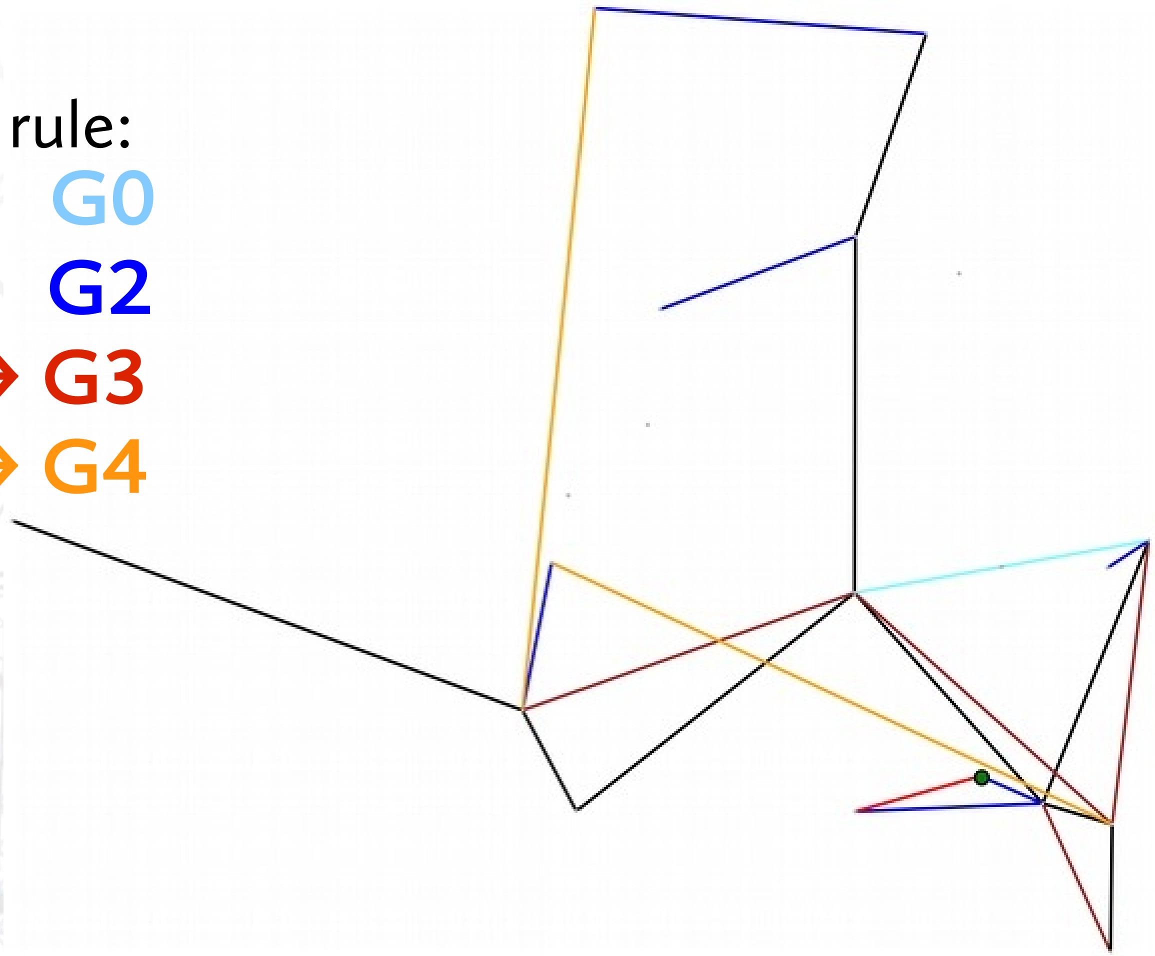

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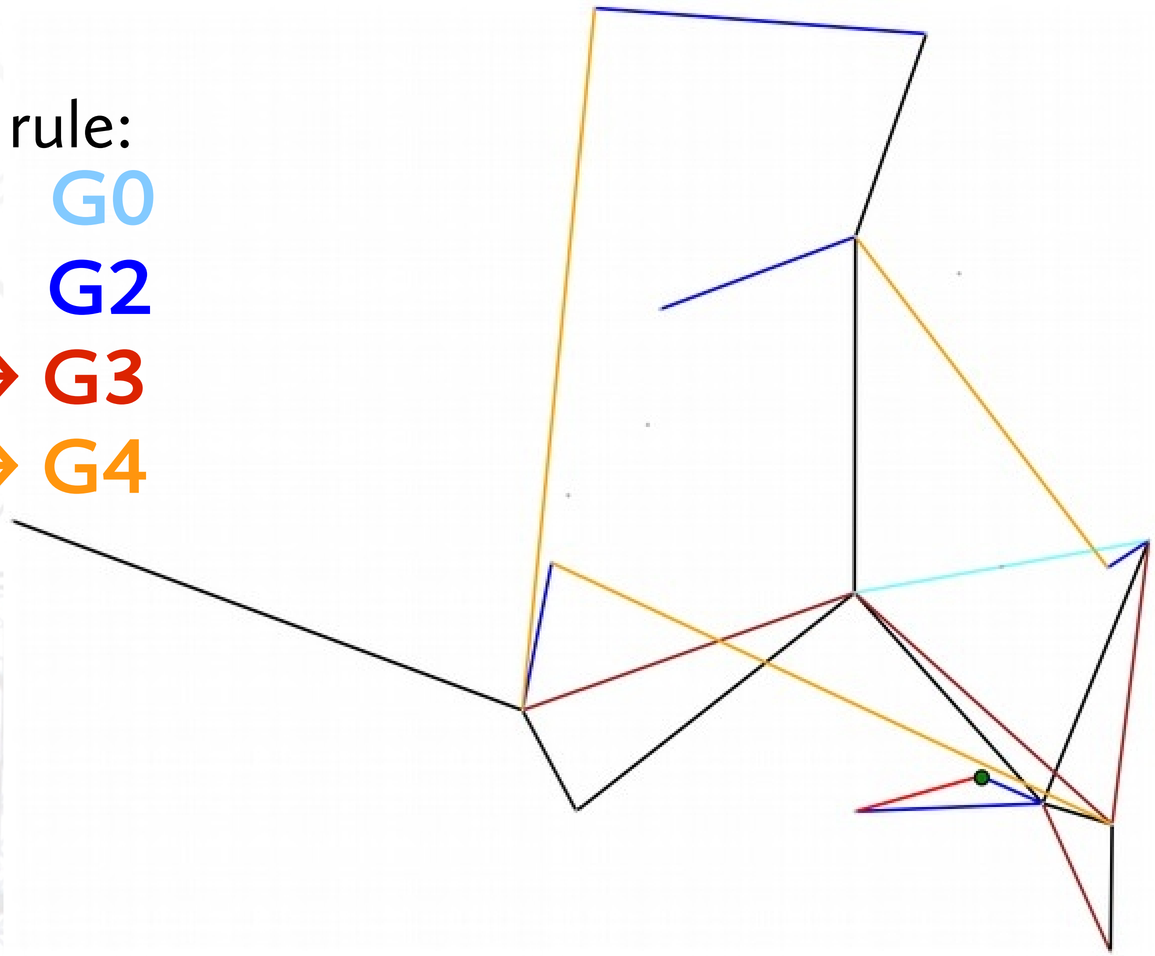

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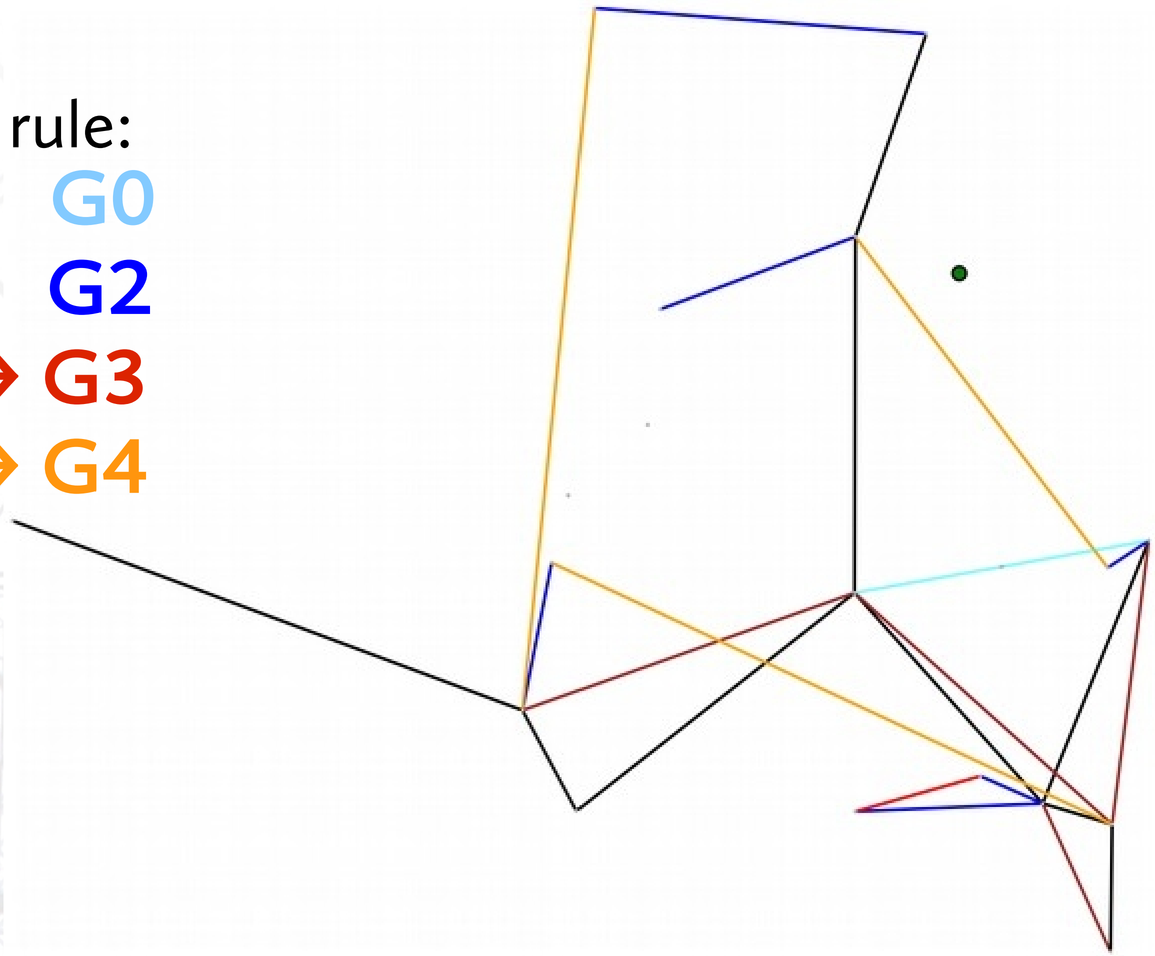

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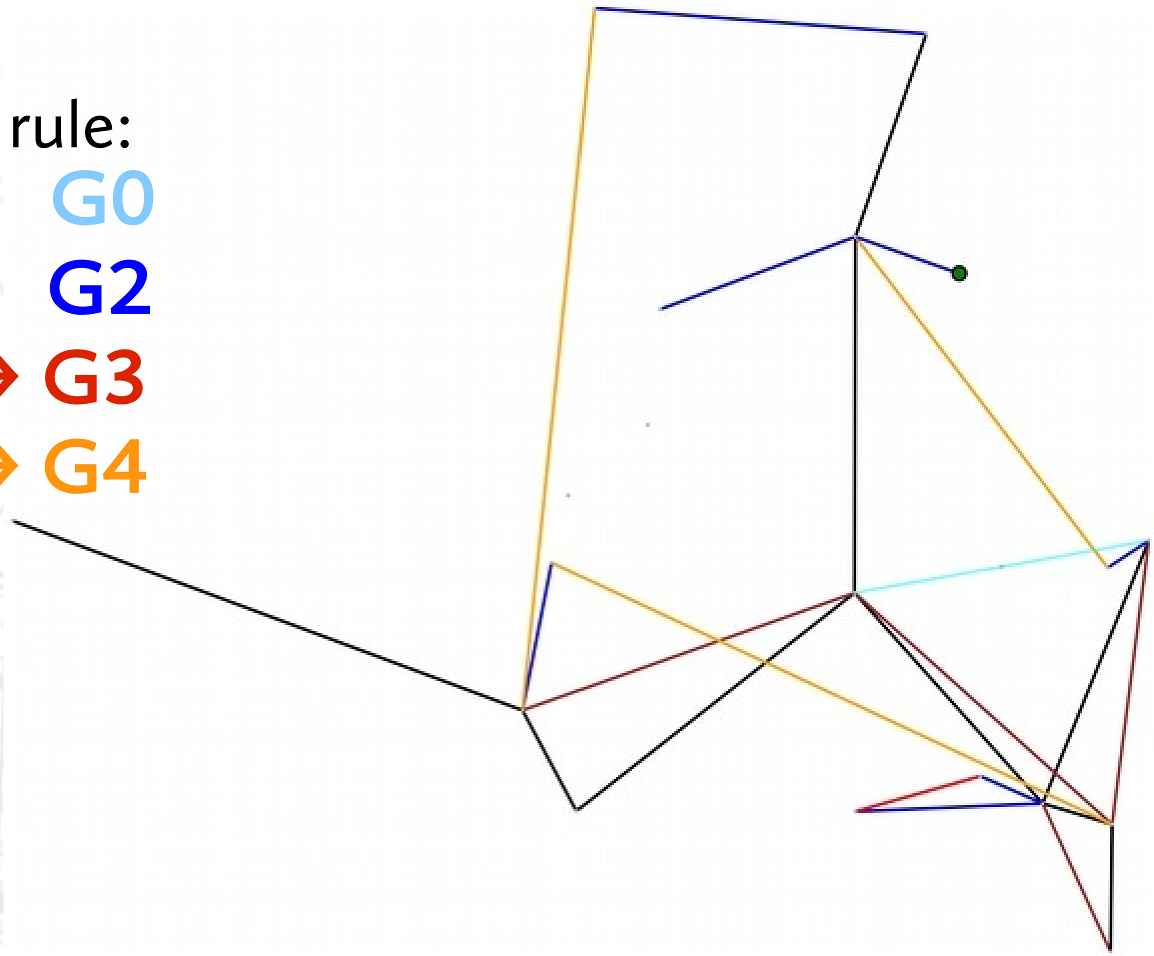

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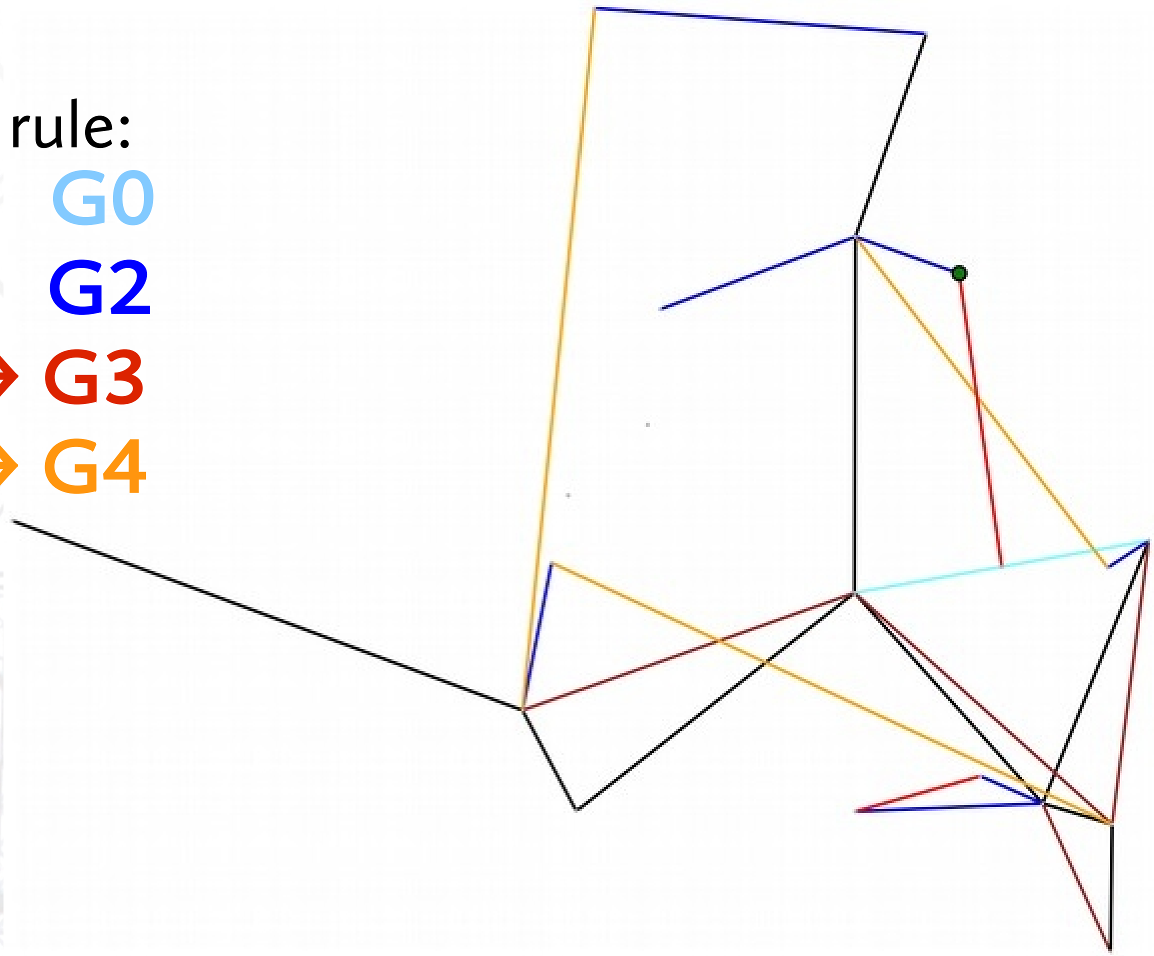

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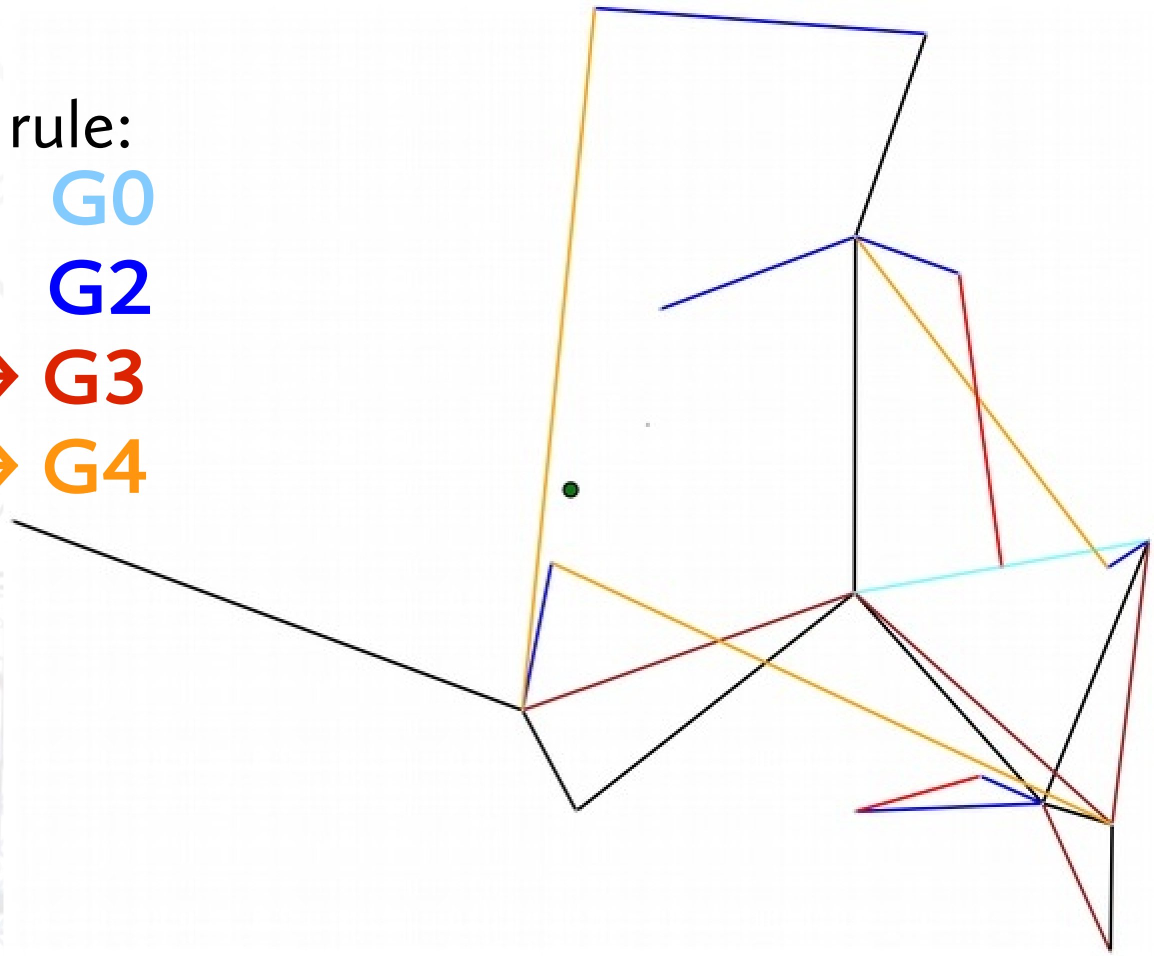

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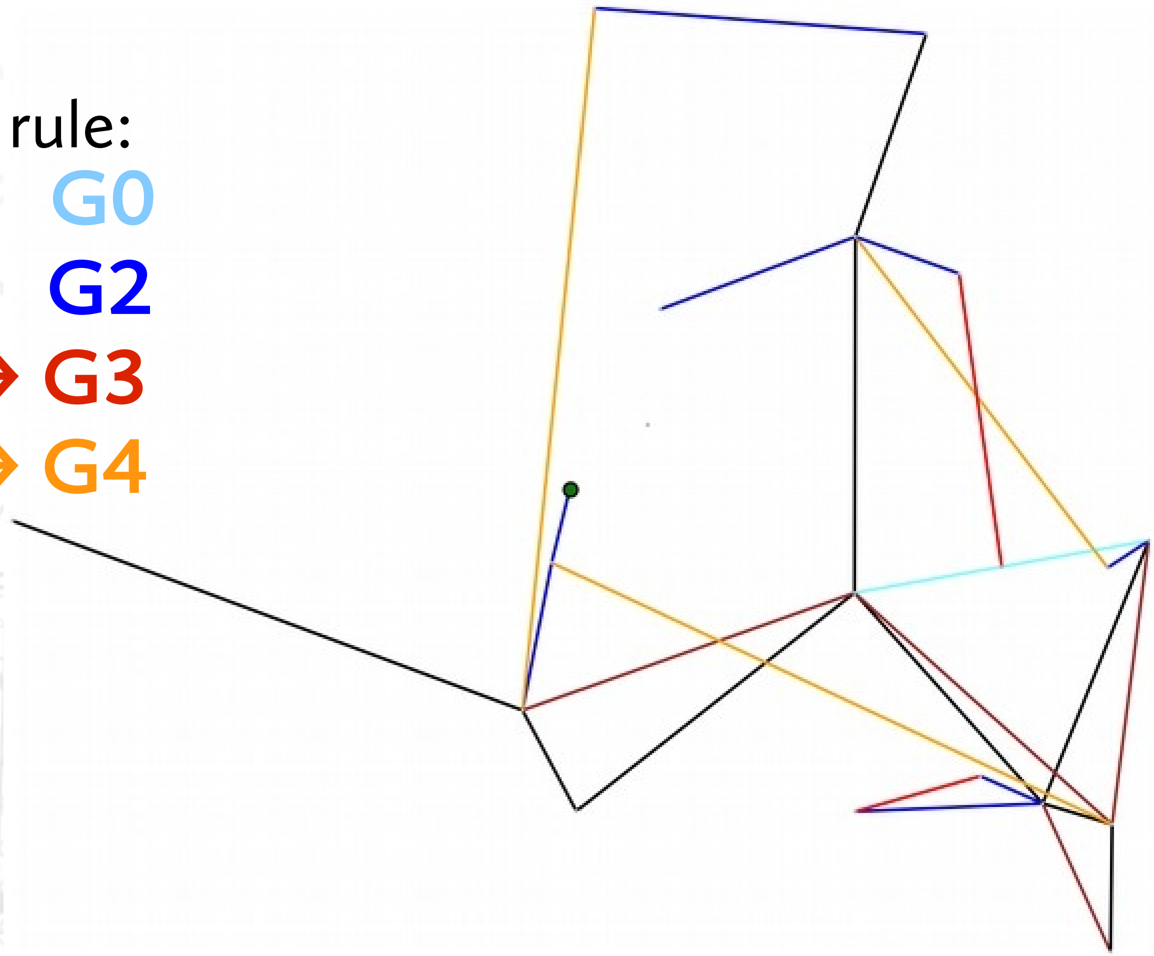

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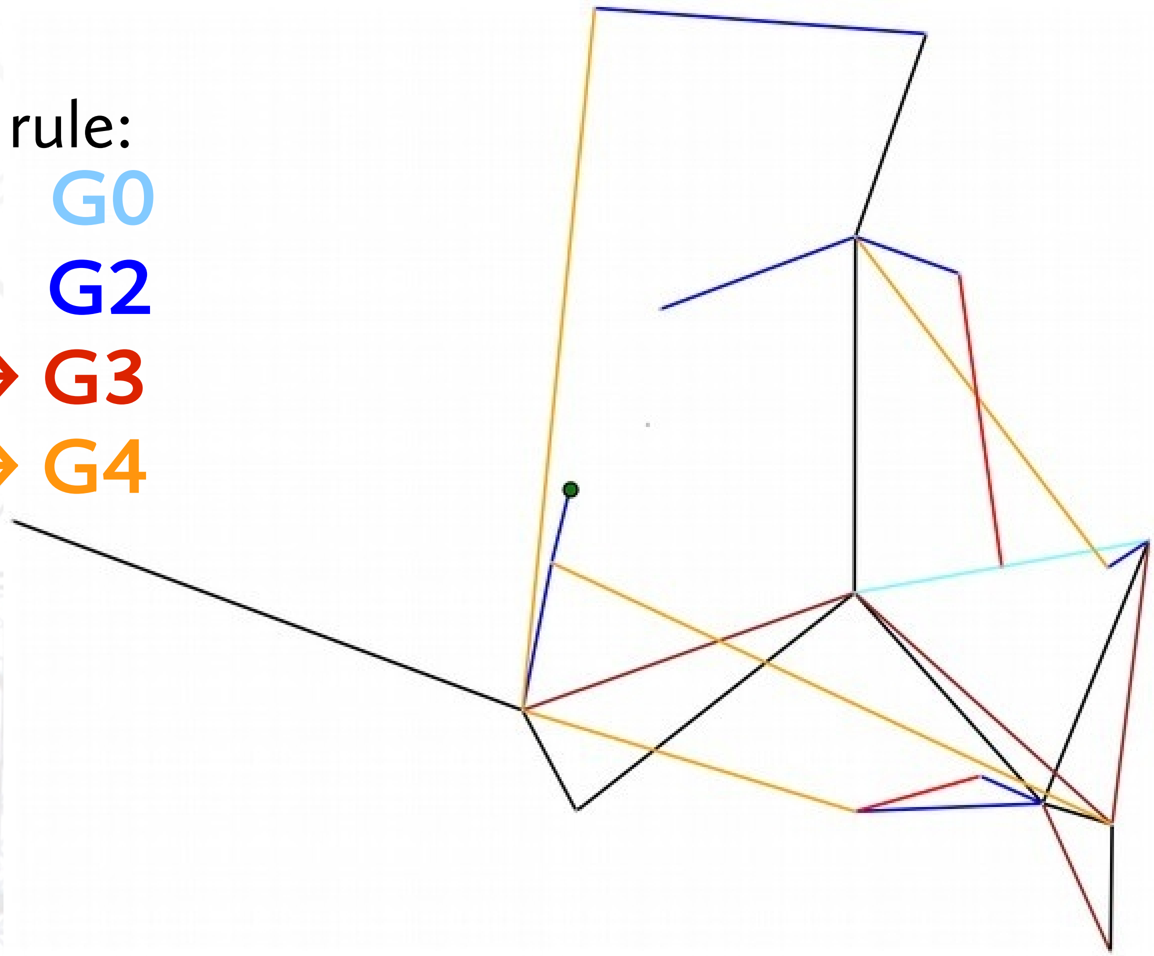

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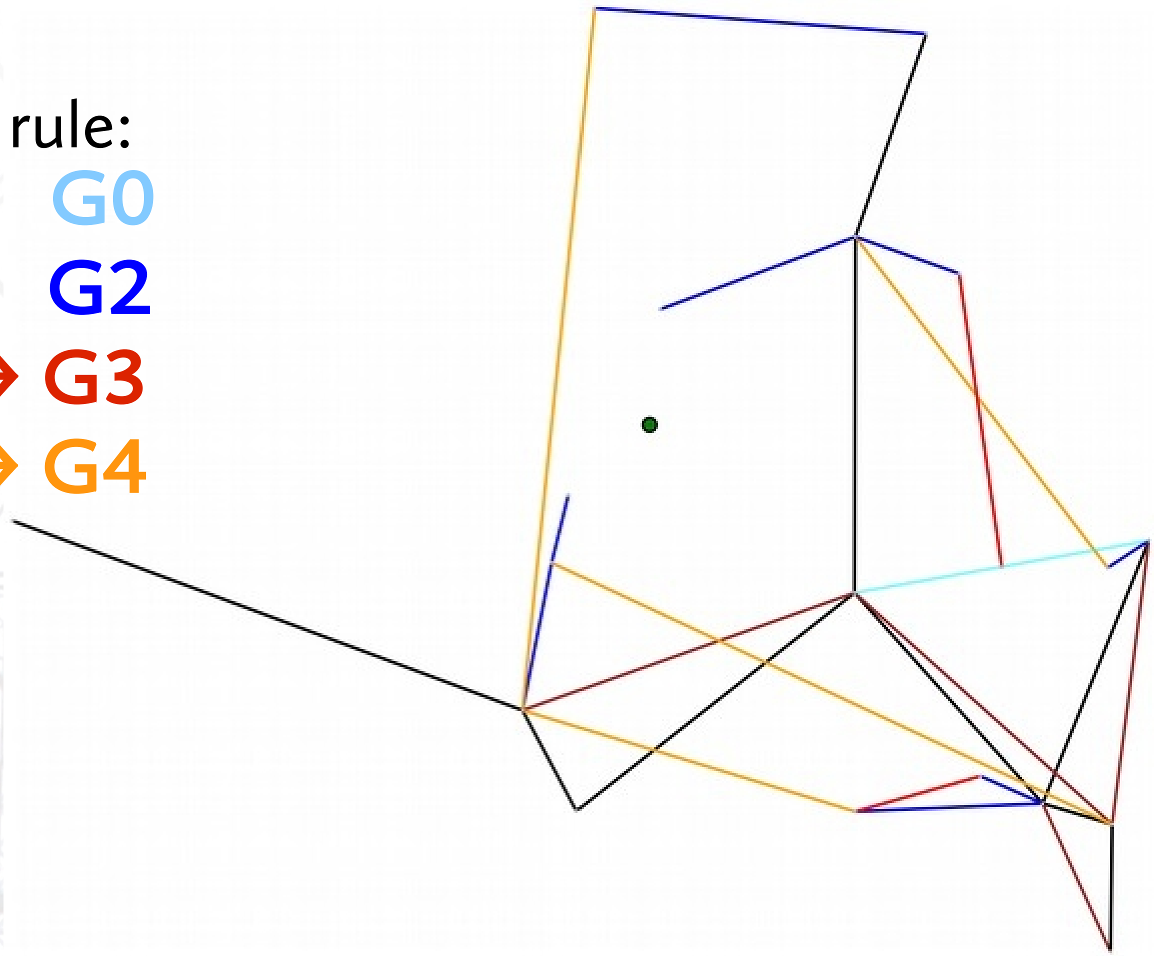

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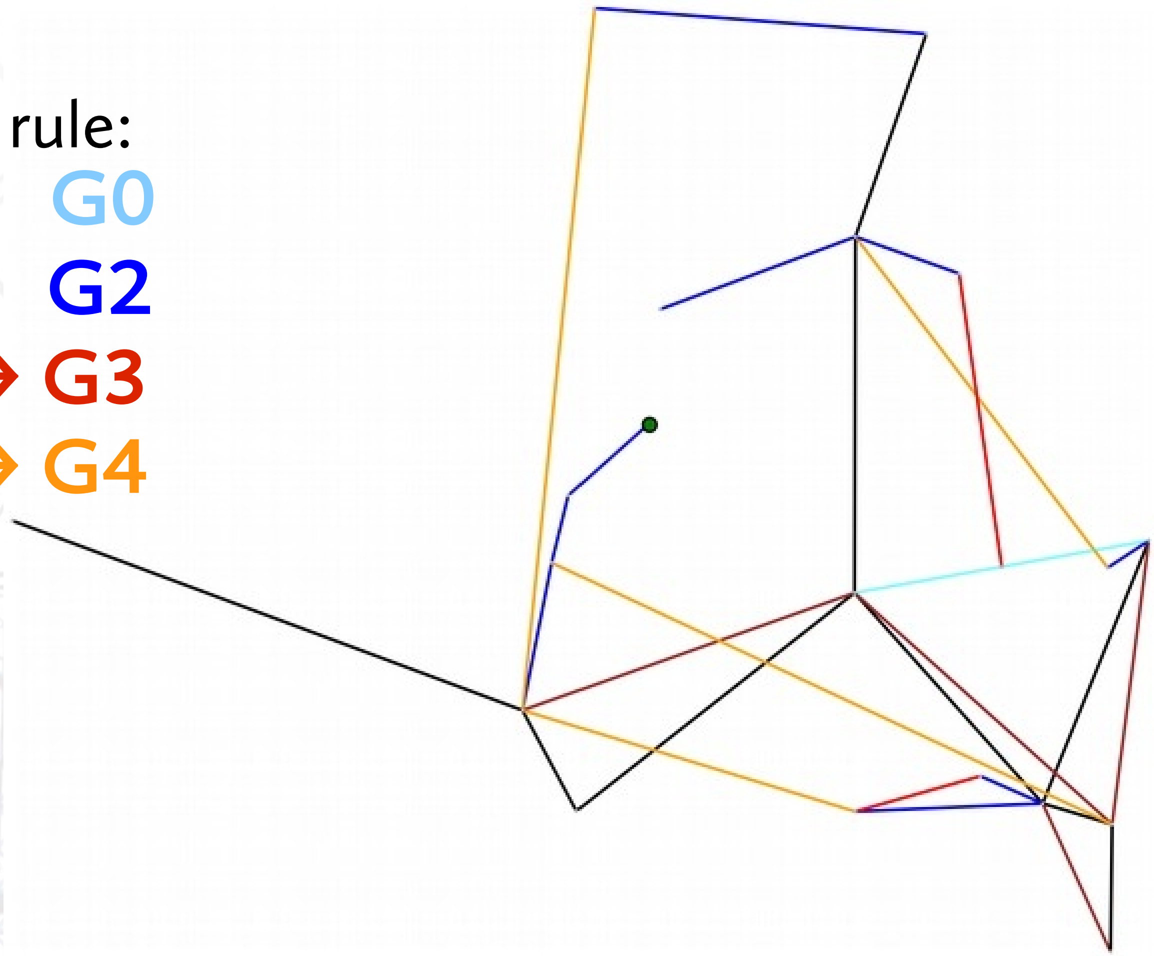


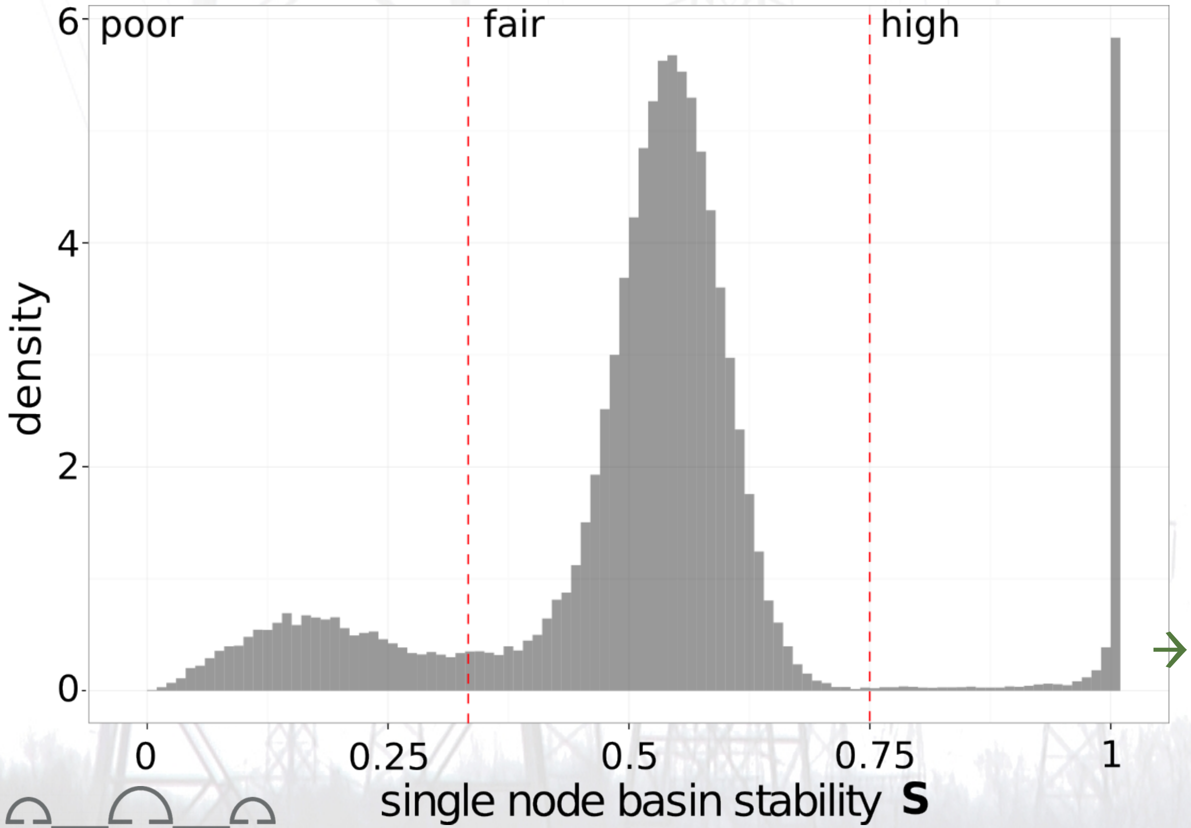
Ref.: Schultz et al., EPJ ST, 2014

3. Ensemble Analysis

Power Grid Ensemble

~600 networks
100 nodes each
$\bar{k}$ = 2.7

$$\ddot{\phi}_i = P_i - \alpha_i \dot{\phi}_i - \sum_{j=1}^{N} K_{ij} \sin(\phi_i - \phi_j)$$



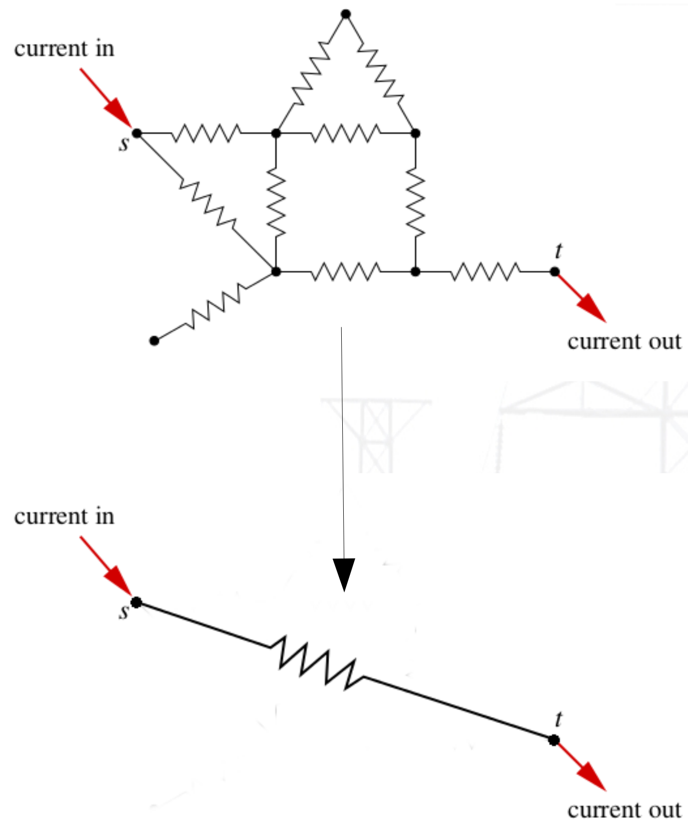
realistic choice:

$$K_{ij} \propto 1/d_{ij}$$

→ classification of SNBS

Effective Resistances
vs. network distance

- resistance of a single edge, virtually replacing all paths between **s** and **t** in an equivalent circuit
- replace path-based observables



Exp. $ERCC_i = 1/\langle ER_{ij} \rangle_j$

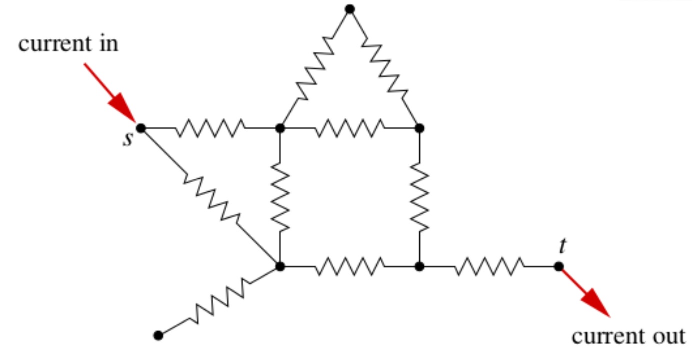
$$ER_{ij} = (b_i - b_j)^\top \mathbf{L}^{-1} (b_i - b_j) = L^{-1}_{ii} - L^{-1}_{ij} - L^{-1}_{ji} + L^{-1}_{jj}$$

Ref.: Klein&Randic 1993, Dörfler&Bullo 2010

Newman's Current Flow Betweenness

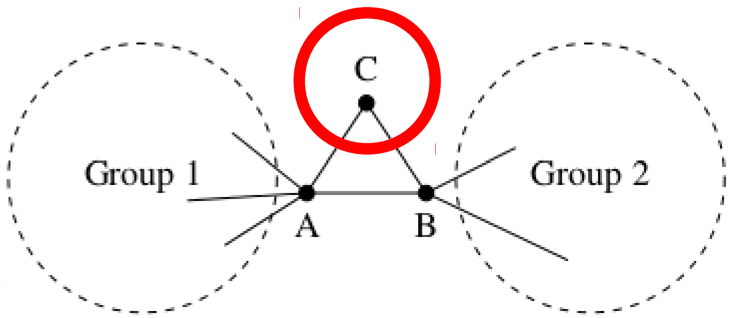
vs. shortest path betweenness

- Based on virtual current through node i according to Kirchhoff's laws
- Generalised for arbitrary admittance



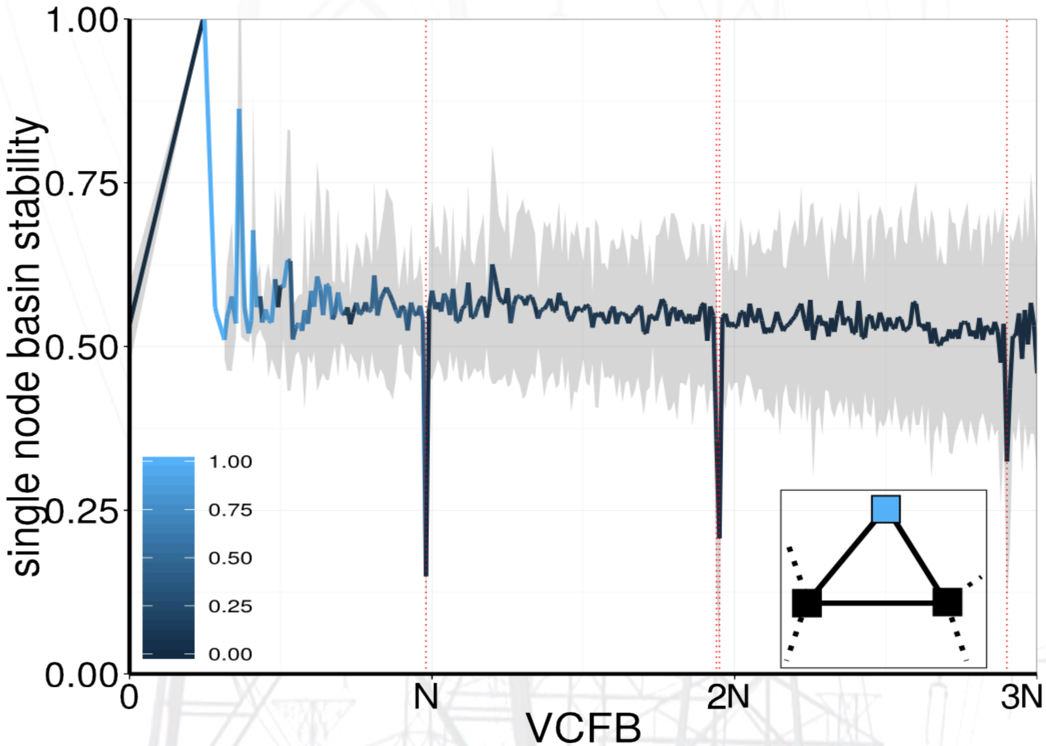
$$VCFB_i = \frac{2}{n(n-1)} \sum_{s<t} I_i^{st}$$

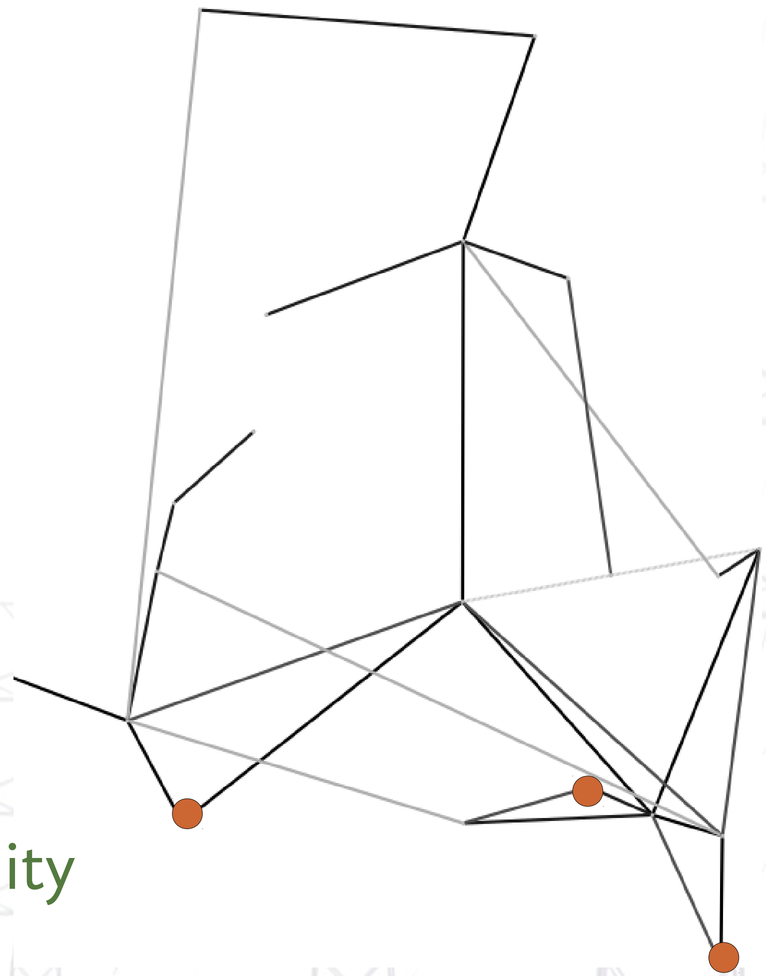
$$I_i^{st} = \frac{1}{2} \sum_j Y_{ij} |V_i - V_j|$$



Ref.: Newman, Social Networks (27), 2005

Nodes on Detours

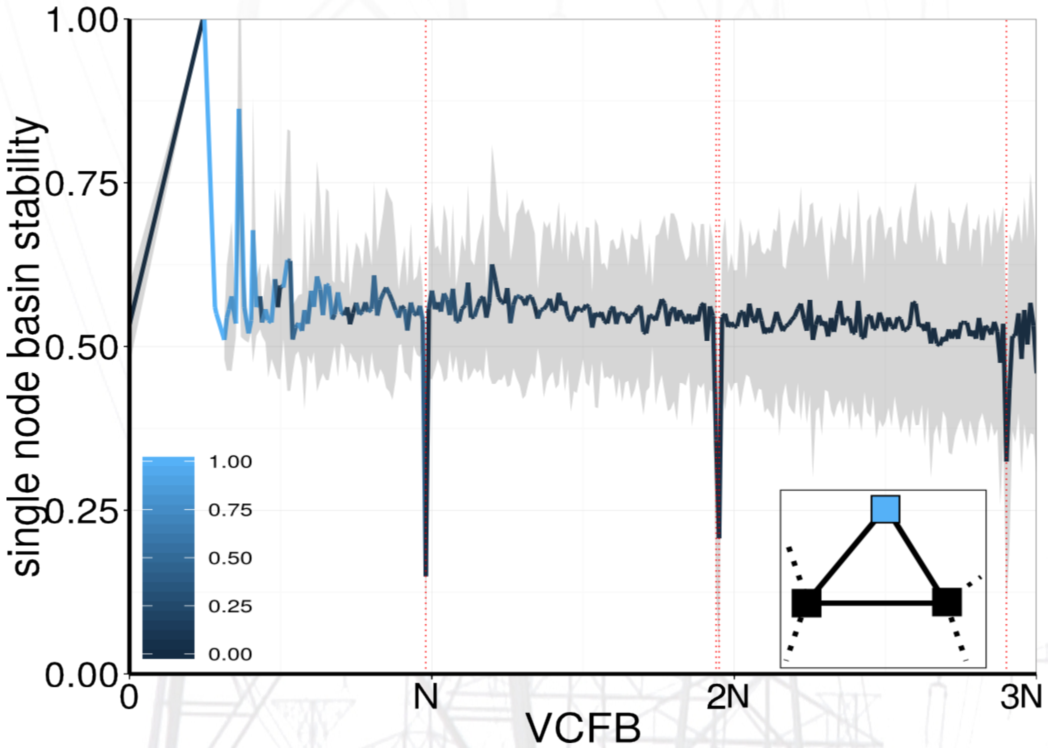


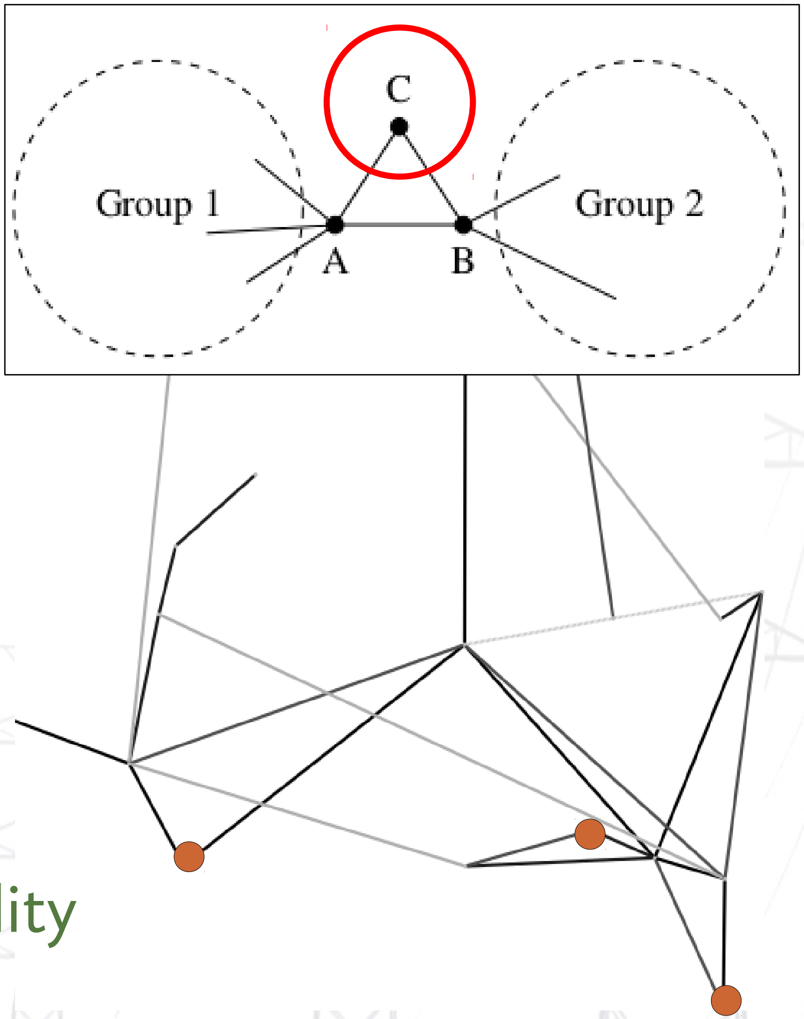


- Indicates fair to high basin stability
- Identified using VCFB

Ref.: Newman, Social Networks (27), 2005
Ref.: Schultz et. al., NJP 16, 125001 (2014)

Nodes on Detours





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Predicting Poor Stability from the Topology

$\tilde{k}_i$

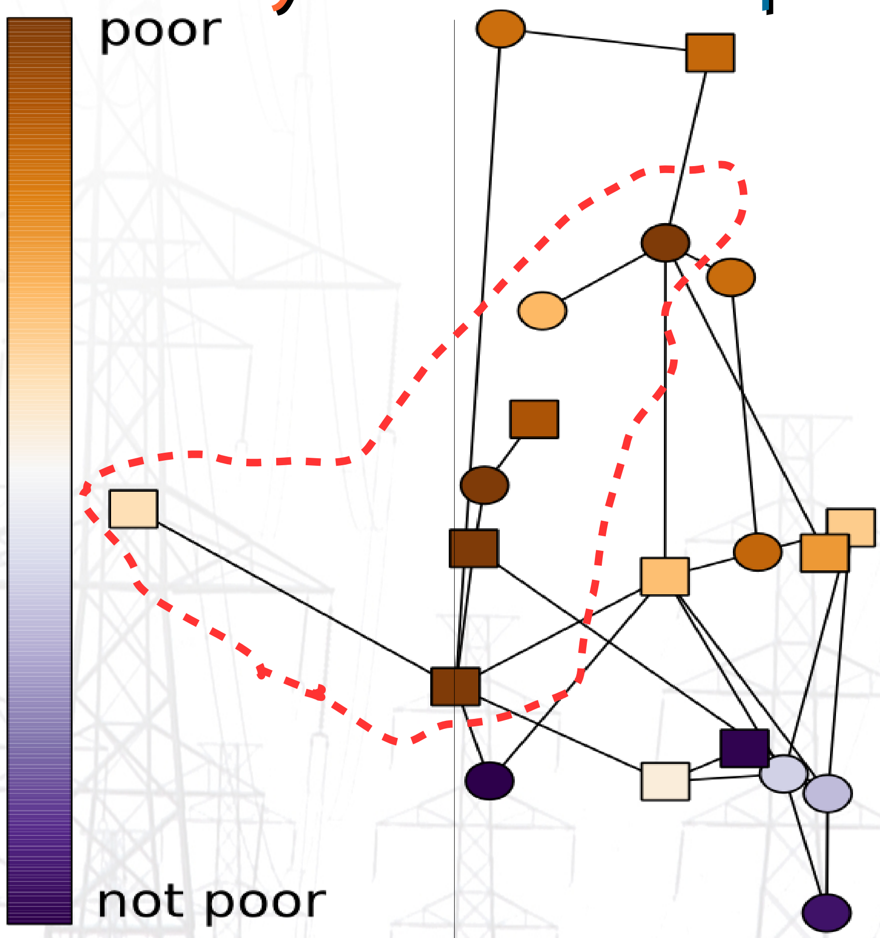
$\tilde{k}_i^N$

$\tilde{C}_i$

e_i

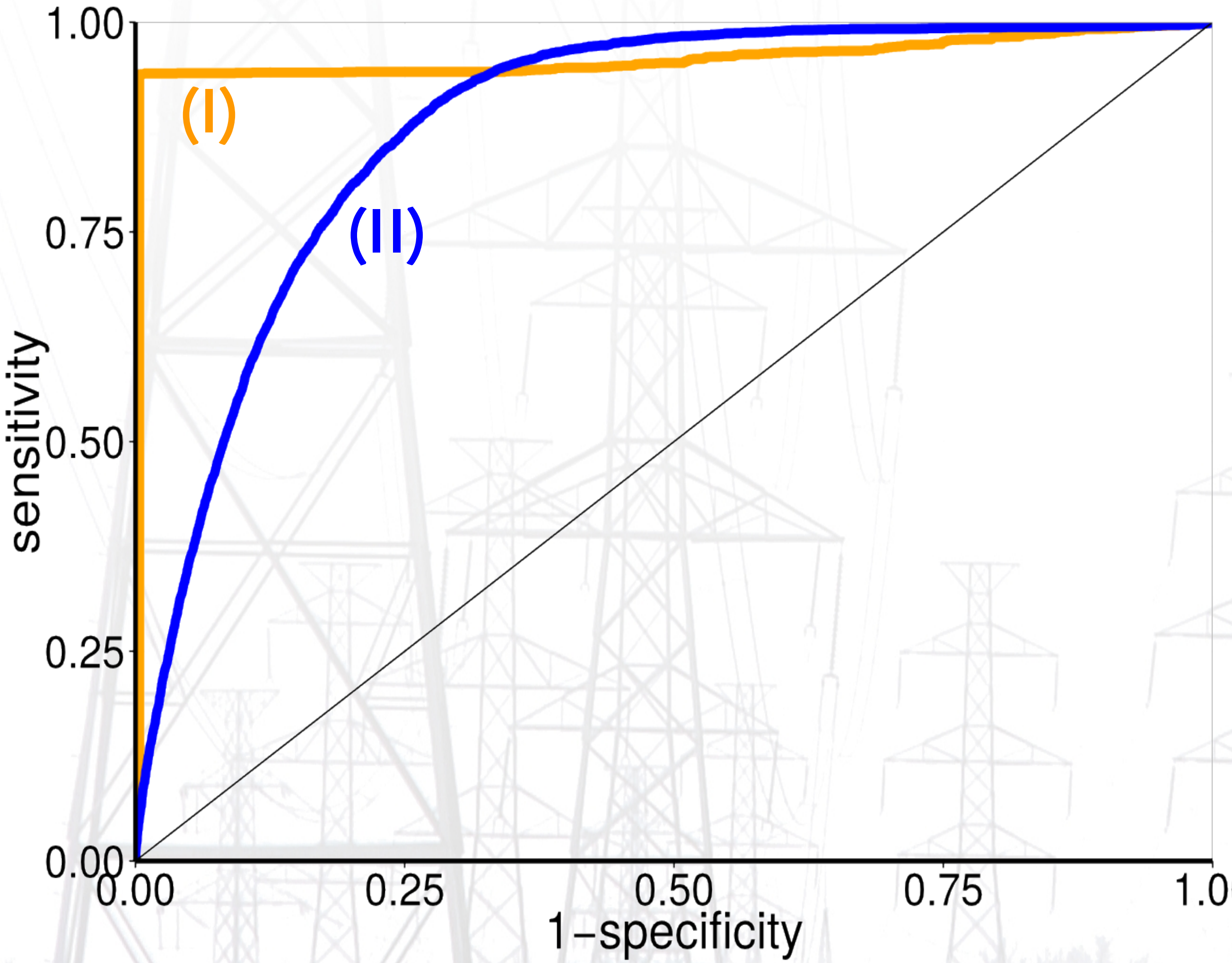
$VCFB_i$

$ERCC_i$



prediction of poor basin stability

Receiver Operating Characteristic



(0) detour nodes

(I) dead ends

Sen: 94%

Spec: 99%

(II) all others

Sen: 84%

Spec: 77%

Ref.: Schultz et. al., NJP 16, 125001 (2014)

Take Home Messages

Random topology model capable to create realistic power grid networks

Spatial information to estimate model parameters, surrogate networks for hypothesis testing, ensemble analysis etc.

Effective resistances and current-flow betweenness reveal additional information for electrical networks

It is possible to predict the weak points using only structural information

Dead ends diminish while detour motifs enhance network resilience

Publications

Detours Around Basin Stability in Power Networks
Paul Schultz, Jobst Heitzig, and Jürgen Kurths
New Journal of Physics 16 (2014) 125001
DOI: 10.1088/1367-2630/16/12/125001

A Random Growth Model for Power Grids and Other Spatially Embedded Infrastructure Networks
Paul Schultz, Jobst Heitzig, and Jürgen Kurths
Eur. Phys. J. Special Topics on "Resilient power grids and extreme events" (2014)
DOI: 10.1140/epjst/e2014-02279-6