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The Multiple-orientability Thresholds for Random Hypergraphs

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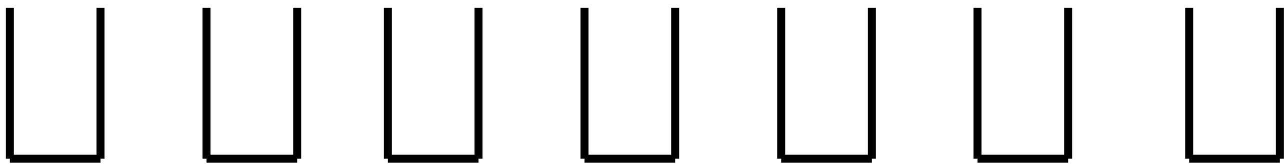
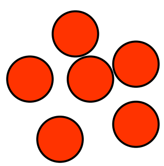
Joint work with Nikolaos Fountoulakis and Konstantinos Panagiotou

Introduction

n bins





m balls

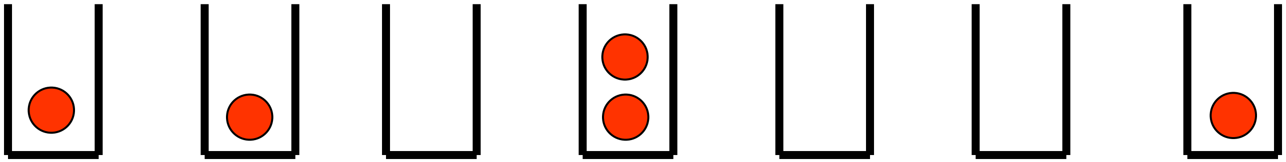
k choices

- **Each ball chooses uniformly at random k bins**

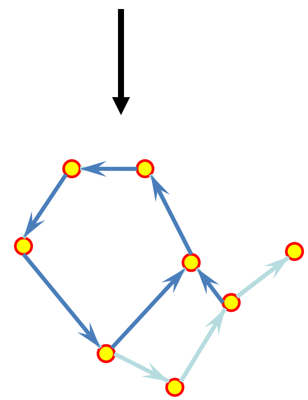
s bin capacity

- **Each bin can hold at most s balls**

Orientability of Hypergraph



n vertices **m** edges **s** orientation **k** vertices in each edge



s - orientation

- A mapping of each edge to one of its vertices
- Each vertex is assigned at most *s* edges

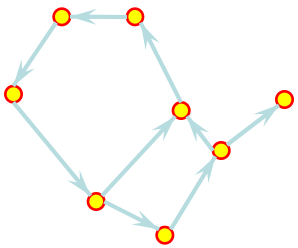
Density = #edges / #vertices

What is the **Critical Density** for an s-orientation to exist ?

The Basic Notions

- *k*-uniform hypergraph
  - each edge contains *k* distinct vertices
  - simple
- Random hypergraphs
  - $H_{n,p,k}$: each edge is present with probability *p*
  - $H_{n,m,k}$: contains exactly *m* edges
- *s*-orientation
  - a mapping of the edges to the vertices
  - each vertex is assigned at most *s* edges

$G_{n,p}, G_{n,m}$
if k = 2



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Main Result

Theorem. Let $k \geq 3$, $s \geq 2$. There is a $c^*_{k,s}$ such that

$$\Pr\left[H_{n,\lfloor cn \rfloor,k} \text{ is } s\text{-orientable}\right] \overset{(n\to\infty)}{\longrightarrow} \begin{cases} 0, & \text{if } c > c^*_{k,s} \\ 1, & \text{if } c < c^*_{k,s} \end{cases}.$$

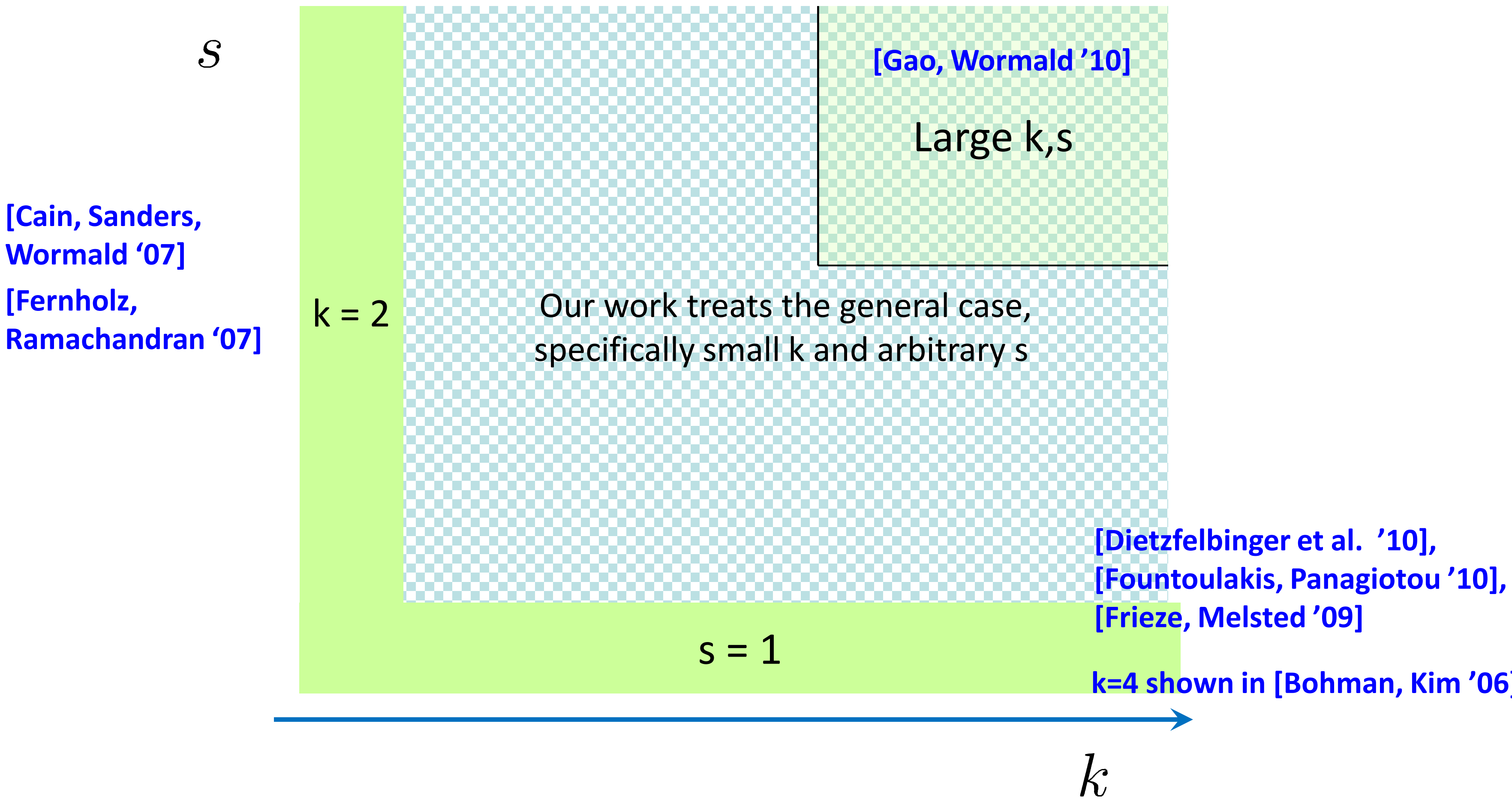
We determine $c^*_{k,s}$ explicitly in k and s

$c^*_{4,2} \approx 1.99$

$c^*_{3,2} \approx 1.97$

$c^*_{3,3} \approx 2.99$

The General Case



Proof Idea

- Basic Lemma : *H* is s-orientable iff all its subgraphs have density at most s.

  (proof on the next slide)

- Main Observation : *H* is s-orientable iff (s+1)- core is s-orientable

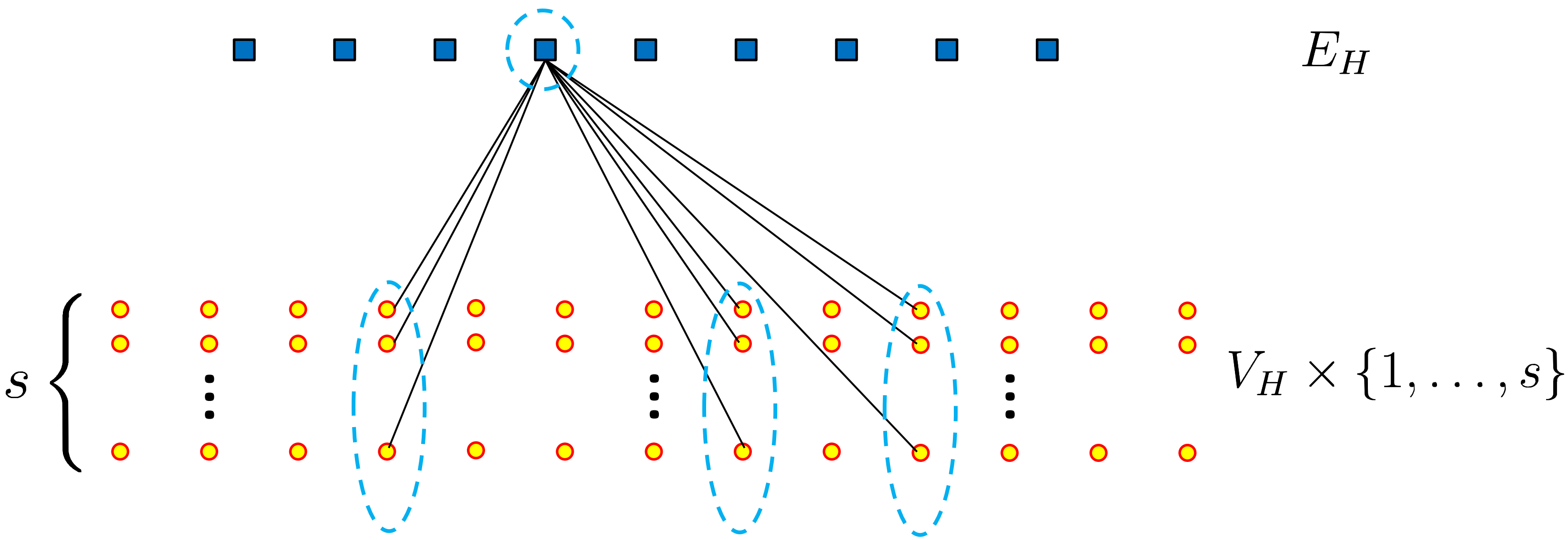
  (proof : Recall definition of the core)

core

Maximum subgraph with mininum degree $s + 1$

Basic Lemma

- **Lemma.** *H* is *s*-orientable iff all its subgraphs have density at most *s*.
- Proof. $\exists$ perfect matching $\Leftrightarrow \exists s-$ orientation
  $\Leftrightarrow \forall E \subseteq E_H \ : \ |N(E)| \geq |E|$
  $\Leftrightarrow \forall$ subgraphs the density $\leq s$



Conclusion and Open Questions

- The threshold for the s-orientability of random k-uniform hypergraphs coincides with the threshold that the (s+1)-core has density s.
- We showed that below the threshold all subgraphs have density < s.
- What happens at the critical density?
- What is the size of the largest subgraph at critical density?
- When does the first subgraph of density at least s appear?

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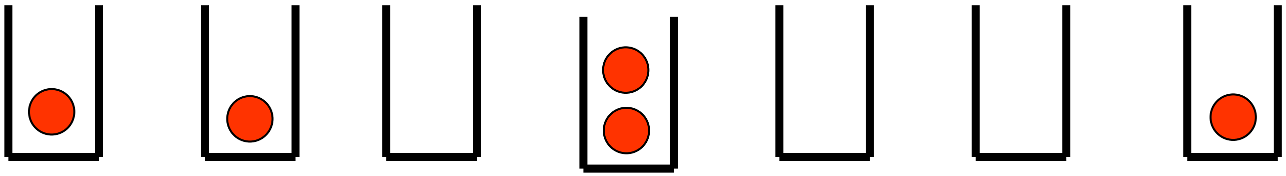

THANK YOU !