Dynamical triangulations (and quadrangulations) in statistical physics

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Outline

1. Dynamical triangulations as discrete approach to quantum gravity
2. Dynamical triangulations in statistical physics: computer simulations of disordered systems
3. Dynamical triangulations and quenched disorder
4. Dynamical triangulations and annealed disorder

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The dynamical triangulations approach

Einstein-Hilbert action is perturbatively non-renormalisable $\Rightarrow$ look for non-perturbative approaches

Quantum gravity

Path-integral quantisation of (pure) gravity:

$$\begin{aligned} Z &= \int \mathcal{D}[g] e^{-S_{EH}[g]} \\ &= \int \frac{\mathcal{D}g}{\mathrm{Vol(Diff)}} \int \mathcal{D}_g x \; e^{-S_{EH}[x,g]}, \end{aligned}$$

with the Einstein-Hilbert action

$$S_{EH}[g] = \frac{1}{16\pi G_N} \int d^D x \sqrt{|g|} \, (-R + 2\Lambda)$$

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Many questions

- What is $\int \mathcal{D}[g]$ supposed to mean?
- What about reparametrization invariance?
- Action is unbounded from below.
- Can this be rotated back to the Lorentzian sector?

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Lattice regularisation

Approximate integral by sum over discretised hyper-surfaces:

$$\int \mathcal{D}[g] \rightarrow \sum_{\mathcal{T}}$$

Two approaches:

- *Regge calculus*: discretise manifold with simplicial complex; sum runs over the *edge lengths*; connectivity fixed.
- *Dynamical triangulations* (DTRS): vary *connectivity*, keeping the edge (cut-off) lengths uniformly fixed.

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Discretized actions

Classical Regge calculus (sic!) gives discretized Einstein-Hilbert actions (for fixed topology):

$$\begin{aligned} S_{EH} &= \kappa_4 N_4 - \kappa_2 N_2 & (4D), \\ S_{EH} &= \kappa_3 N_3 - \kappa_1 N_1 & (3D), \\ S_{EH} &= \kappa_2 N_2 & (2D), \end{aligned}$$

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Two dimensions

Due to the Gauß-Bonnet theorem,

$$\int_{\mathcal{M}} d^2 x R = 4\pi\chi = 8\pi(1-h)$$

one has for pure quantum gravity in 2D:

$$\begin{aligned} Z(\mu) &= \sum_{N=1}^{\infty} e^{-\mu N} Z(N), \\ Z(N) &= \sum_{T \in \mathcal{T}_N} \frac{1}{C_N}, \end{aligned}$$

where $C_N = \mathrm{Vol}(\mathrm{Aut}(T))$.

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Matrix models

Consider the matrix integral

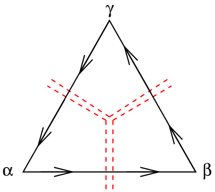
$$W(g,N) \equiv \int \mathrm{d}\phi\, e^{-\frac{1}{2}\mathrm{Tr}\,\phi^2 + \frac{g}{3\sqrt{N}}\mathrm{Tr}\,\phi^3} \equiv \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{g}{3\sqrt{N}}\right)^k \left\langle \mathrm{Tr}\,\phi^{3k} \right\rangle ,$$

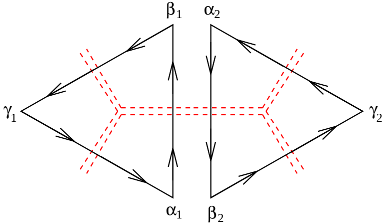
with $\mathcal{N} \times \mathcal{N}$ Hermitian matrix ϕ and the measure

$$\mathrm{d}\phi \equiv \prod_{\alpha\le\beta} \mathrm{d}\,\mathrm{Re}\,\phi_{\alpha\beta} \prod_{\alpha<\beta} \mathrm{d}\,\mathrm{Im}\,\phi_{\alpha\beta}.$$

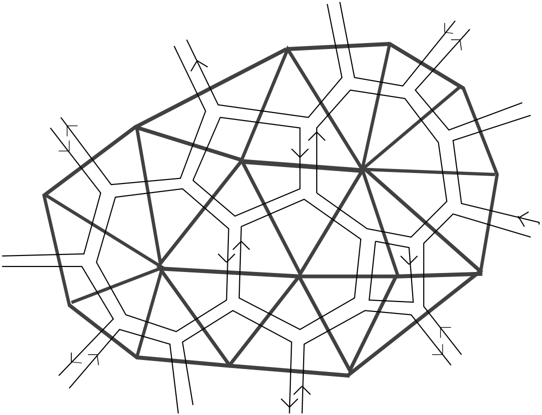
Then, the propagator (two-point function) is

$$\langle \phi_{\alpha\beta}\phi_{\alpha'\beta'} \rangle = \int \mathrm{d}\phi\, e^{-\frac{1}{2}\sum_{\alpha\beta}|\phi_{\alpha\beta}|^2} \phi_{\alpha_\beta}\phi_{\alpha'\beta'} = \delta_{\alpha\beta'}\delta_{\beta\alpha'}$$





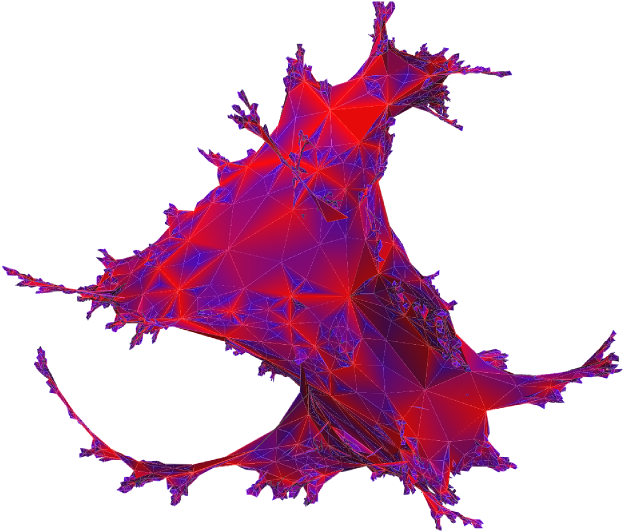
Matrix models



Pure ϕ^3 model (Brézin, Itzykson, Parisi, Zuber, 1978):

$$Z_N = \frac{8^N \Gamma(\frac{3}{2}N)}{(N+2)!\Gamma(\frac{1}{2}N+1)}$$

How does it look like?



non-trivial Hausdorff dimension $d_h = 4$

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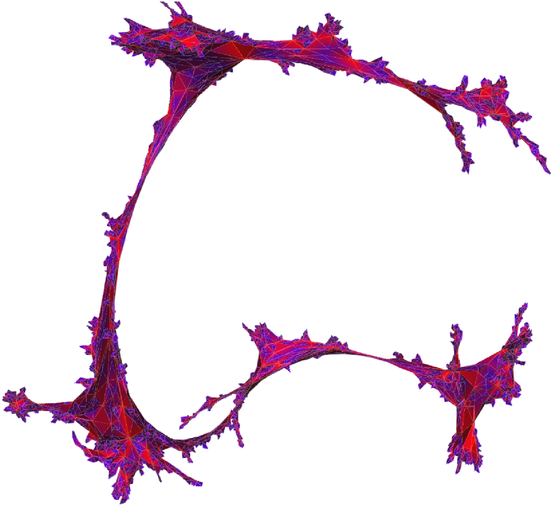


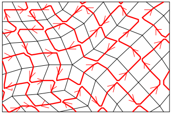
non-trivial Hausdorff dimension $d_h = 4$

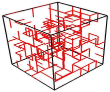
Outline

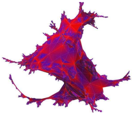
Disorder in statistical physics

Disorder

- **Annealed**
  - **KPZ/DDK**
    Dressed exponents from KPZ/DDK formula
- **Quenched**
  - **Uncorrelated**
    Harris criterion:
    $\alpha > 0$
    – random bond model
  - **Correlated**
    Luck criterion:
    $(1-\alpha)/(2-\alpha) < \beta$
    – QG graphs
    – Voronoi/Delaunay





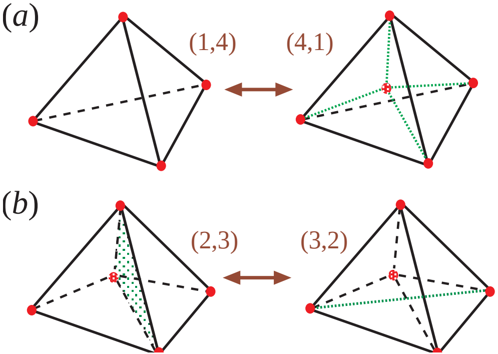


Simulation method

For a a general simplicial complex, define the (k, l) moves,

$$a_1 \dots a_l \overline{b_1 \dots b_k} \to \overline{a_1 \dots a_l} b_1 \dots b_k,$$

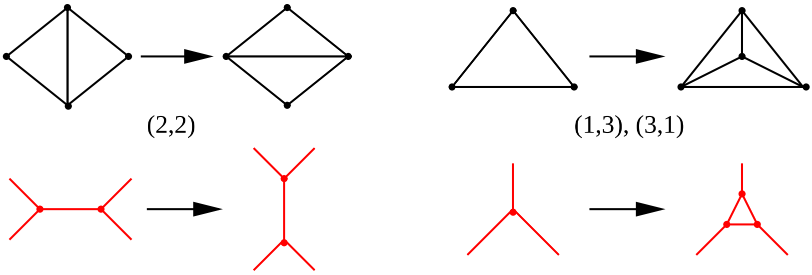
where $k + l = D + 2$, $k = 1, \dots, D + 1$ and $a_1 \dots a_l \overline{b_1 \dots b_k} \in K$, $b_1 \dots b_k \notin K$.



These can be shown to be ergodic in the space of homeomorphic simplicial manifolds (for $d < 4$).

Simulation method

In two dimensions:



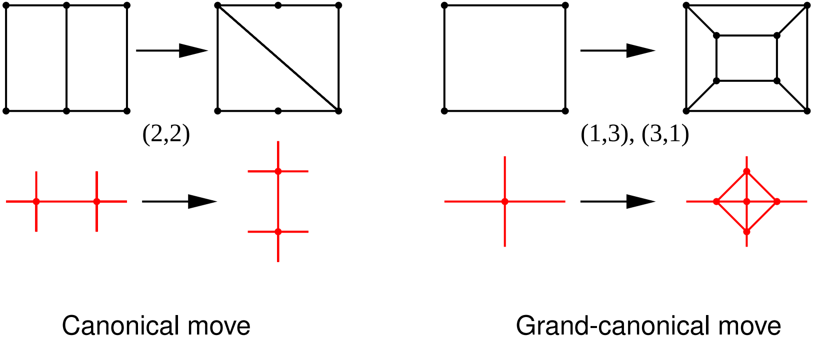
Canonical move

Grand-canonical move

Canonical move alone is ergodic for simulations in the canonical ensemble.

Simulation method

What about ϕ^4 graphs and quadrangulations?



Moves not ergodic in general!

Simulation method

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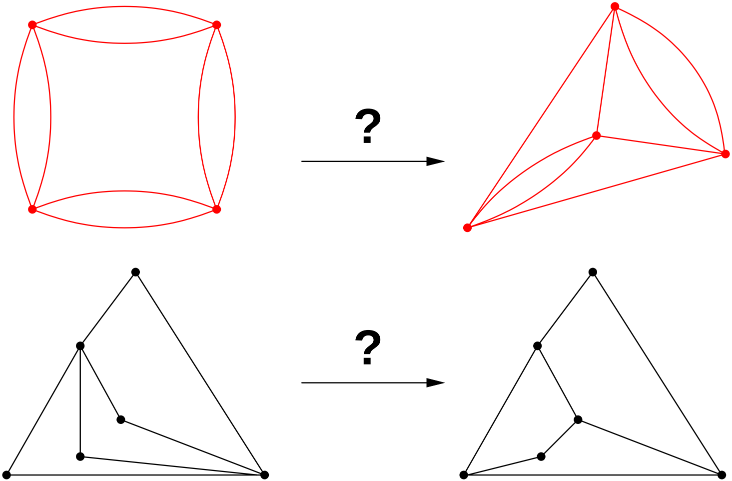
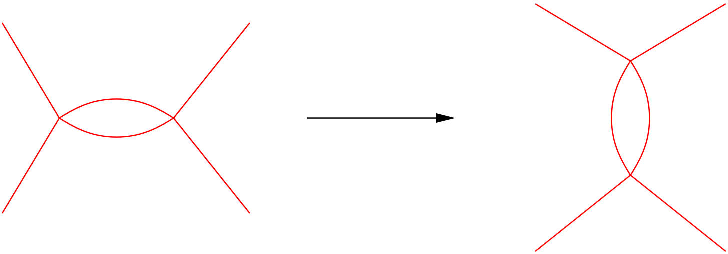


Simulation method

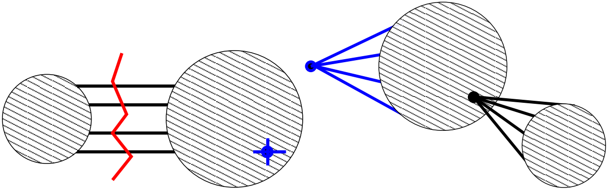
What about ϕ^4 graphs and quadrangulations?

$\Rightarrow$ One needs two-link flip to restore ergodicity



Simulation method

Non-local dynamics: "*minimal-neck baby universe surgery*"



J. Ambjørn, B. Durhuus, and T. Jonsson, Quantum Geometry -- A Statistical Field Theory Approach (Cambridge University Press, Cambridge, 1997).

J. Ambjørn, M. Carfora, and A. Marzuoli, The Geometry of Dynamical Triangulations (Springer, Berlin, 1997).

Outline

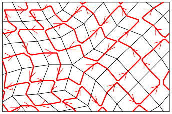
Disorder in statistical physics

Disorder

Annealed | **Quenched**

KPZ/DDK

Dressed exponents from KPZ/DDK formula

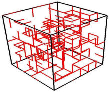


Uncorrelated

Harris criterion:

$\alpha > 0$

− random bond model



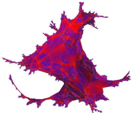
Correlated

Luck criterion:

$(1-\alpha)/(2-\alpha) < \beta$

− QG graphs

− Voronoi/Delaunay



The Harris-Luck criterion

Effects of coupling spin models to random graphs instead of regular lattices:

- For sufficient connectivity, ordered phase should persist (at least for ferromagnets).
- Order of transition and universality class might change:
  - Regular lattice: Harris criterion
    Variance of the coupling over a correlation volume:

    $$\sigma_R(J) \sim R^{-d/2} \Rightarrow \sigma_\xi(J) \sim \xi^{-d/2} \sim t^{\nu d/2},$$

    Disorder is relevant if:

    $$\nu d/2 < 1, \quad \alpha > 0$$

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Effects of coupling spin models to random graphs instead of regular lattices:

- For sufficient connectivity, ordered phase should persist (at least for ferromagnets).
- Order of transition and universality class might change:
  - Regular lattice: Harris criterion, $\alpha > 0$.
  - Random graph: consider average co-ordination number in patch of size R,

$$J(R) \equiv \frac{1}{B(R)} \sum_{i \in P} q_i.$$

Then,

$$\sigma_R(J) \equiv \langle |J(R) - J_0| \rangle / J_0 \sim \langle B(R) \rangle^{-(1-\omega)} \sim R^{-d_h(1-\omega)},$$

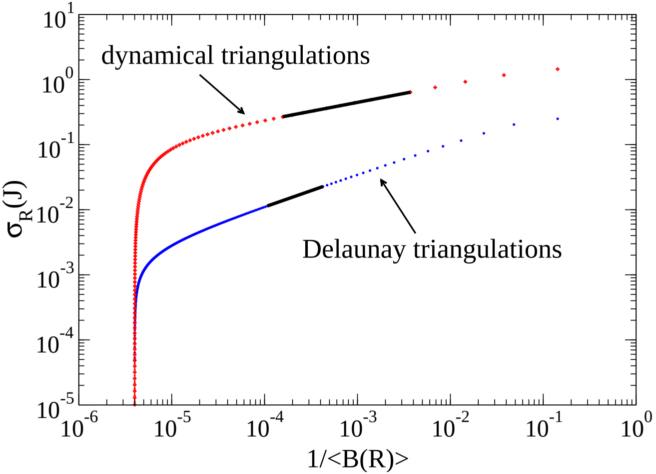
and disorder is relevant if $d_h \nu (1-\omega) > 1$, where ω is the *wandering exponent* of the random structure, with $\omega = 1 - a/2d_h,$.

Equivalently, disorder is relevant if

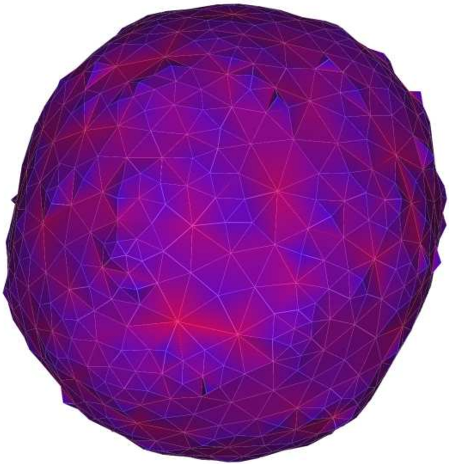
$$\alpha > \frac{1-2\omega}{1-\omega}.$$

Wandering exponents

Decay of the averaged fluctuation of coordination numbers:



Voronoi-Delaunay triangulations



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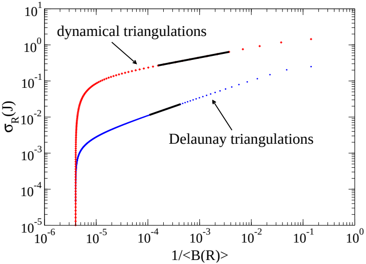

- Dynamical triangulations: $\omega = 0.7473(98)$ ($\omega = 3/4$?) $\Rightarrow$ disorder relevant for
$$\alpha \gtrsim -2.$$
- Voronoi-Delaunay triangulations: $\omega = 0.50096(55)$, exponentially decaying correlations and the Harris criterion is recovered, i.e.,
$$\alpha > 0.$$

Simulation results

- Dynamical triangulations:
  - Exact result for percolation: $\alpha = -2/3$, $\beta = 5/36$, $\gamma = 43/18 \Rightarrow \alpha = -2$, $\beta = 1/2$, $\gamma = 3$.
  - Monte Carlo simulations for the $q = 2, 3, 4$ states Potts models show a change in universality class.
  - First-order phase transition in $q = 10$ Potts model is softened to a continuous transition.
  - Full agreement with relevance criterion.

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  - Full agreement with relevance criterion.
- Voronoi-Delaunay triangulations:
  - No change for the 2D Ising model, but marginal since $\alpha = 0$.
  - Surprisingly, however, also apparently no change for $q = 3$ Potts model with $\alpha = 1/3 > 0$, in contradicition to relevance criterion.

Lattice	$x_\epsilon(1/2\nu)$	$x_\epsilon(\alpha/2\nu)$	$x_\sigma(\beta/2\nu)$	$x_\sigma(\gamma/2\nu)$
Voronoi	0.8003(67)	0.7799(27)	0.1234(27)	0.1282(12)
Regular	0.8000	0.8000	$0.133\bar{3}$	$0.133\bar{3}$

  - Possible connection to structure of weakly connected regions.

Frustration from dynamical triangulations

Effect on ferromagnets is rather weak. What about *antiferromagnets*, where frustration comes to play?

Regular lattices

- **square lattice**: Néel order with phase transition equivalent to the ferromagnet as seen from the Mattis transformation of one sub-lattice
- **triangular lattice**: paramagnet down to zero temperature (Wannier, 1950)

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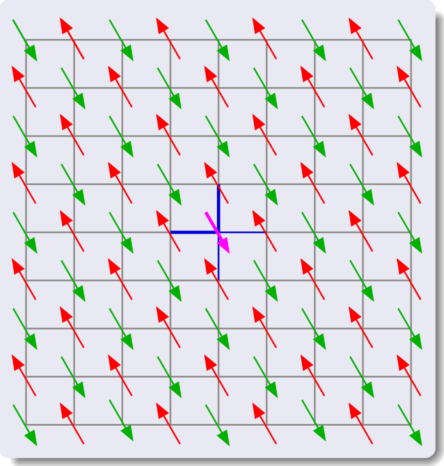

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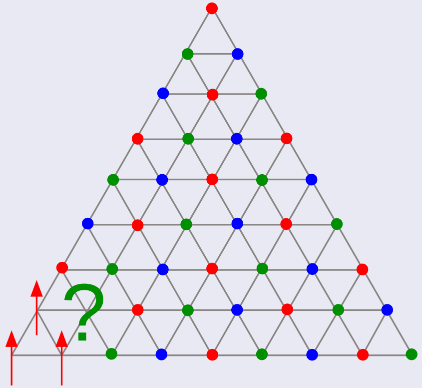

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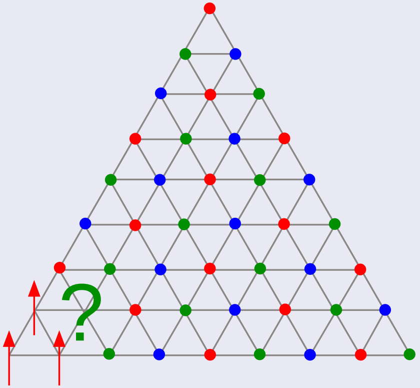
order depends on *bipartiteness*



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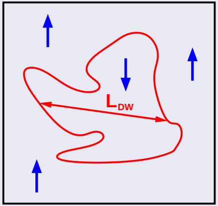
Fat graphs

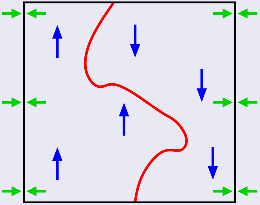
Type	Bipartite	Annealed	Quenched
quadrangulations	✓	equivalent to FM	equivalent to FM
triangulations	–	all triangles are frustrated, even at $T = 0$ $\Rightarrow$ PM everywhere	PM everywhere
ϕ^4 graphs	–	ground state is square lattice with Néel order $\Rightarrow$ finite-T phase transition?	spin-glass order at $T = 0$?
ϕ^3 graphs	–	GS is honeycomb lat. with Néel order	spin-glass order at $T = 0$?

Spin stiffness and zero-temperature scaling

Ferromagnet (Peierls)



$\Delta E \sim L^{d-1}$ resp. L^{d-2}



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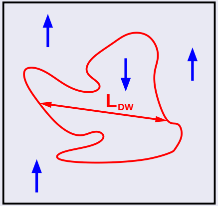

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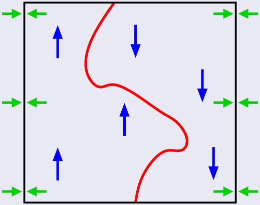

Spin glass (Bray/Moore, 1987)

Distribution of couplings evolving under RG transformations, asymptotic width scales as

$$J(L) \sim JL^{\theta(d)}.$$

Spin-stiffness exponent θ determines lower critical dimension. For $\theta < 0$,

$$\xi \sim T^{-\nu}, \quad \nu = -1/\theta.$$

Numerically, θ can be determined from inducing droplets or domain walls with a change of *boundary conditions*,

$$\Delta E = |E_{\mathrm{AP}} - E_{\mathrm{P}}| \sim L^{\theta}.$$

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2D Ising

- ground-state problem is polynomial $\rightarrow$ large systems tractable

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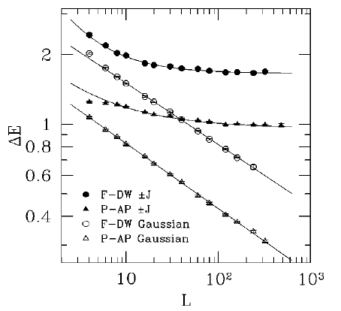

(Hartmann/Young, 2001)

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- ground-state problem is polynomial → large systems tractable, $\theta \approx -0.28$ resp. $\theta = 0$

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Antiferromagnet and spin glass on DTRS

Spin stiffness for the random-lattice case.

KPZ/DDK

If the spin glass on a regular lattice corresponds to a $c = 0$ CFT, then

$$\tilde{\Delta} = \frac{\sqrt{1 + 24\Delta} - 1}{4}.$$

Conjecture for θ_s/d_h:

Bonds	Regular	KPZ $c = 0$
$\pm J$	0	0
Gauss	-0.1422	-0.0886

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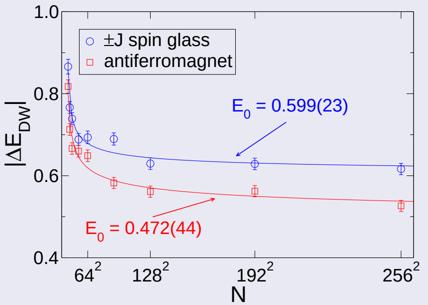

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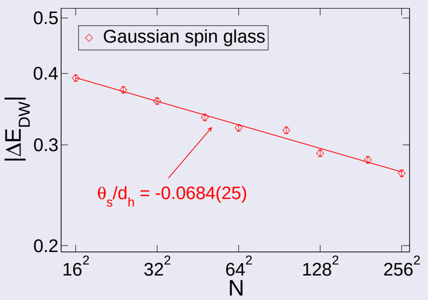

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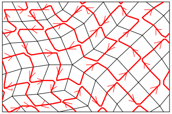
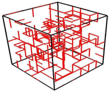
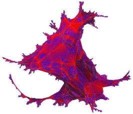
- **Annealed**
  - **KPZ/DDK**
    Dressed exponents from KPZ/DDK formula
- **Quenched**
  - **Uncorrelated**
    Harris criterion:
    $\alpha > 0$
    – random bond model
  - **Correlated**
    Luck criterion:
    $(1-\alpha)/(2-\alpha) < \beta$
    – QG graphs
    – Voronoi/Delaunay







The KPZ/DDK framework

Liouville field theory predicts *dressing* of conformal weights of critical matter coupled to quantum gravity:

$$\tilde{\Delta} = \frac{\sqrt{1-c+24\Delta} - \sqrt{1-c}}{\sqrt{25-c} - \sqrt{1-c}}$$

i.e., in terms of statistical mechanics: disorder is relevant in all those cases. E.g., for the (2D) Ising model:

	α	β	γ
regular lattice	0	1/8	7/4
DTRS	-1	1/2	2

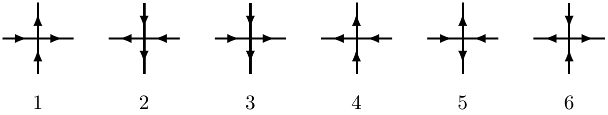
For $c \leq 1$, this framework breaks down. $c = 1$ is marginal case with different realizations:

- single massless scalar field
- 4-states Potts model
- 6-vertex model

Marginality entails logarithmic corrections to all scaling relations.

Vertex models on random graphs

Allow six arrow configurations on the square lattice:



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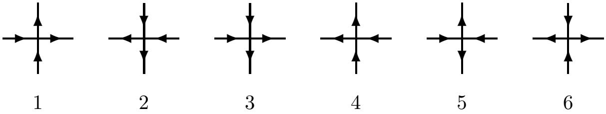

Vertex weights

$$\omega_i = \exp\left(-\epsilon_i/k_b T\right), \quad i = 1, \ldots, 6,$$
$$a = \omega_1 = \omega_2,$$
$$b = \omega_3 = \omega_4,$$
$$c = \omega_5 = \omega_6,$$

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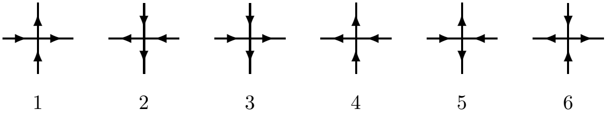

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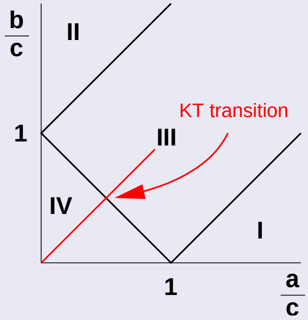
Phase diagram for the square lattice



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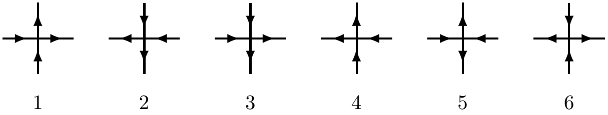
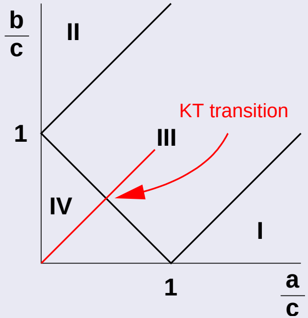


F model

$$a = b = e^{-K}, \quad c = 1$$

- Undergoes a Kosterlitz-Thouless phase transition on the square lattice.
- Is marginal with $c = 0$ in the KPZ/DDK framework.
- What happens on a DTRS?

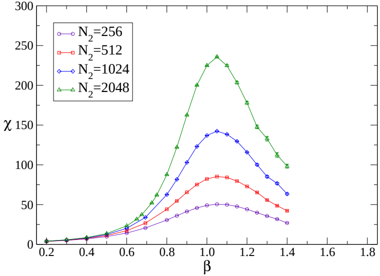
Phase diagram for the square lattice



Simulation results

Monte Carlo simulations of the six-vertex F model coupled to dynamical quadrangulations.

Polarizability



Results

- Critical *high*-temperature phase terminating at $\beta_c = \beta = \ln 2$.
- Kosterlitz-Thouless critical point at β_c with additional *logarithmic corrections*.
- Critical exponents

$$\begin{aligned} \gamma/d_h\nu &= 0 \\ \beta/d_h\nu &= 1/2 \end{aligned}$$

- Hausdorff dimension $d_h = 4$, independent of temmperature.
- String-susceptibility exponent shifted from $\gamma = -1/2$ to $\gamma = 0$.

Frustration from dynamical triangulations

Effect on ferromagnets is rather weak. What about *antiferromagnets*, where frustration comes to play?

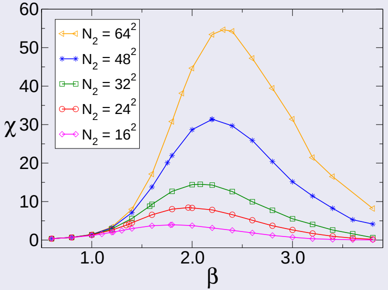
Fat graphs

Type	Bipartite	Annealed	Quenched
quadrangulations	✓	equivalent to FM	equivalent to FM
triangulations	–	all triangles are frustrated, even at $T = 0$ $\Rightarrow$ PM everywhere	PM everywhere
ϕ^4 graphs	–	ground state is square lattice with Néel order $\Rightarrow$ finite-T phase transition?	spin-glass order at $T = 0$?
ϕ^3 graphs	–	GS is honeycomb lat. with Néel order	spin-glass order at $T = 0$?

Ising antiferromagnet on ϕ^3 graphs

Monte Carlo simulation of the combined system (Pachner moves plus spin flips).

Magnetic susceptibility



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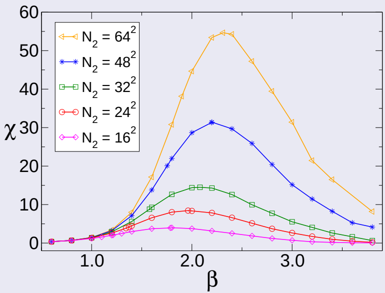
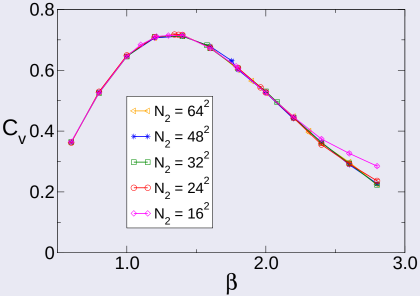


Specific heat



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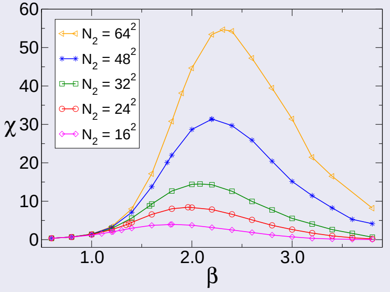
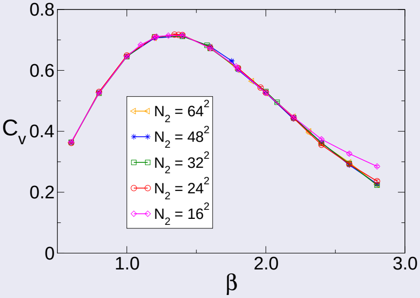


Specific heat

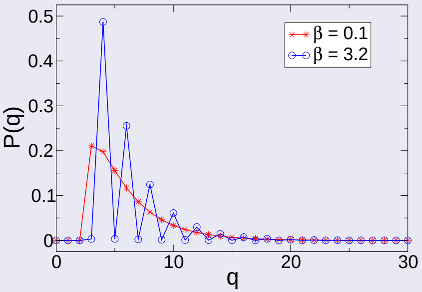


Kosterlitz-Thouless type phase transition?

Forced bipartite phase

Antiferromagnetic interaction forces graphs into a bipartite phase composed of squares, hexagons etc.

Co-ordination number distribution



Conclusions

- Dynamical triangulations (and quadrangulations) provide an ideal laboratory for studying the effects of connectivity disorder on spin models.
- Quenched disorder:
  - Connectivity disorder from DTRS relevant for virtually all types of coupled matter.
  - Ferromagnets experience a change of critical exponents, but ordered phase is stable.
  - Frustration exerted through DTRS on antiferromagnets changes critical behaviour, but might also wipe out ordered phase.
  - Antiferromagnet becomes equivalent to $\pm J$ spin glass on triangulations.
- Annealed disorder:
  - Critical exponents always change according to KPZ/DDK formula, no Fisher renormalization.
  - In ferromagnets, ordered phase is stable against the random perturbation.
  - Frustration in induced in antiferromagnets yields wide range of behavior from pure paramagnets to disorder-induced bipartite phases.

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You

Thank you for your attention!