Selected aspects of urban complexity

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Kolloquium Theoretische Physik
Carl von Ossietzky Universität Oldenburg
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UNTERSTÜTZT VON / SUPPORTED BY
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The Nobel Prize in Physics 2021

“for groundbreaking contributions to our understanding of complex physical systems”

Syukuro Manabe	1/4
Klaus Hasselmann	1/4
Giorgio Parisi	1/2

Niklas Elmehed/Nobel Prize Outreach

“for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales”

Complex systems

complicated vs. complex

we can understand a mechanical watch (Ottino 2004)

not so complex systems

→ emergence

different properties at each level of complexity

e.g. chemistry obeys laws of physics but we cannot infer chemistry from them (Anderson 1972, Strogatz et al. 2022)

Cities as complex systems – urban complexity

Cities are attractive despite many negative characteristics

difficulties in managing them

due to high degree of complexity

Interacting entities (people, infrastructure)

Emergent properties, in addition to sum of isolated properties

- discrete dynamics and cellular automata
- networks
- dynamical systems
- agent-based modeling
- scaling

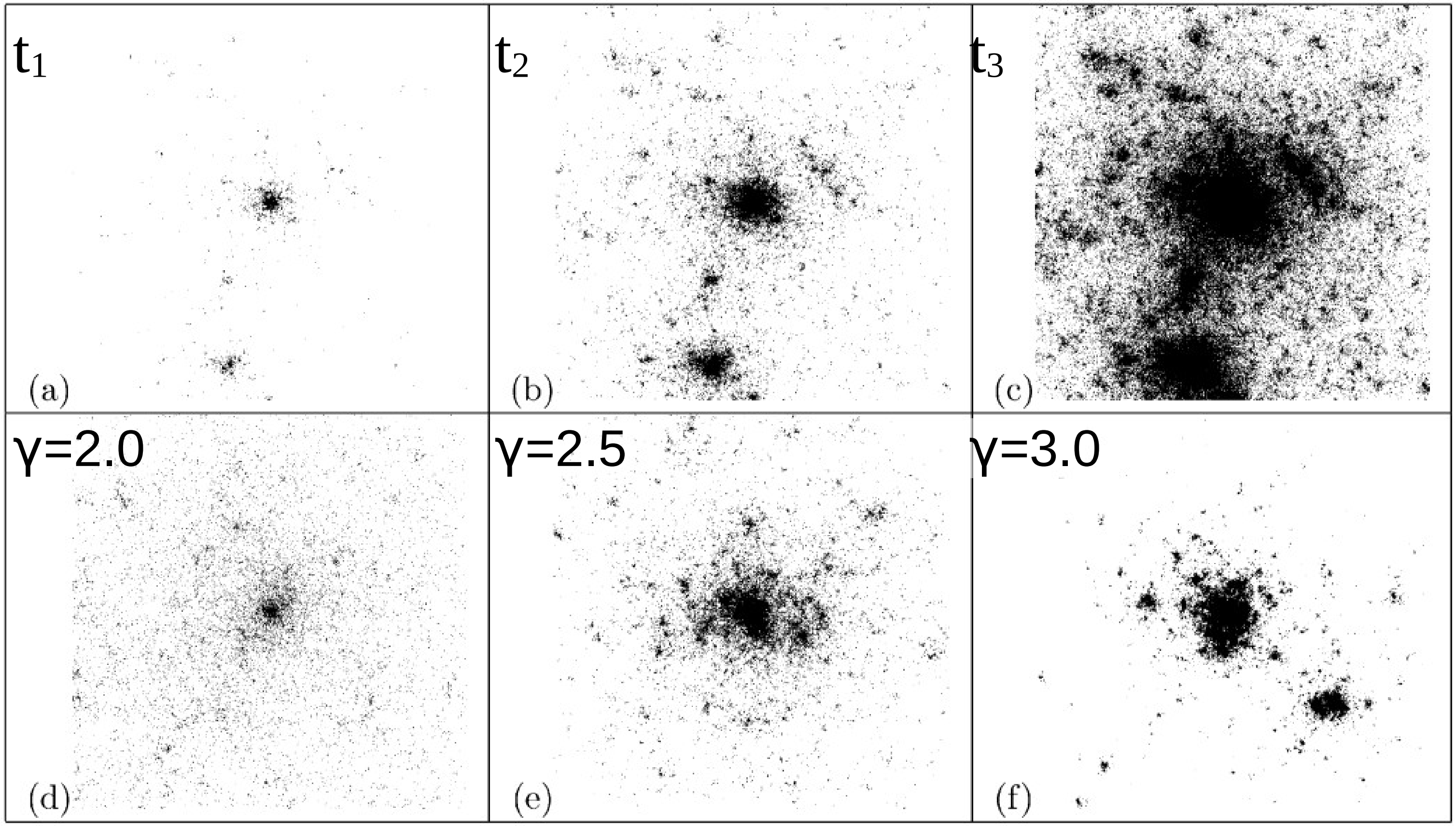
Cities as complex systems – urban complexity

Selected aspects

1. Gravitation Growth
2. Urban Percolation
3. City Size Distributions & Urban Scaling

Gravitational Growth

Gravitational Growth



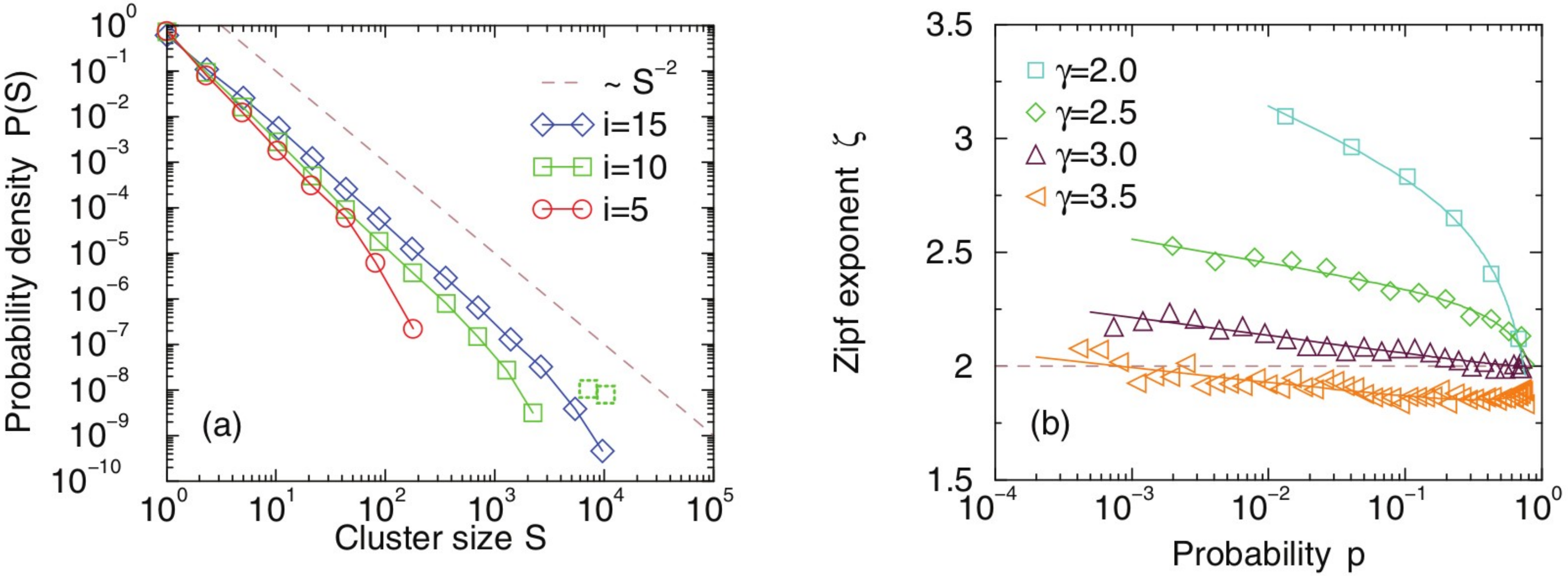
$$q_i \sim \frac{\sum_j w_j d_{ij}^{-\gamma}}{\sum_j d_{ij}^{-\gamma}}, \quad i \neq j$$

Rybski et al.

PRE, 2013

Fig. 1

Gravitational Growth: cluster size distribution

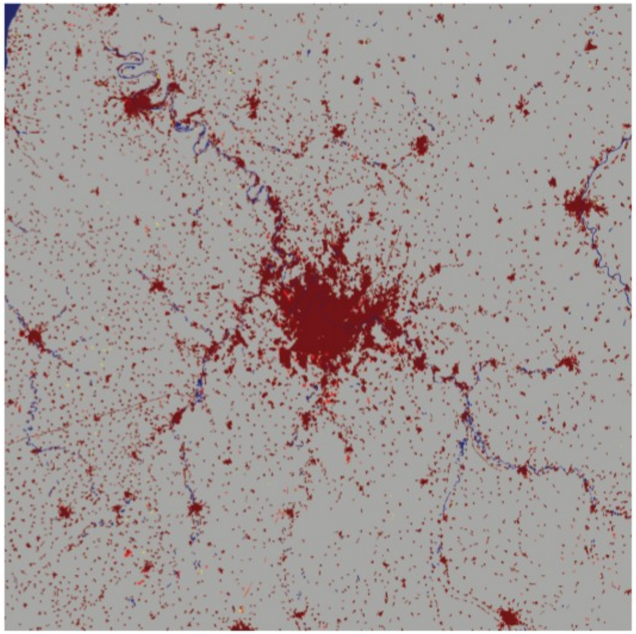


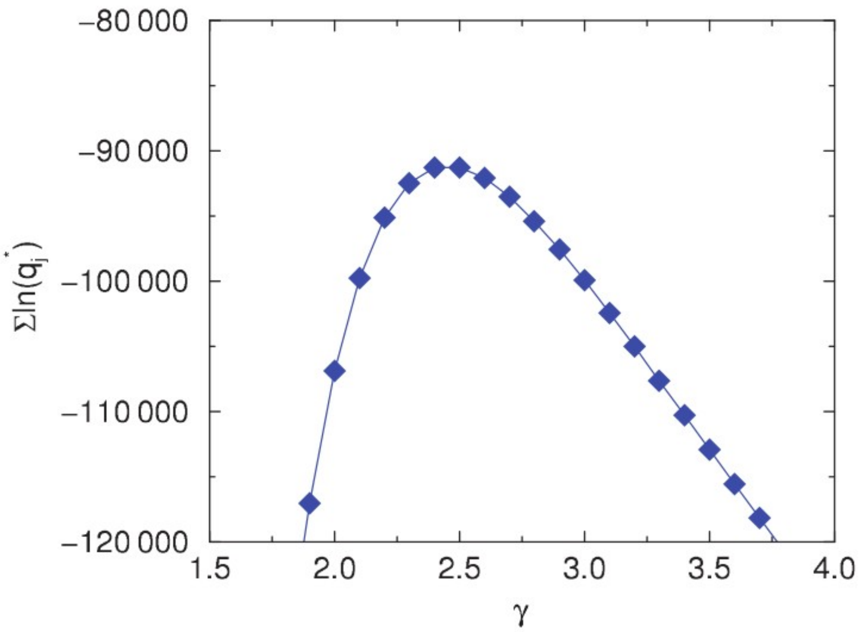
Rybski et al.

PRE, 2013

Fig. 1

Real-world data



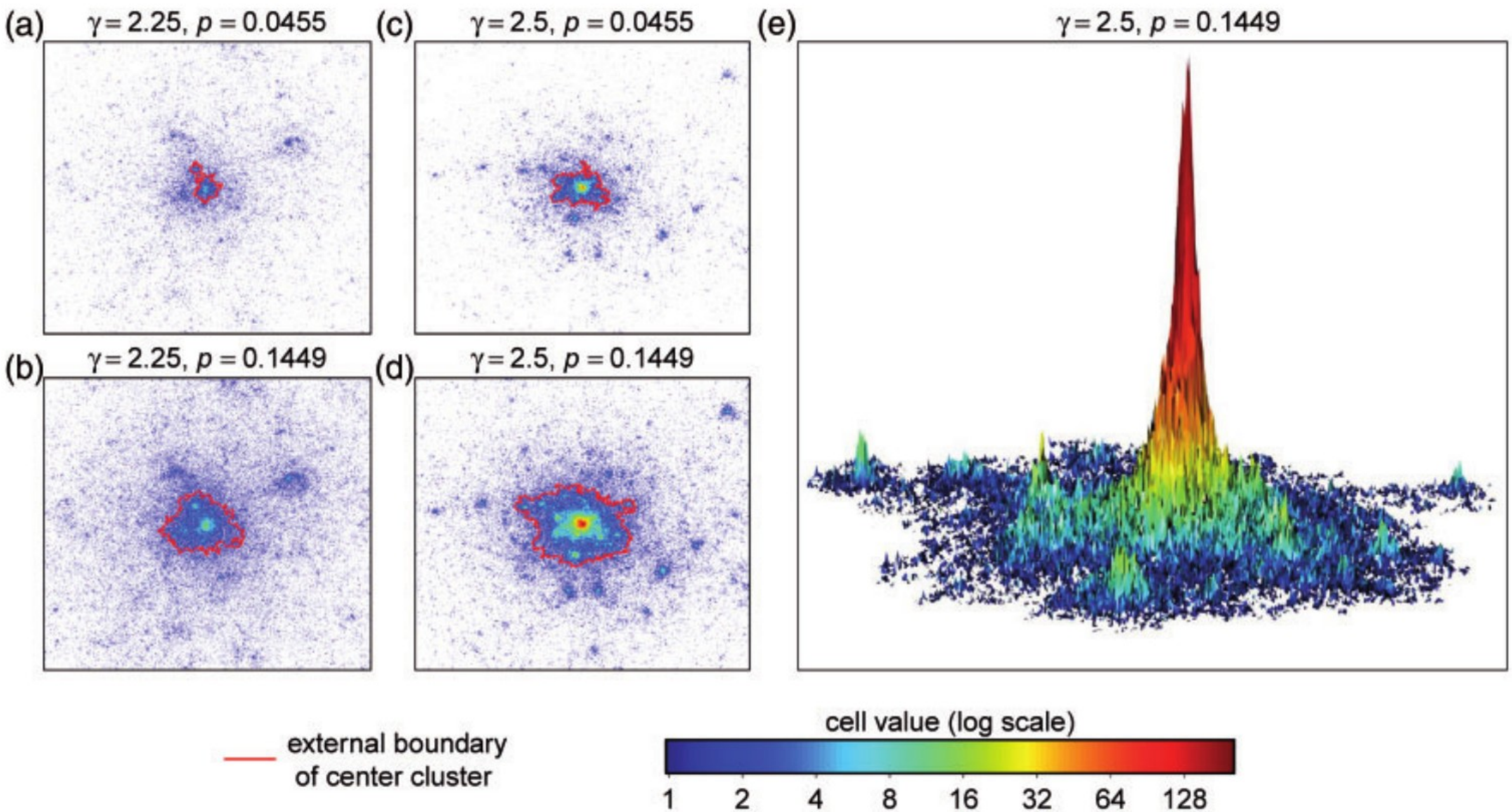


Rybski et al.

PRE, 2013

Fig. 1

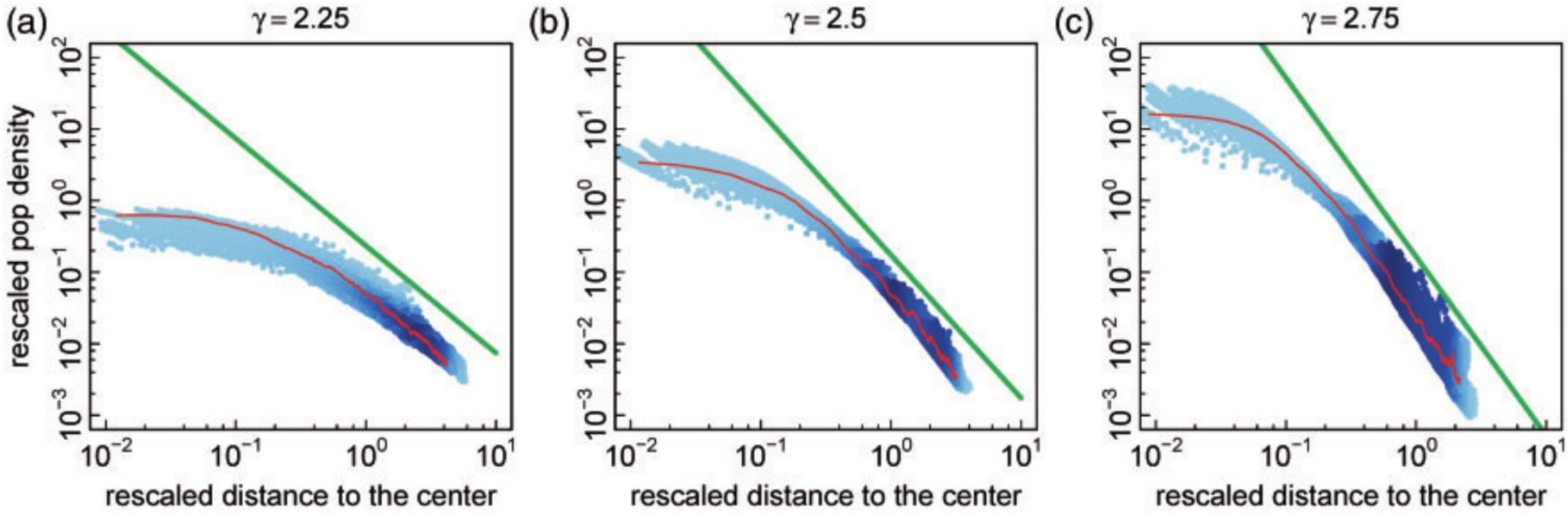
Cumulative version



Y.Li et al. 2021

Population density

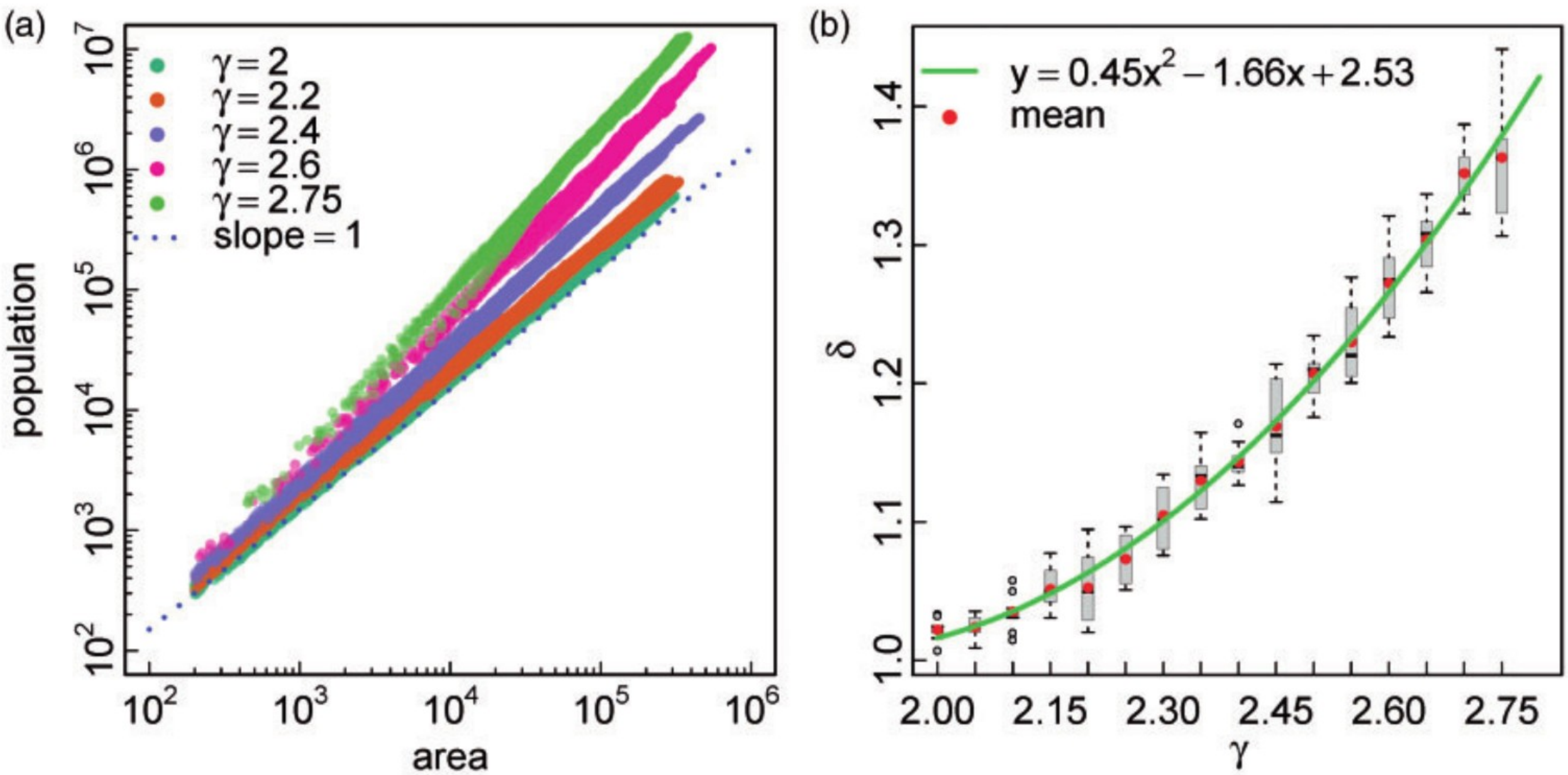
Y.Li et al. 2021



Rescaled after Lemoy & Caruso, 2020.

$$\frac{D(r)}{S^{1/3}} \sim \left(\frac{r}{S^{1/3}}\right)^{-\alpha} \qquad \alpha = 2\gamma - 3$$

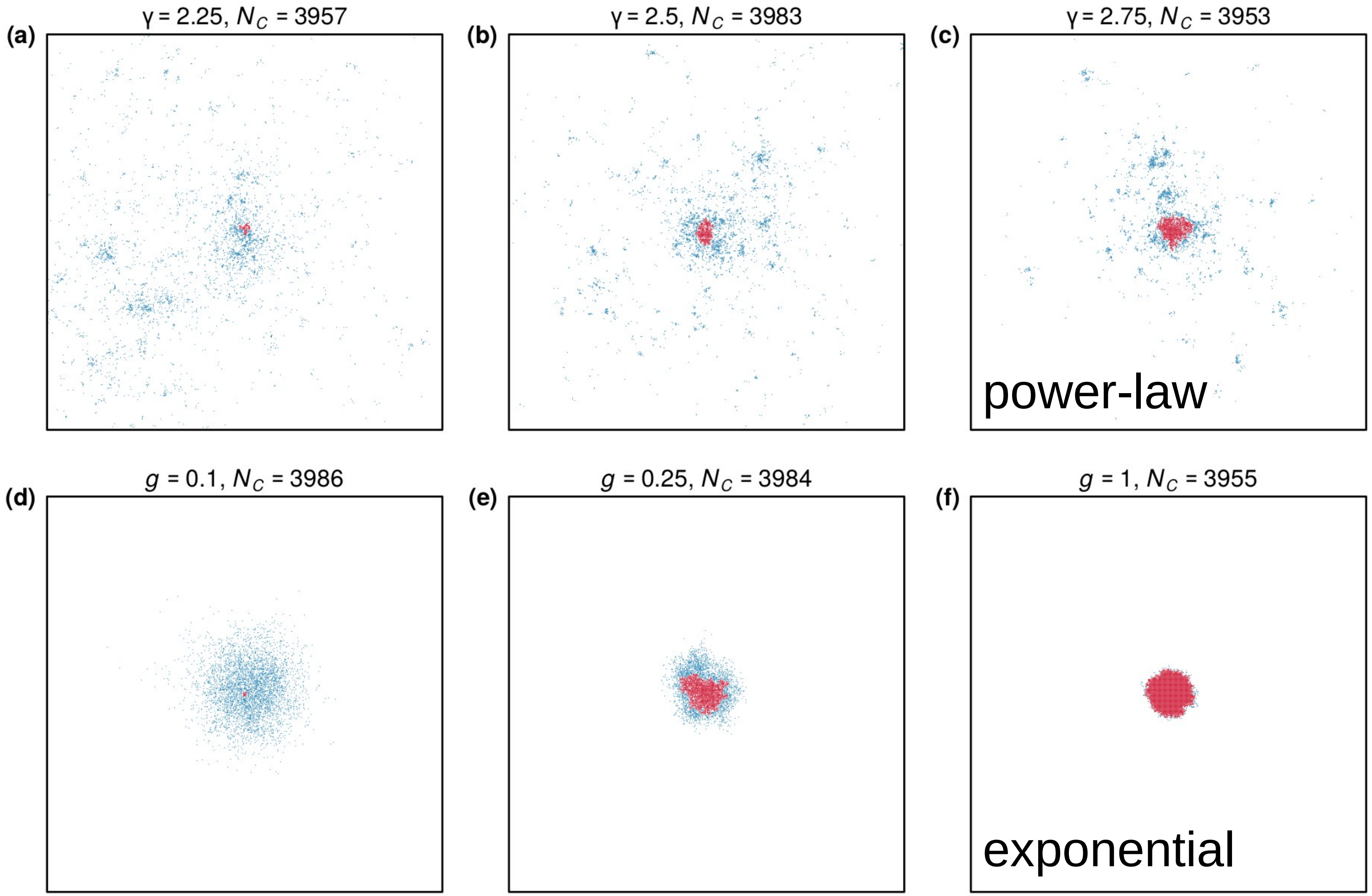
Fundamental allometry



Y.Li et al. 2021

Comparison with exponential

Rybski & Li, 2021



Urban Percolation

PNAS PNAS PNAS

Laws of population growth

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Edited by Michael F. Goodchild, University of California, Santa Barbara, CA, and approved September 29, 2008 (received for review July 31, 2008)

An important issue in the study of cities is defining a metropolitan area, because different definitions affect conclusions regarding the statistical distribution of urban activity. A commonly employed method of defining a metropolitan area is the Metropolitan Statistical Areas (MSAs), based on rules attempting to capture the notion of city as a functional economic region, and it is performed by using experience. The construction of MSAs is a time-consuming process and is typically done only for a subset (a few hundreds) of the most highly populated cities. Here, we introduce a method to designate metropolitan areas, denoted "City Clustering Algorithm" (CCA). The CCA is based on spatial distributions of the population at a fine geographic scale, defining a city beyond the scope of its administrative boundaries. We use the CCA to examine Gibrat's law of proportional growth, which postulates that the mean and standard deviation of the growth rate of cities are constant, independent of city size. We find that the mean growth rate of a cluster by utilizing the CCA exhibits deviations from Gibrat's law, and that the standard deviation decreases as a power law with respect to the city size. The CCA allows for the study of the underlying process leading to these deviations, which are shown to arise from the existence of long-range spatial correlations in population growth. These results have sociopolitical implications, for example, for the location of new economic development in cities of varied size.

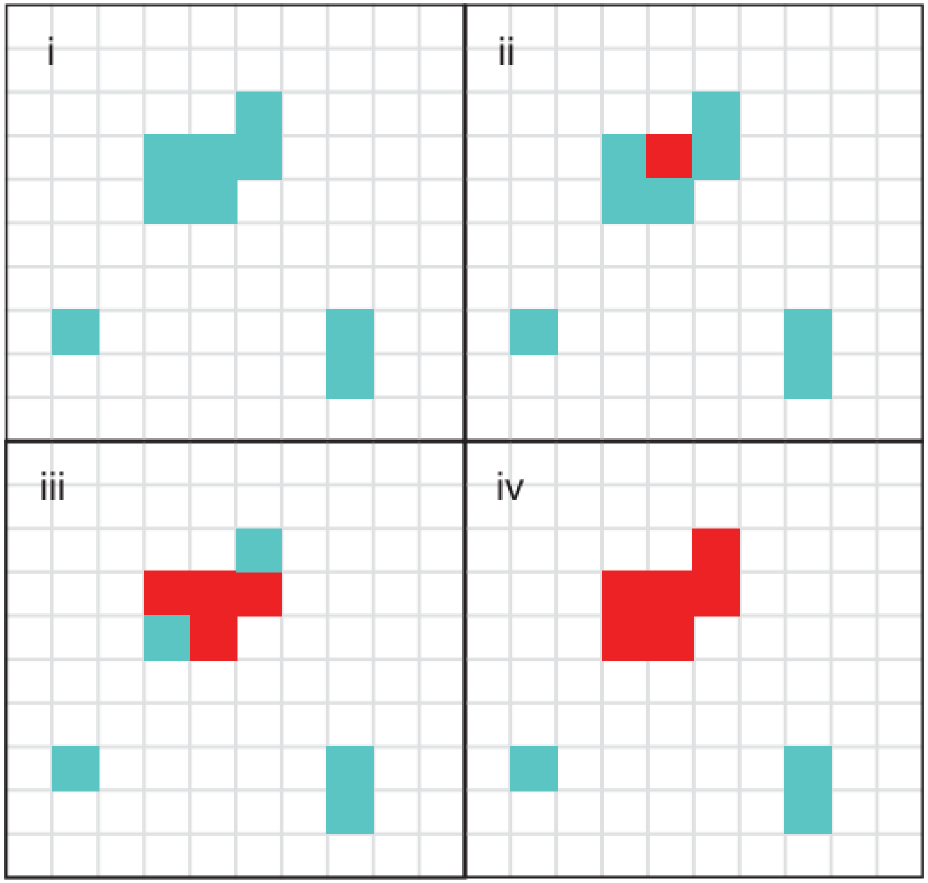
scaling | statistical analysis | urban growth

In recent years there has been considerable work on how to

country and continental levels (Great Britain, the United States, and Africa) deviates from Gibrat's law. We find that the mean and standard deviation of population growth rates decrease with population size, in some cases following a power-law behavior. We argue that the underlying demographic process leading to the deviations from Gibrat's law can be modeled from the existence of long-range spatial correlations in the growth of the population, which may arise from the concept that "development attracts further development." These results have implications for social policies, such as those pertaining to the location of new economic development in cities of different sizes. The present results imply that, on average, the greatest growth rate occurs in the smallest places where there is the greatest risk of failure (larger fluctuations). A corollary is that the safest growth occurs in the largest places having less likelihood for rapid growth.

The analyzed data consist of the number of inhabitants, $n_i(t)$, in each cell i of a fine geographical grid at a given time, t. The cell size varies for each dataset used in this study. We consider three different geographic scales: on the smallest scale, the area of study is Great Britain (GB: England, Scotland and Wales), a highly urbanized country with a population of 58.7 million in 2007, and an area of 0.23 million km^2. The grid is composed of 5.75 million cells of 200 m by 200 m (8). At the intermediate scale, we study the USA (continental United States without Alaska), a single country nearly continental in scale, with a population of 303 million in 2007, and an area of 7.44 million km^2. The original USA data consists of 59,456 sites defined by Federal Information Processing Standards

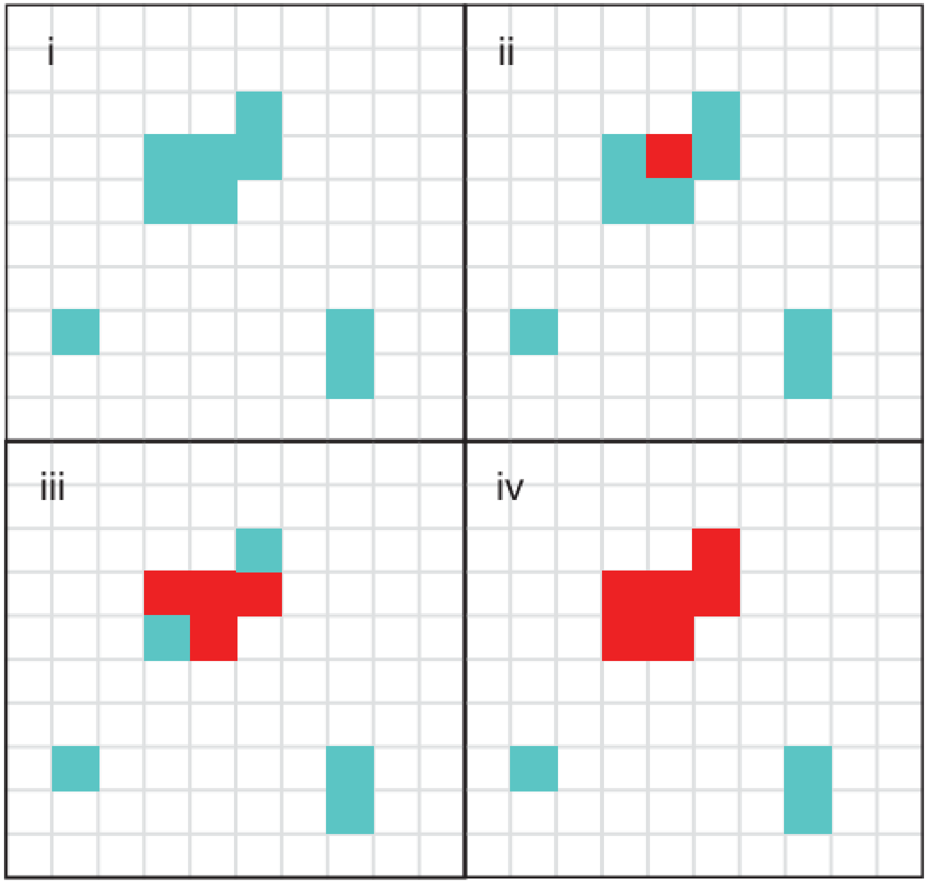
City Clustering Algorithm (CCA) – Rozenfeld et al. AER 2011

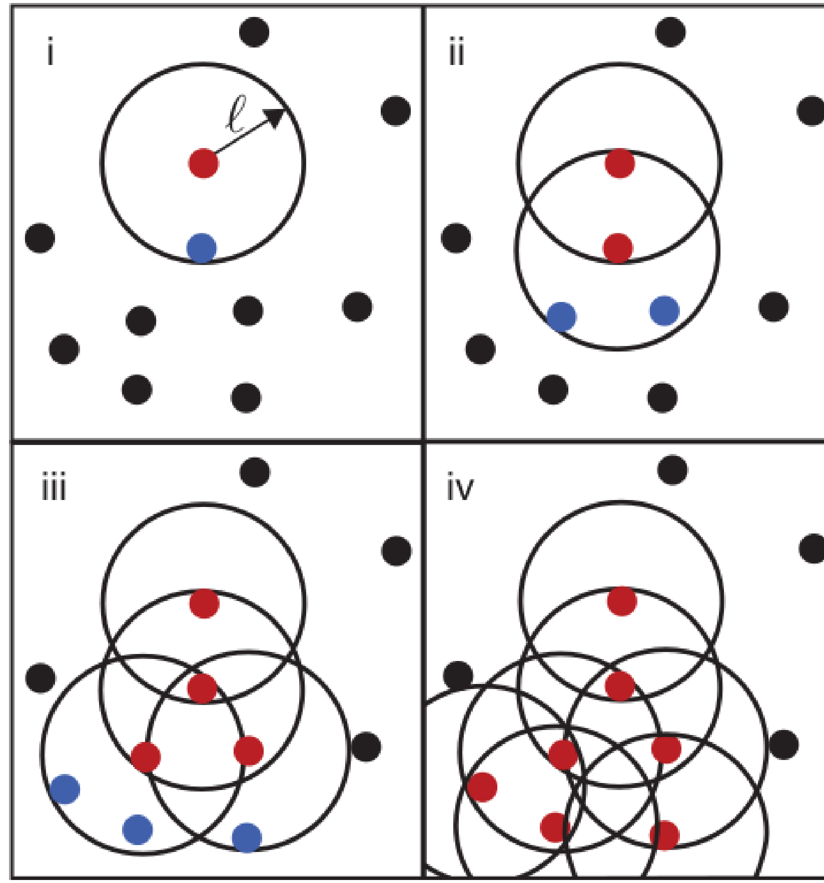


Any two objects are assigned to the same cluster if their distance is short than or equal to a predefined threshold distance.

Generalization of Burning Algorithm
(Stauffer & Aharony 1991;
Hoshen & Kopelman, PRB 1976)

City Clustering Algorithm (CCA) – Rozenfeld et al. AER 2011



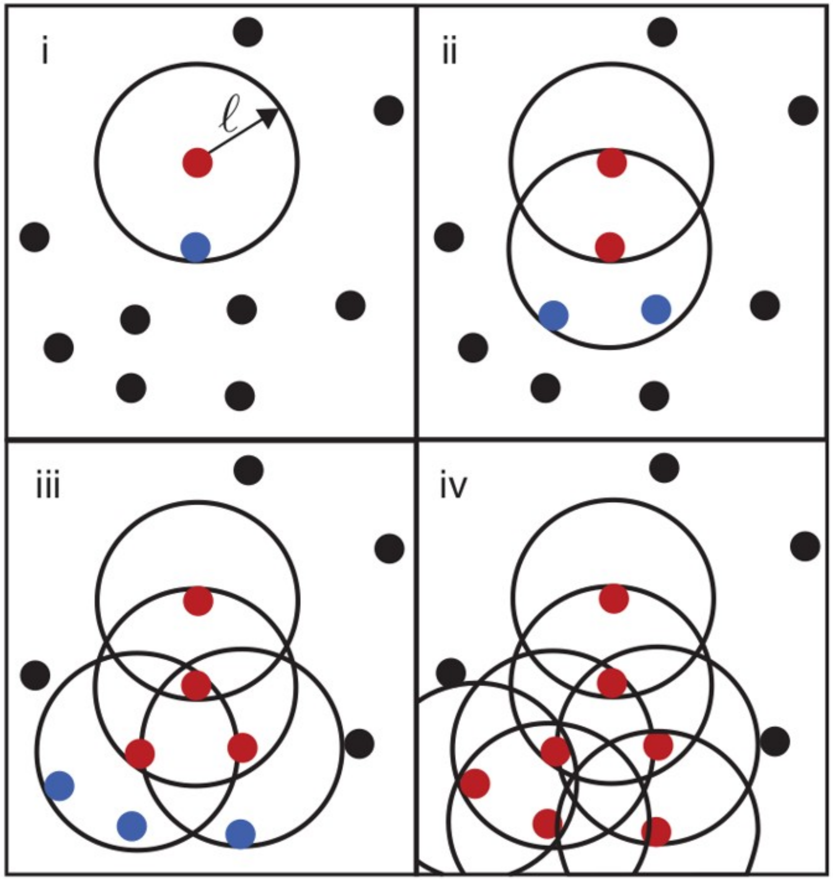


Generalization of Burning Algorithm
(Stauffer & Aharony 1991;
Hoshen & Kopelman, PRB 1976)

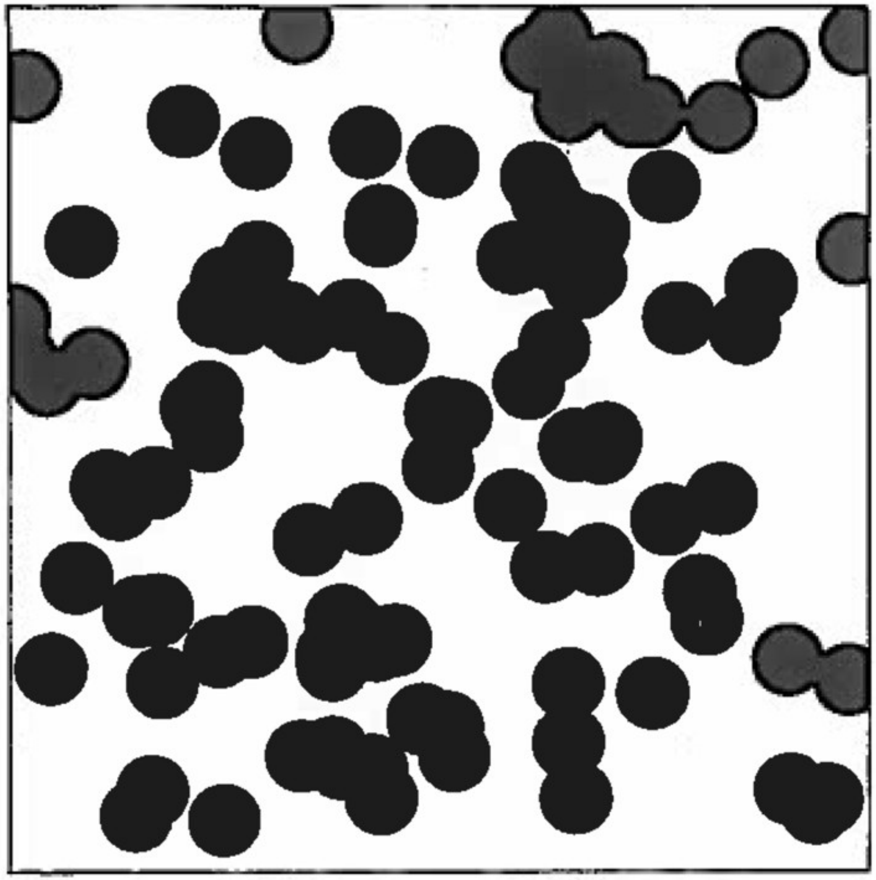
Similar to Random Geometric Graph
(but without network)

→ for large / equivalent

Percolation



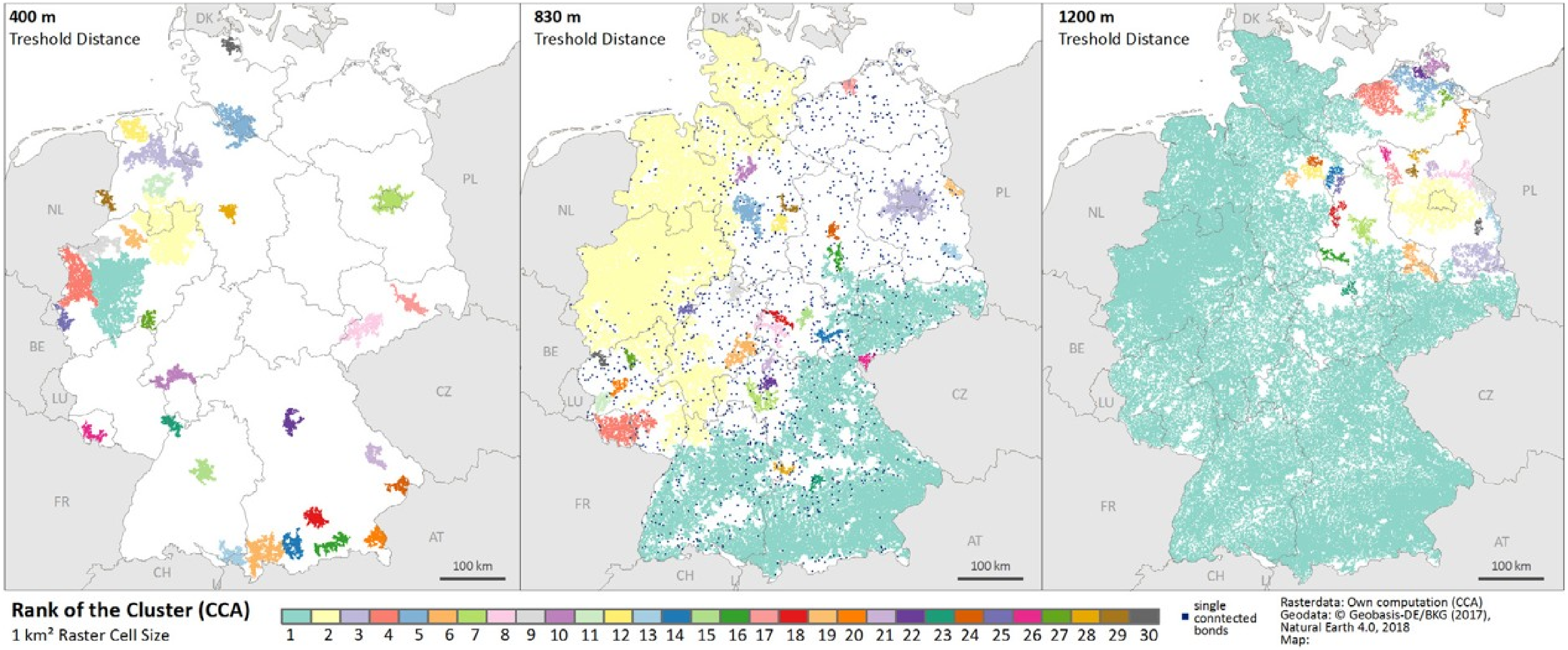
City Clustering Algorithm (point data)
Rozenfeld et al. AER 2011



Continuum Percolation
Bunde & Havlin 1991 (Fig.2.2)

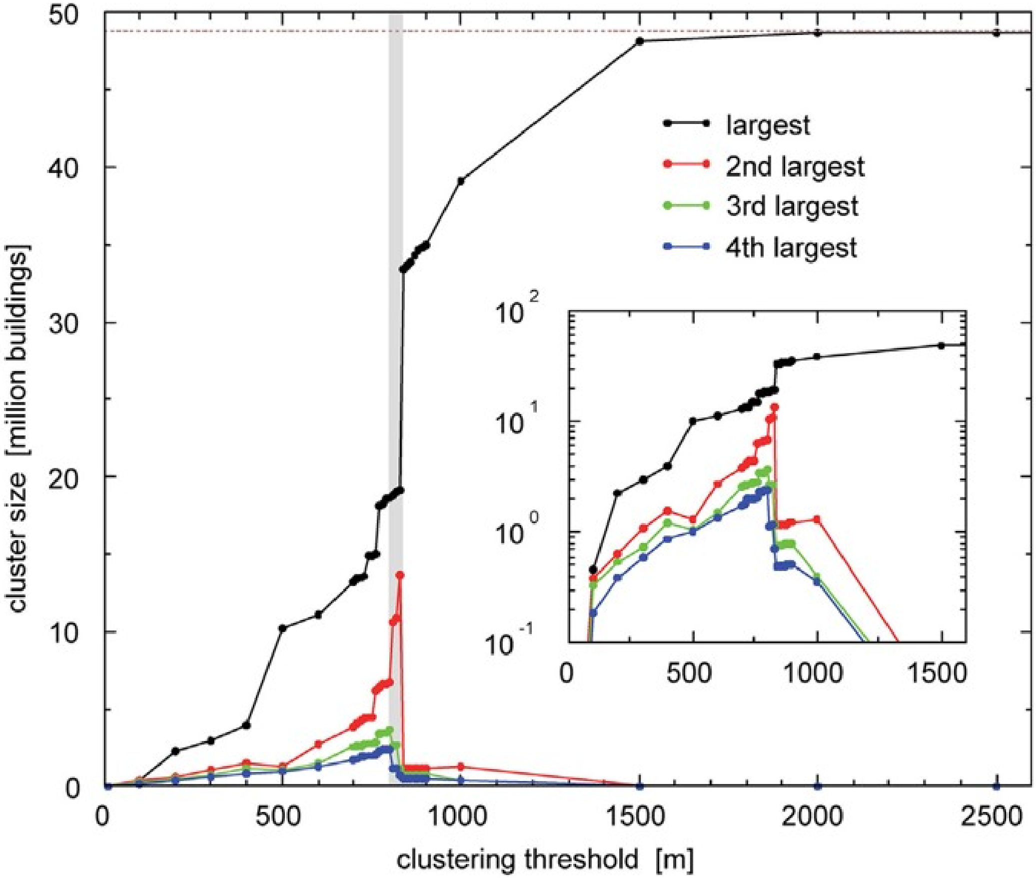
(be aware of factor 2)

Behnisch et al. Land.Urban.Plan 2019



48M building coordinates

Behnisch et al. Land.Urban.Plan 2019



Transition at approx. 830m

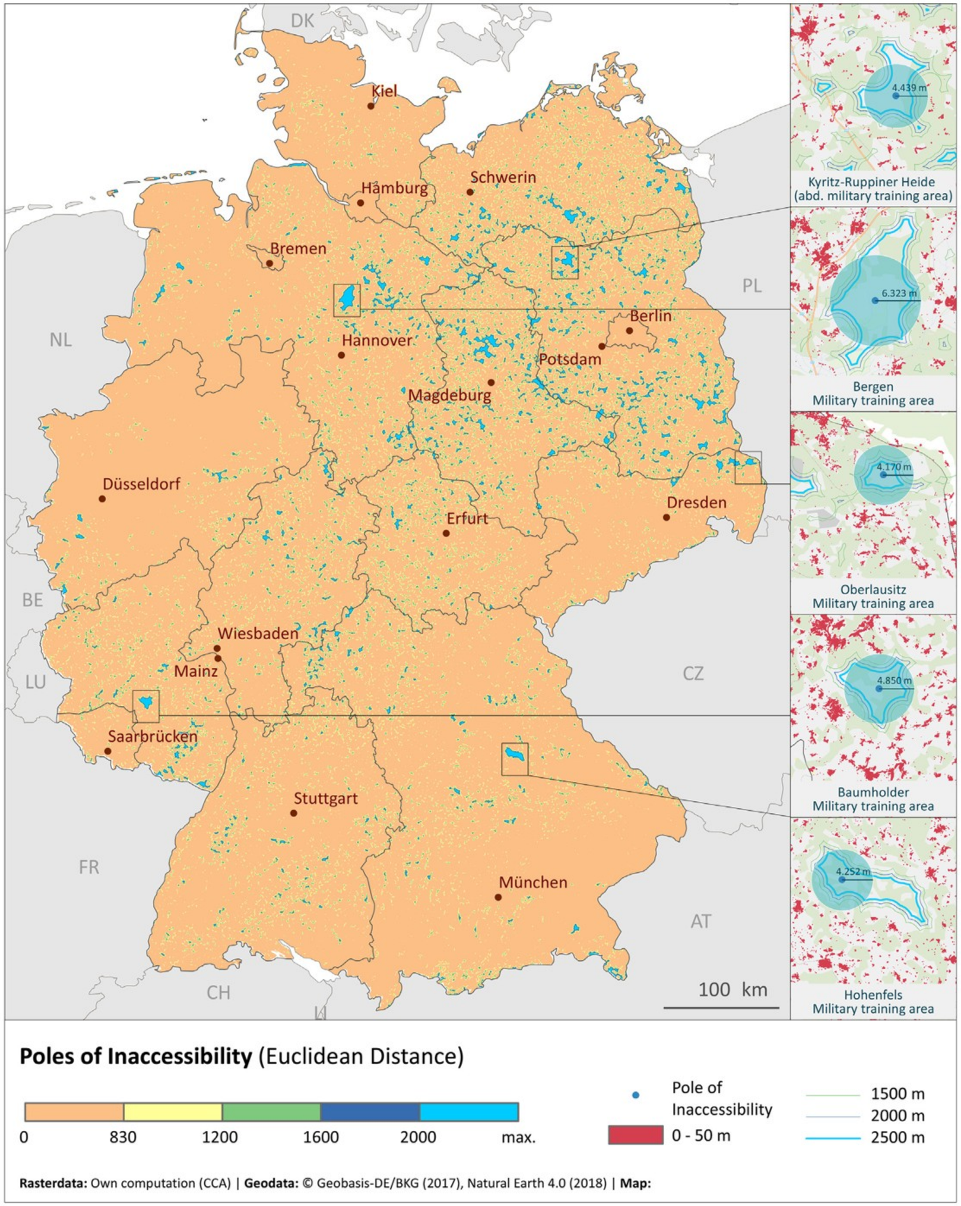
Brandenburg 1450m

Saarland 400m

Threshold-distance 1.5km:

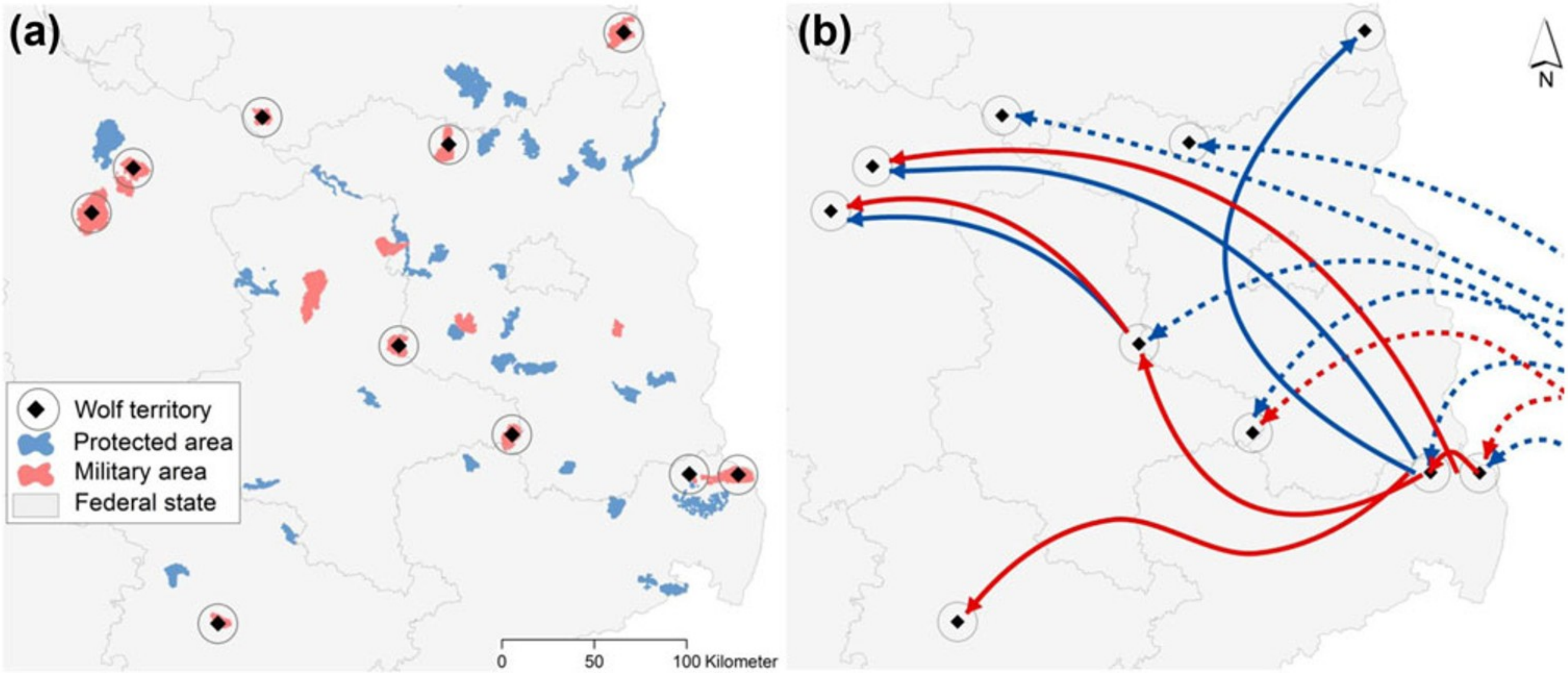
99% building stock

Excursus: Behnisch et al. Land.Urban.Plan 2019



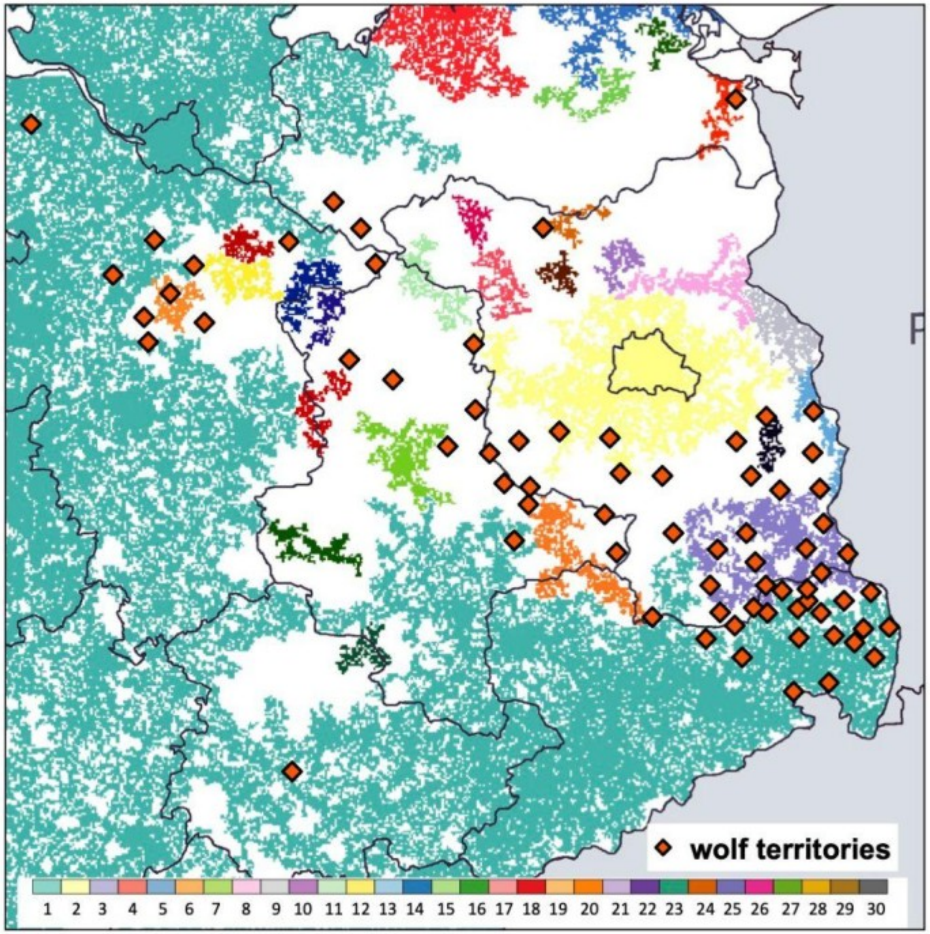
Top 5 "largest" *poles of inaccessibility* are present of former military training sites

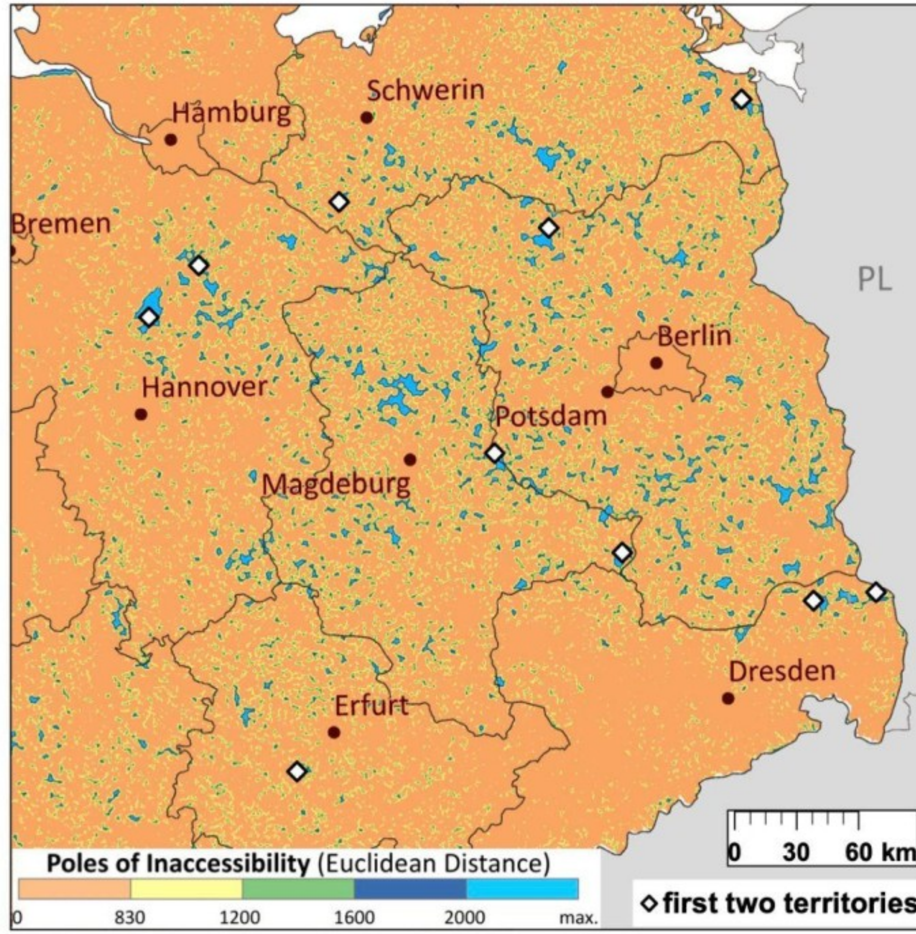
Excursus: Wolves – Reinhardt et al. Conserv.Lett. 2018



Military training areas facilitate the recolonization of wolves in Germany.
The first territories were always established on MTAs.

Excursus: Wolves – Reinhardt et al. Conserv.Lett. 2018

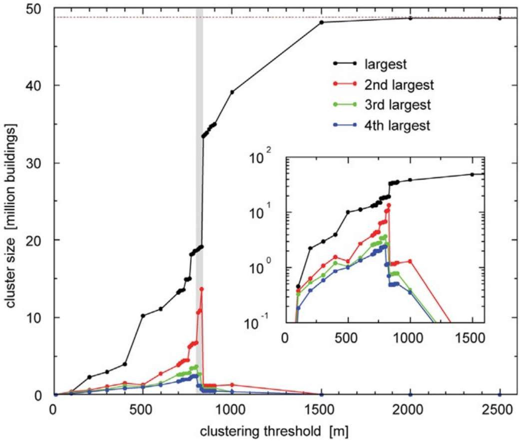




Percolation?

Is *Urban Percolation* aka *CCA Percolation*
a percolation phenomenon in the original sense?

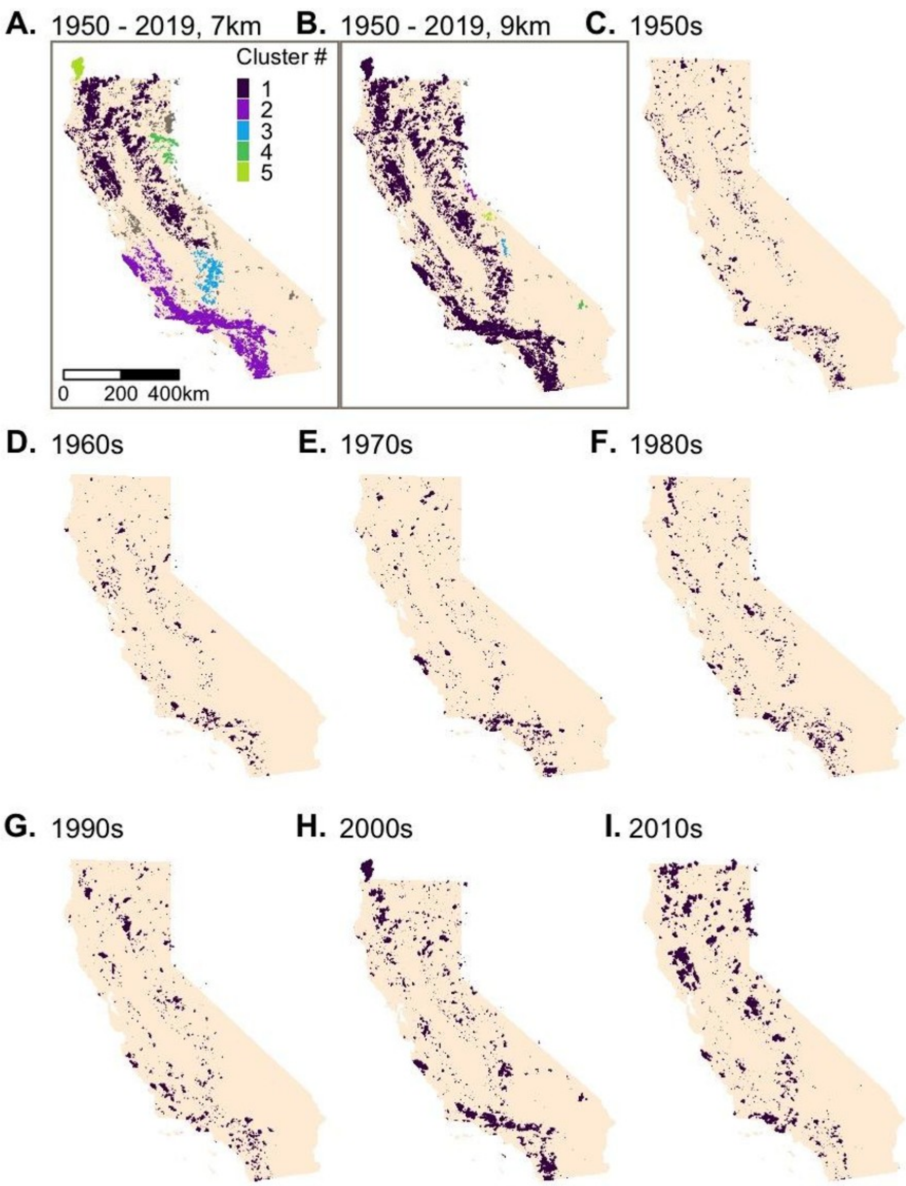
- probabilities are not independent
- most systems are sub-critical
  "large" threshold distances are necessary
  space between objects needs to be bridged



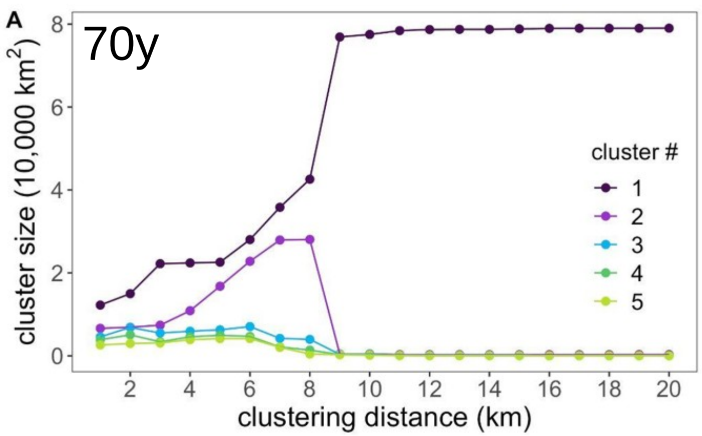
How can we make the connection to "true" percolation?

When do we have a percolation transition for *l*=1?

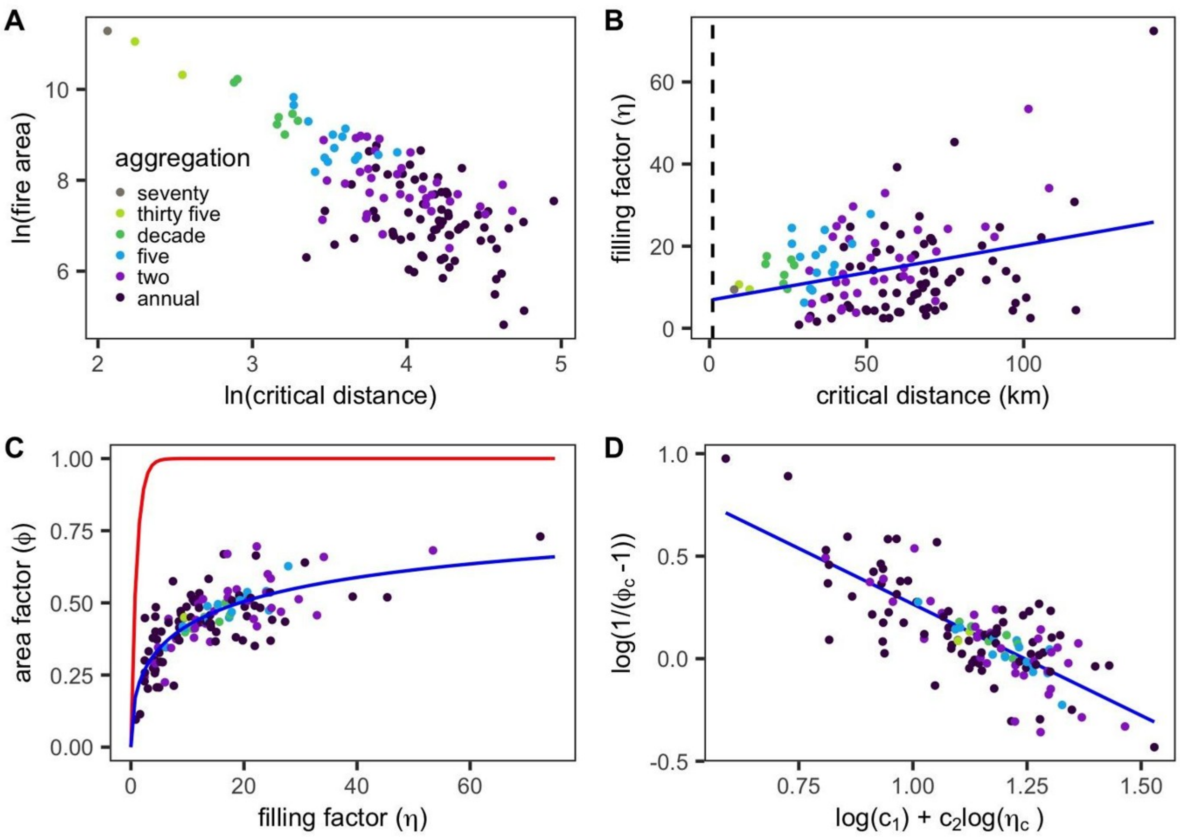
“Predictive Percolation”: Hemond et al. 2023



Fire perimeters
in California
1950-2019
(rasterized 1km)



"Predictive Percolation": Hemond et al. 2023



$$\eta = \frac{\Pi}{4} l_c^2 \frac{n}{N}$$

Filling factor

n: number of objects

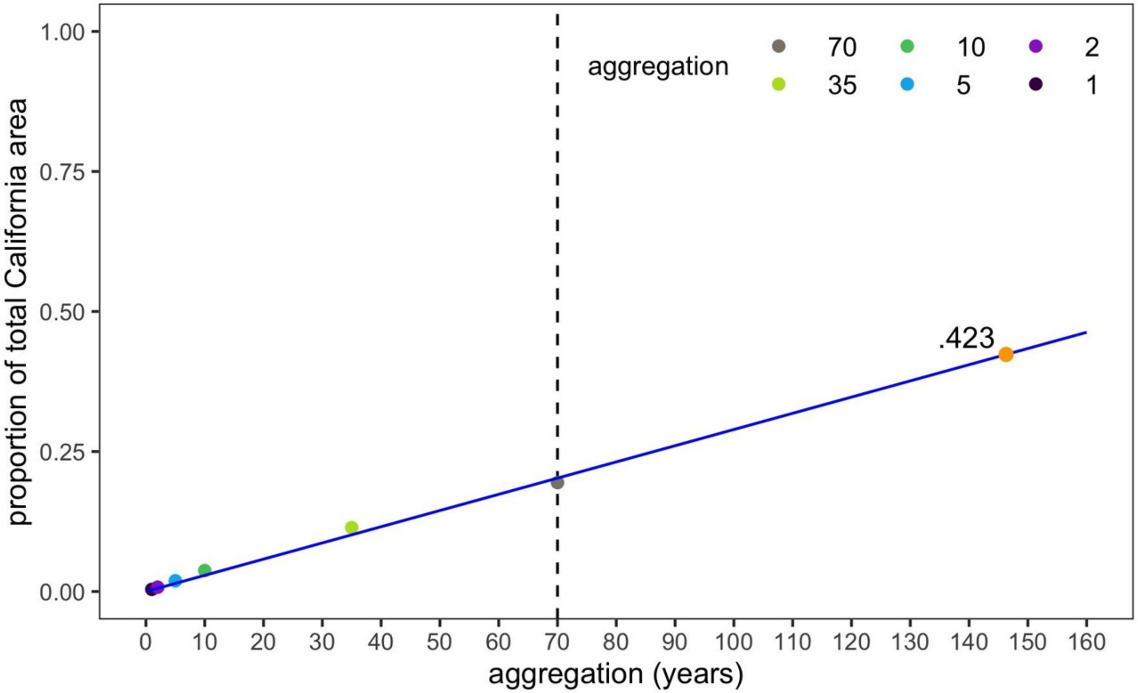
N: total number

$$\phi = \frac{1}{1 + c_1 \eta^{-c_2}}$$

Area factor

$c_{1,2}$: parameters

“Predictive Percolation”: Hemond et al. 2023



η_c^{*}	ϕ_c^{**}	$A_c^{\dagger}$	Years[‡]
10.19 *[2.84; 14.89]*	.423 *[.283; .469]*	173,389 *[115,840; 192,064]*	146.3 *[97.8; 162.1]*

[*] The filling factor

[**] The percolation threshold value based upon Eq. (3)

[†] The fire area in km^2 at the percolation threshold

[‡] The projected number of years until percolation is reached, counting from 1950

City size distribution
& urban scaling

City size distribution

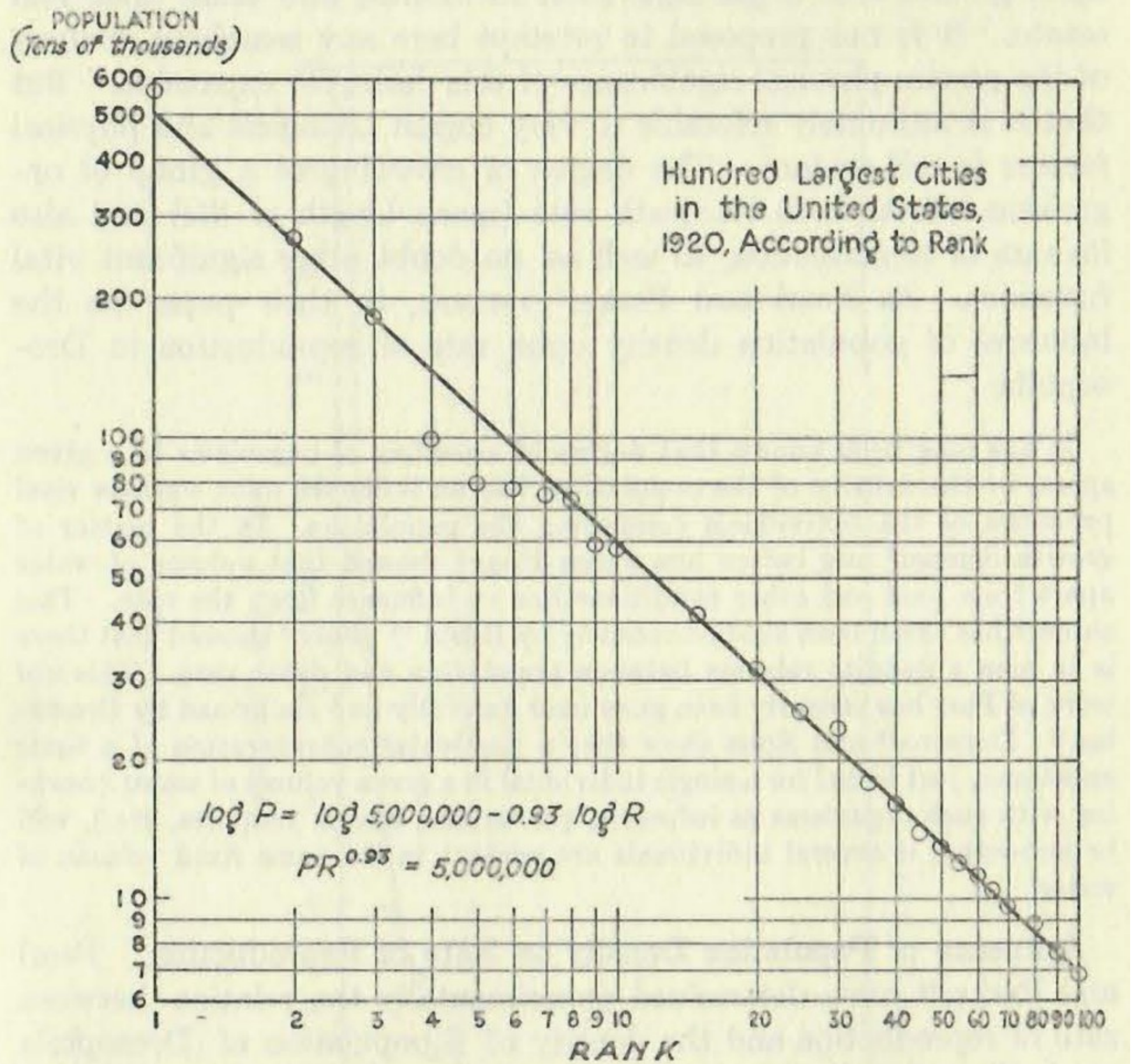


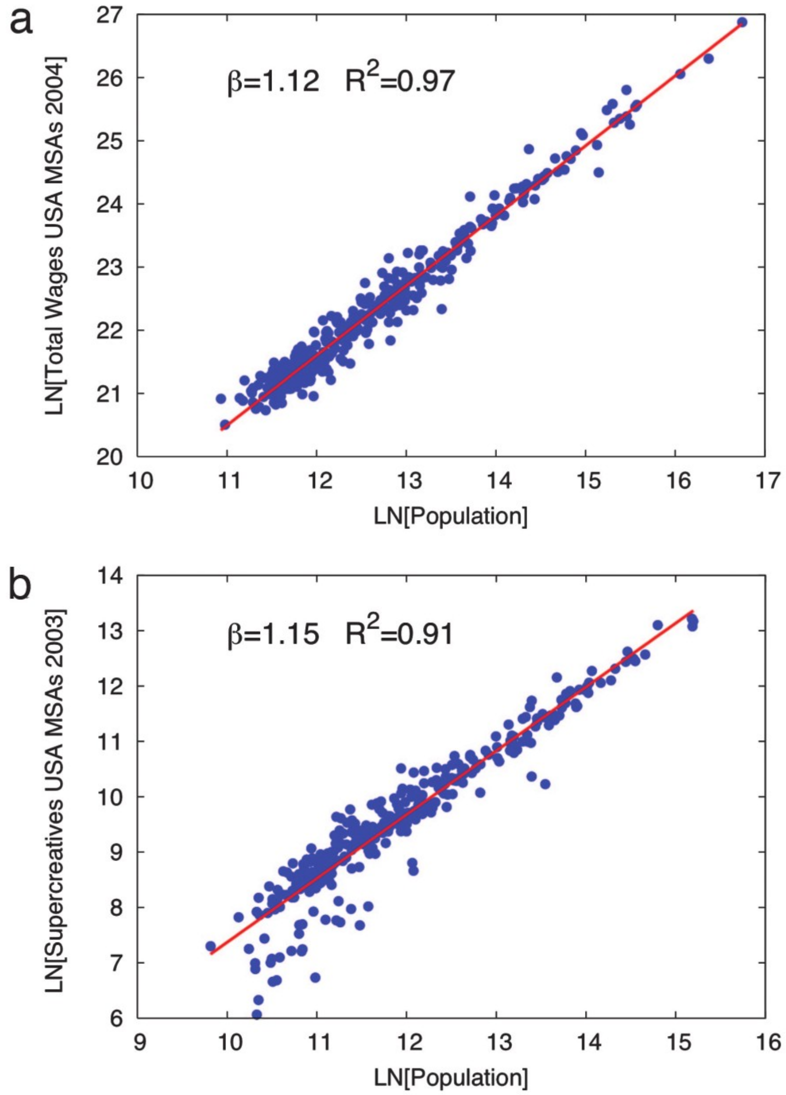
FIG. 60. LAW OF URBAN CONCENTRATION

Lottka 1925

(Auerbach 1913;

Zipf 1949)

Urban scaling



Bettencourt et al.,

PNAS, 2007

socio-economic: $\beta > 1$

personal needs: $\beta = 1$

infrastructure: $\beta < 1$

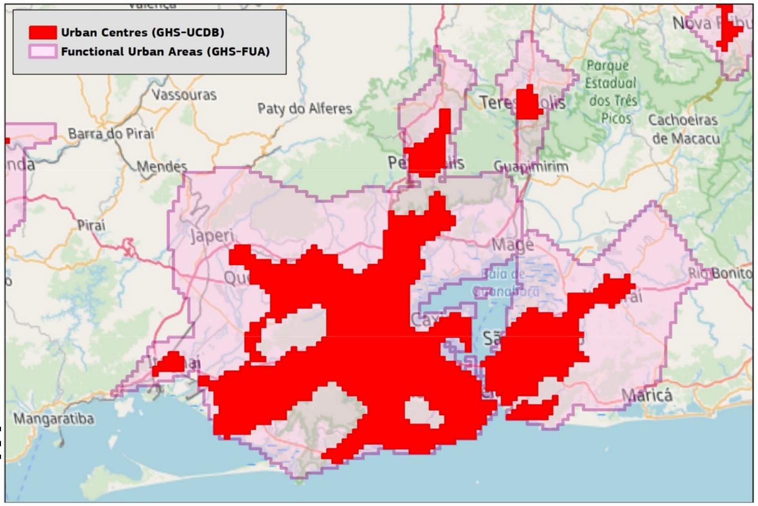
contrary to *Dreisatz*

Data: FUA

Functional Urban Areas (FUA) provided by OECD & EU includes population from GHSL

Gridded GDP from
Kummu et al. Sci Data 2018

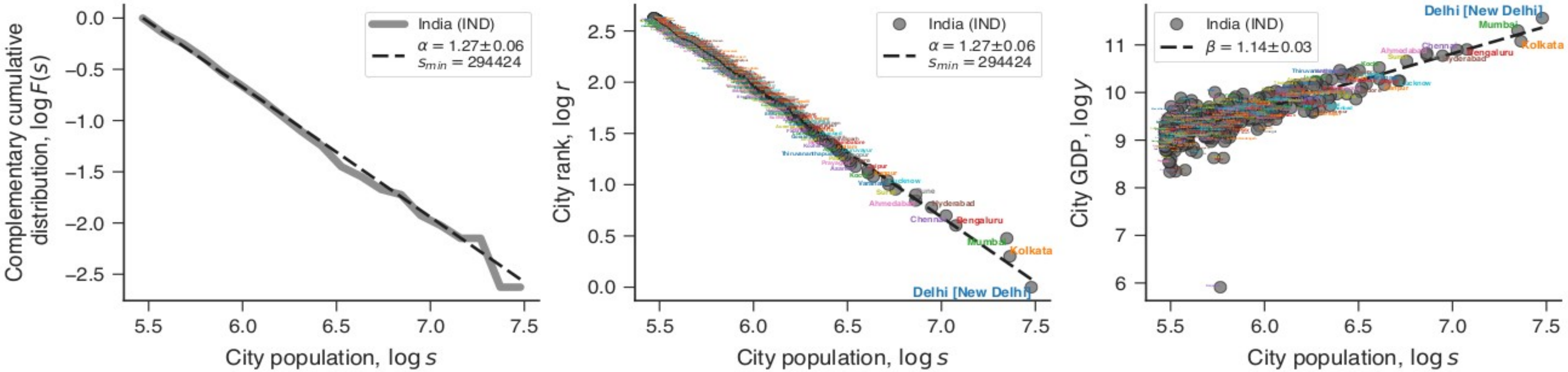
Together:
- Consistently defined Urban Units
- 4571 cities from 96 countries
- population and GDP for each



Schiavina, Marcello; Moreno-Monroy, Ana; Maffenini, Luca; Veneri, Paolo (2019). GHS-FUA R2019A - GHS functional urban areas, derived from GHS-UCDB R2019A, (2015), R2019A. European Commission, Joint Research Centre.

Results: example

Nice example: India (ccdf, rank-plot, urban scaling)

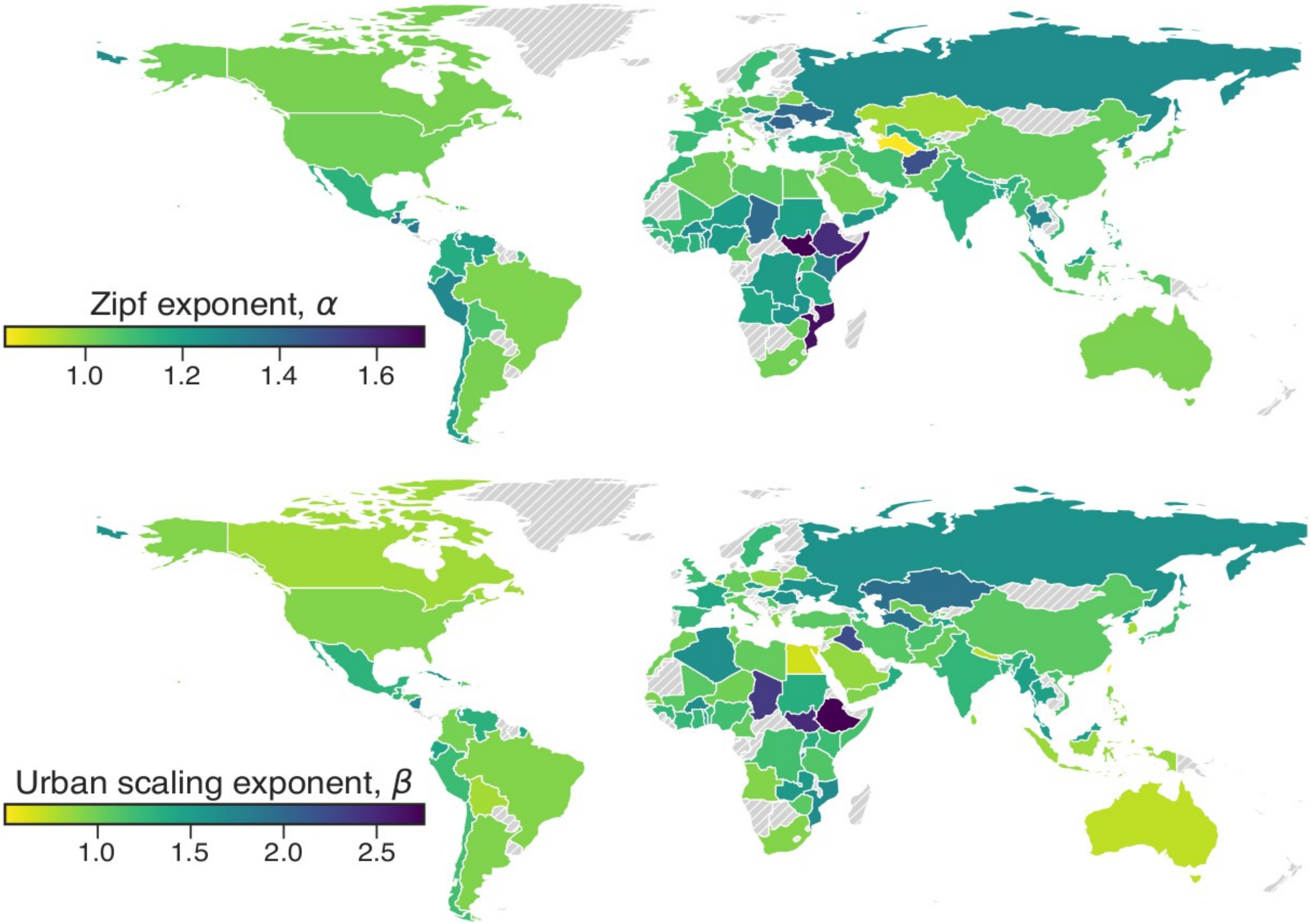


Repeating analysis for 96 countries

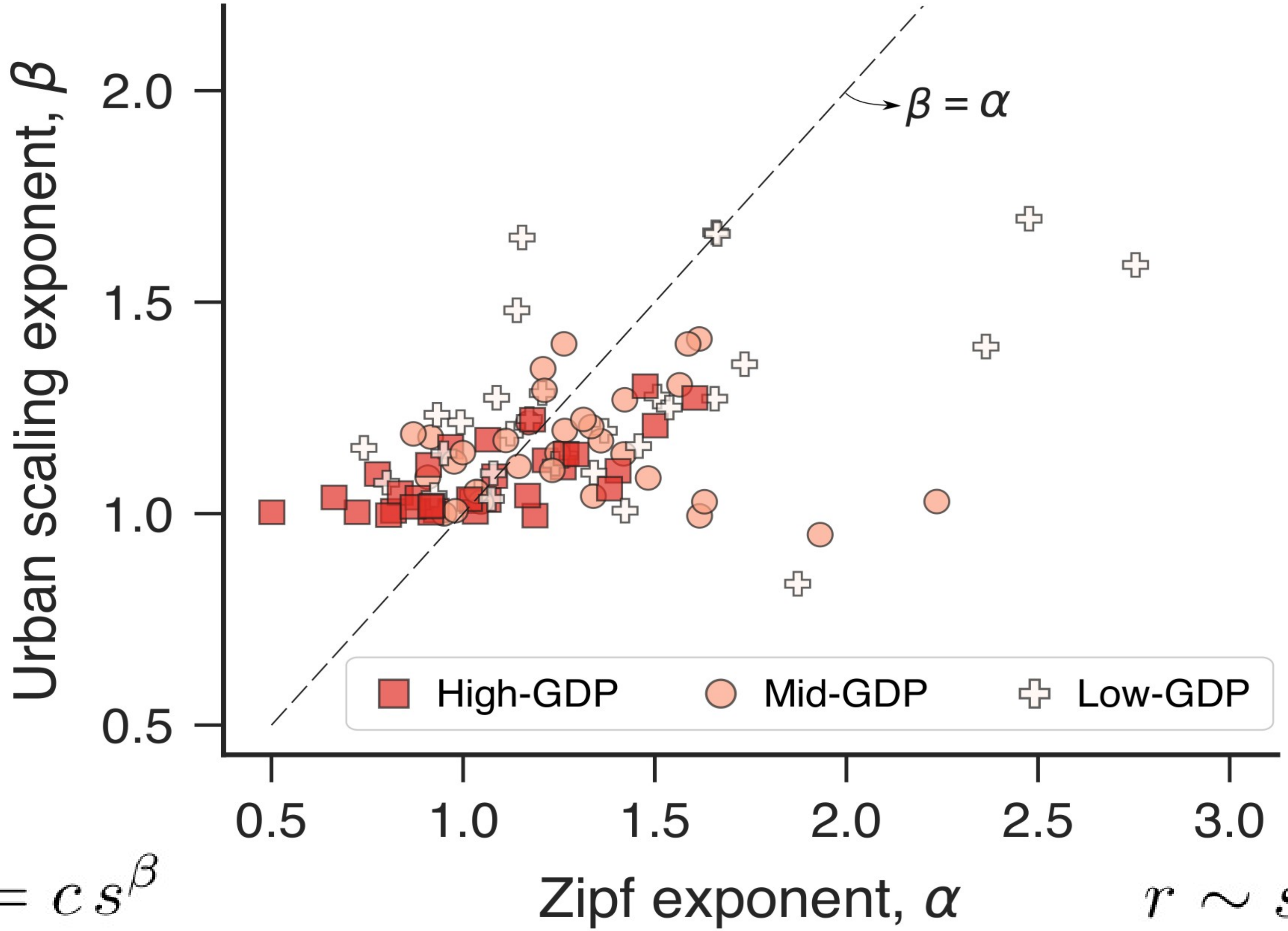
Obtaining Zipf exponent α and urban scaling exponent β

Are there correlations?   Yes

Results: maps



Results: scatter-plot



$$y = c\, s^{\beta}$$

$$r \sim s^{-\alpha}$$

Hypothesis

Global aggregates also scale
i.e. country GDP & population

$$Y = Y_0 S^\gamma$$

Additional global constraints:
smallest & largest city

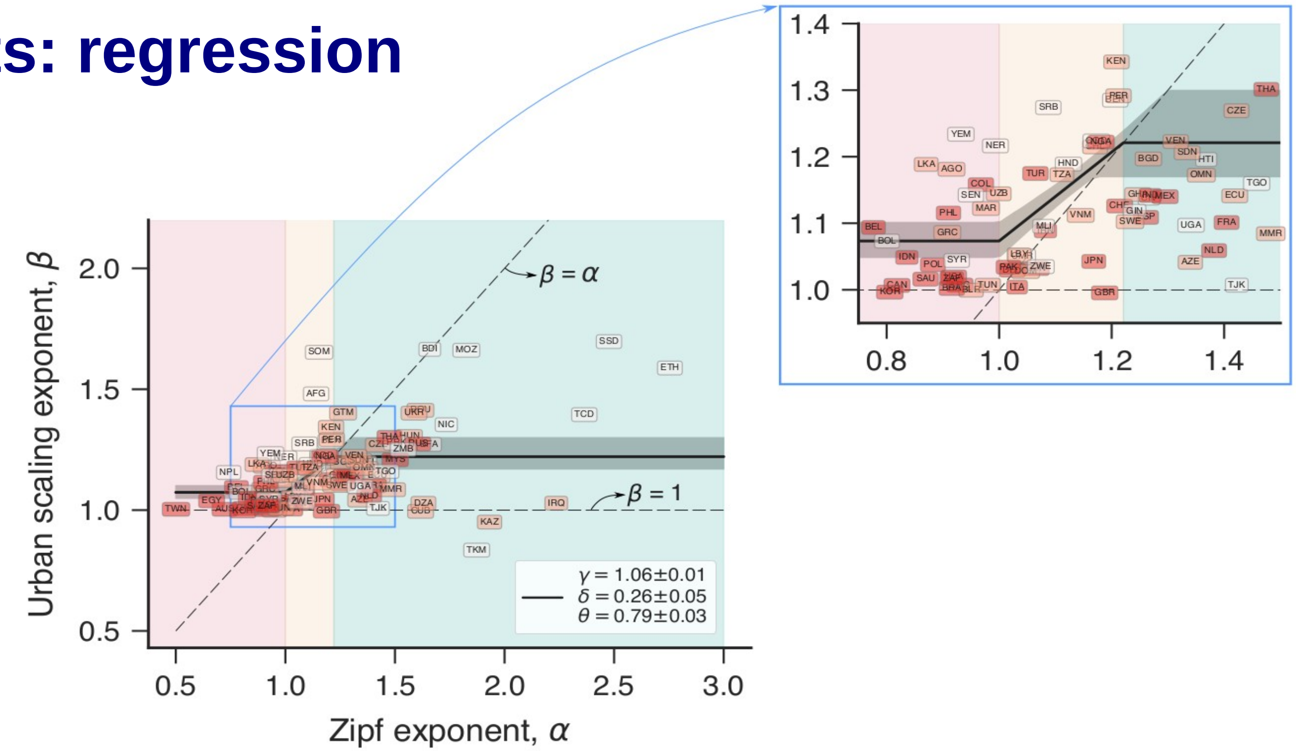
$$\begin{aligned} s_{\min} &= aS^\delta \\ s_{\max} &= bS^\theta \end{aligned}$$

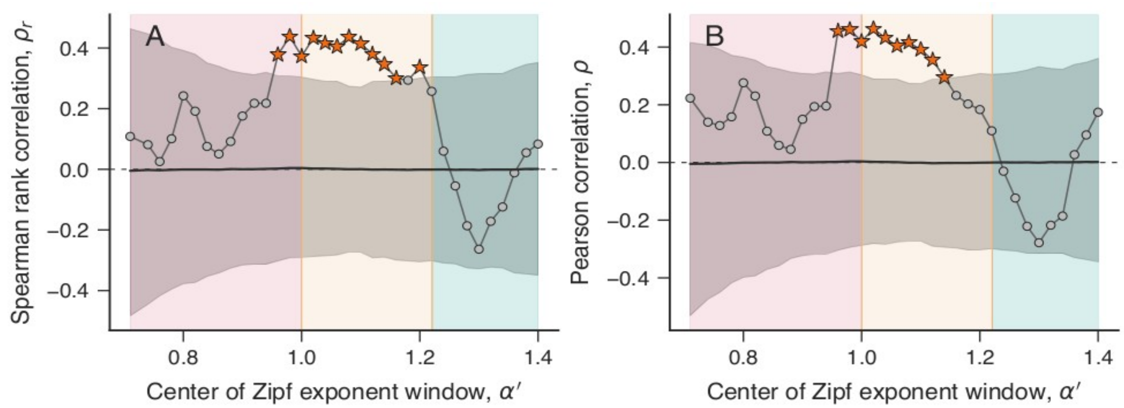
Math leads to:

$$\beta = \begin{cases} 1 + \frac{\gamma - 1}{\theta} & 0 < \alpha \leq 1 \\ \frac{\gamma + \delta - 1}{\theta} + \left(1 - \frac{\delta}{\theta}\right)\alpha & 1 < \alpha < 1 + \frac{\gamma - 1}{\delta} \\ 1 + \frac{\gamma - 1}{\delta} & \alpha \geq 1 + \frac{\gamma - 1}{\delta} \end{cases}$$

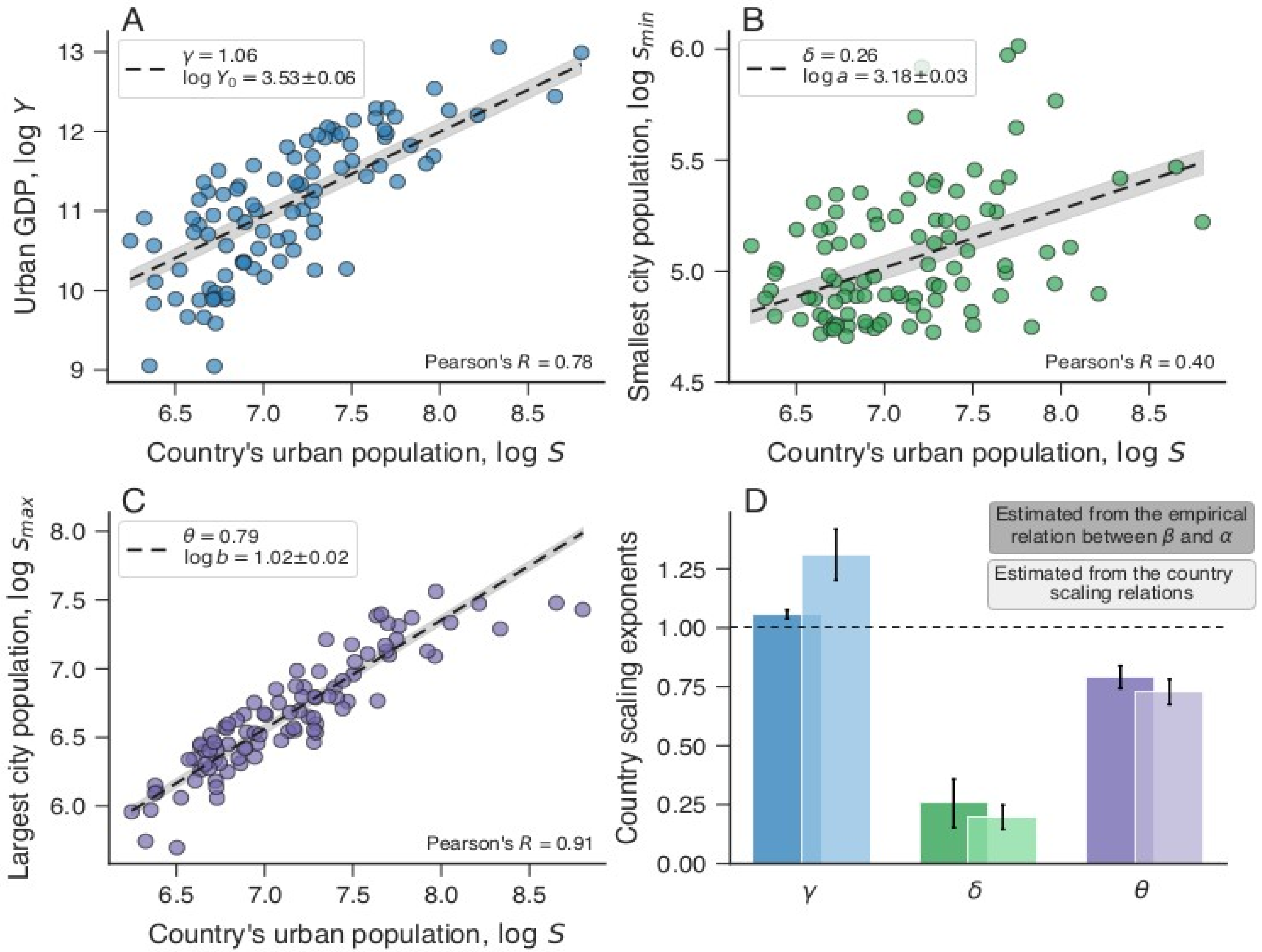
Note: country & global exponents

Results: regression

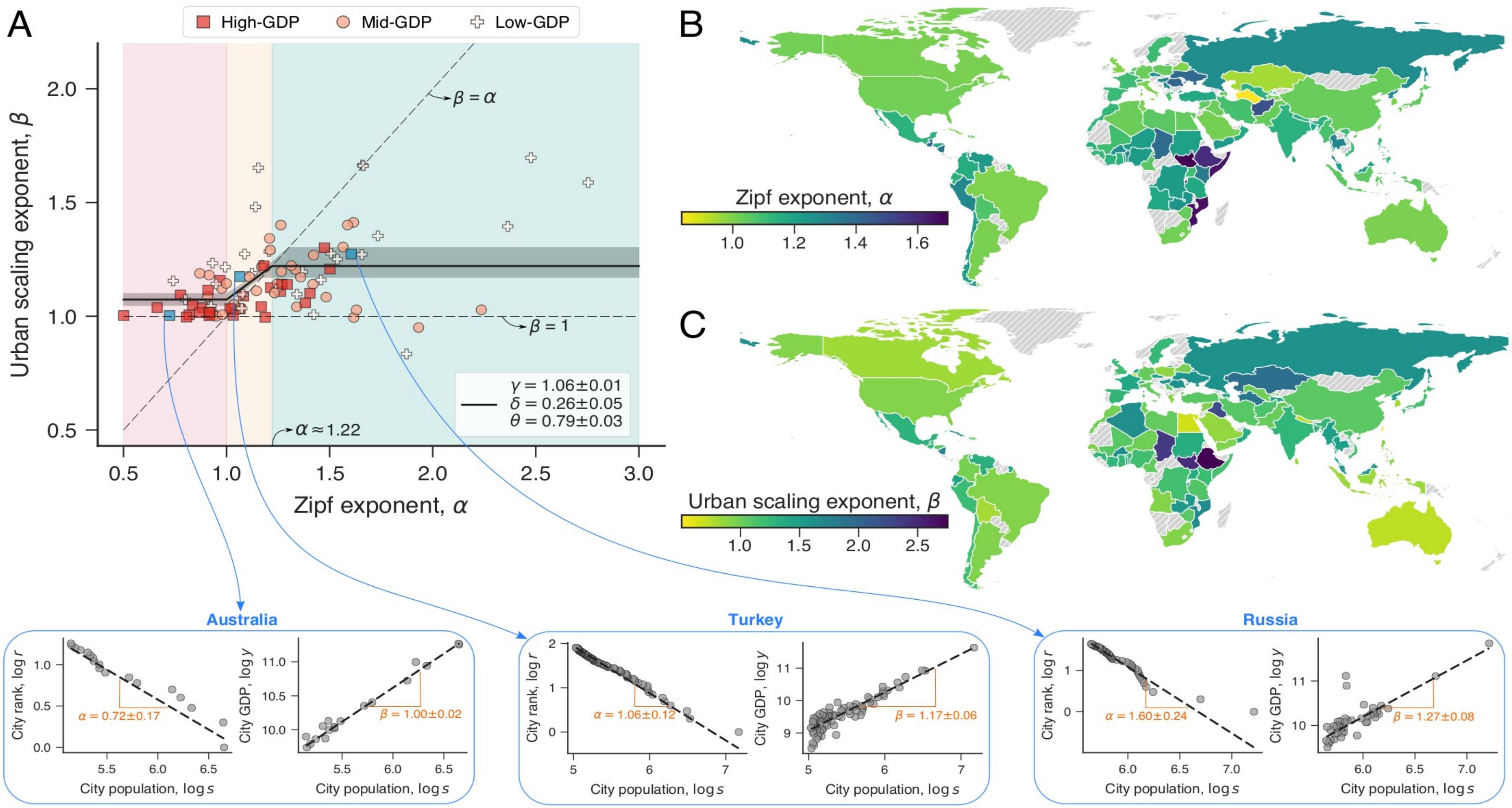




Results: country scaling



Results: overview



Why are the exponent related?

Inter-city interactions

1. Urban system with few large cities (large Zipf exponent)
   diverse specialized companies concentrate in less cities
   pronounced increasing returns to scale
   (large urban scaling exponent)
   vice versa

2. Urban system with pronounced increasing returns to scale
   (large urban scaling exponent)
   population is attracted by large cities
   (moving alters Zipf exponent)
   if they find no job or a less payed one, GDP/cap reduces
   (urban scaling exponent adjusted)
   vice versa

Note: urban scaling is often attributed to *intra*-city interactions

Summary

Zipf’s and urban scaling exponents are correlated

We derive a relation based on country scaling (3 exponents)

No causality (from our analysis & derivation)

Simulations (not shown)

Zipf’s law and urban scaling are two sides of the same coin

Urban scaling does not solely emerge from intra-city processes

Paper:

Ribeiro HV, Oehlers M, Moreno-Monroy AI, Kropp JP, Rybski D (2021)

Association between population distribution and urban GDP scaling.

PLoS ONE 16(1): e0245771.

https://doi.org/10.1371/journal.pone.0245771

Thank you for your attention

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Urban Percolations

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