Complex dynamical systems and our brain

Claudius Gros
Dimitrije Marković, Mathias Linkerhand,
Rodrigo Echeveste, Guillermo Ludueña

Institute for Theoretical Physics
Goethe University Frankfurt, Germany

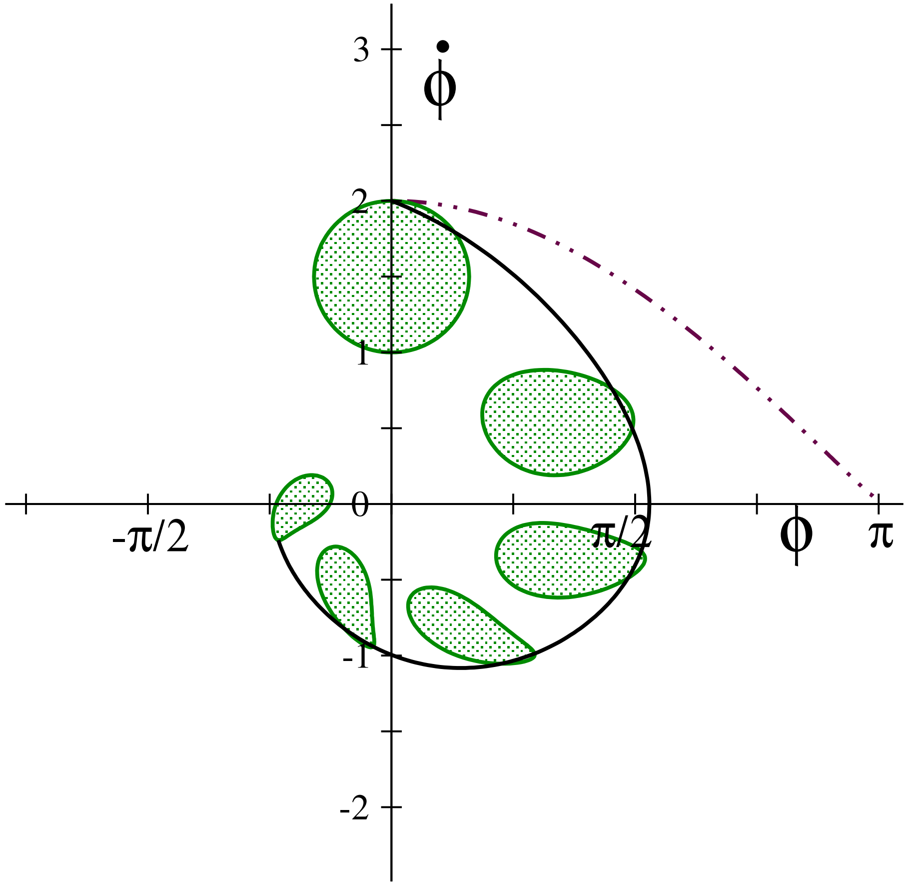
dynamical systems and physics

classical mechanics / the brain

$$\dot{x}_i = f_i(x_1, .., x_N | \gamma_1, .., \gamma_M)$$

- dynamical variables: x_i
  parameters: y_j

- Newton: $\dot{x} = v$
  $\dot{v} = F/m$



tranjectories in phase space: $(\phi, \dot{\phi})$
pendulum $\ddot{\phi} = -\gamma\dot{\phi} - \sin\phi$

overview

the brain as a complex dynamical system

- slow & fast dynamical variables
- diffusive emotional control

generating functional for dynamical networks

- polyhomeostasis / synaptic flux optimization
- transient state dynamics

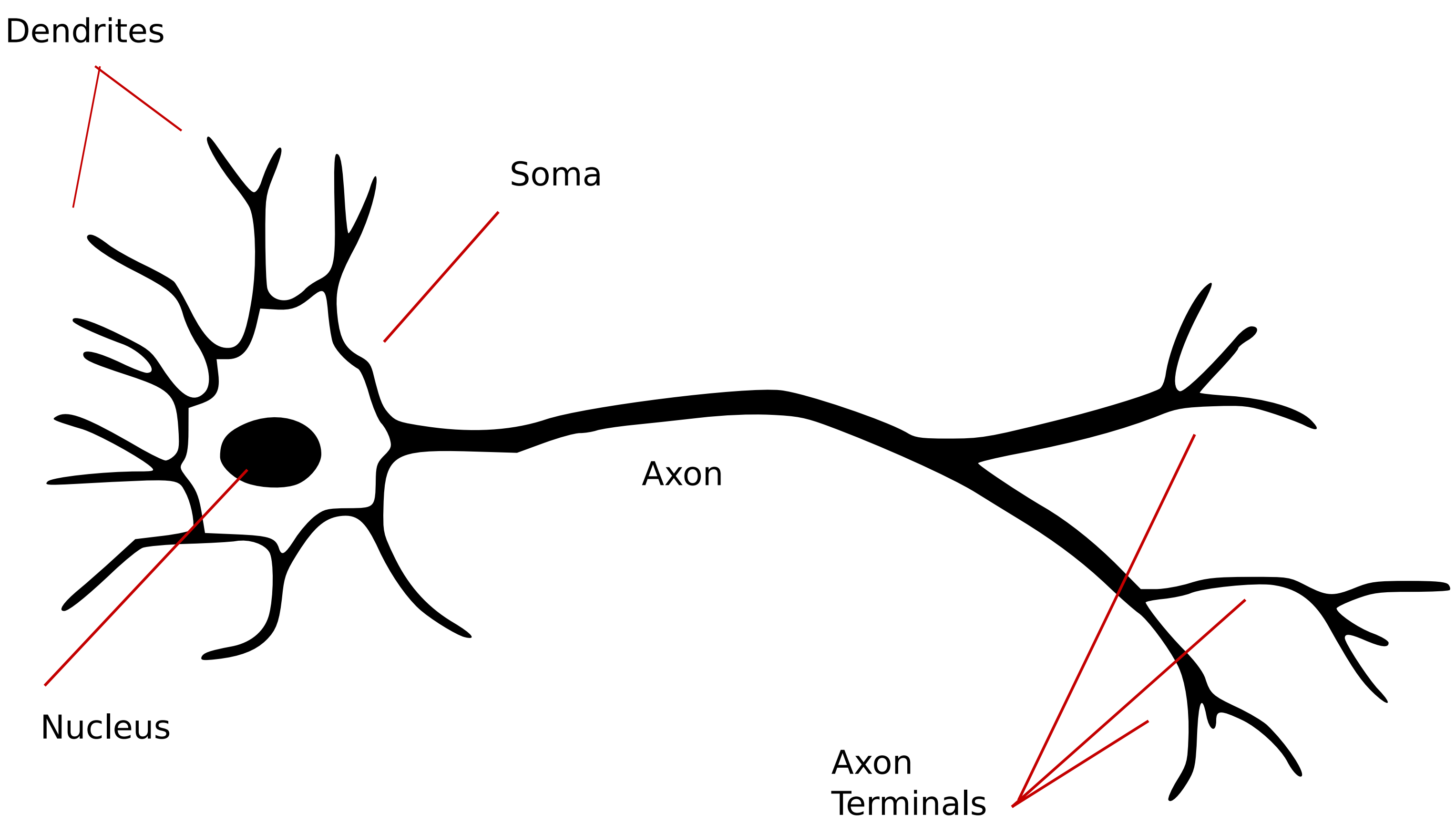
perspective

- complexity barriers in the sciences

100 billion neurons

$\approx 10^4$ inputs (synapses) per neuron

input (dentrites) ▷ output (axon)

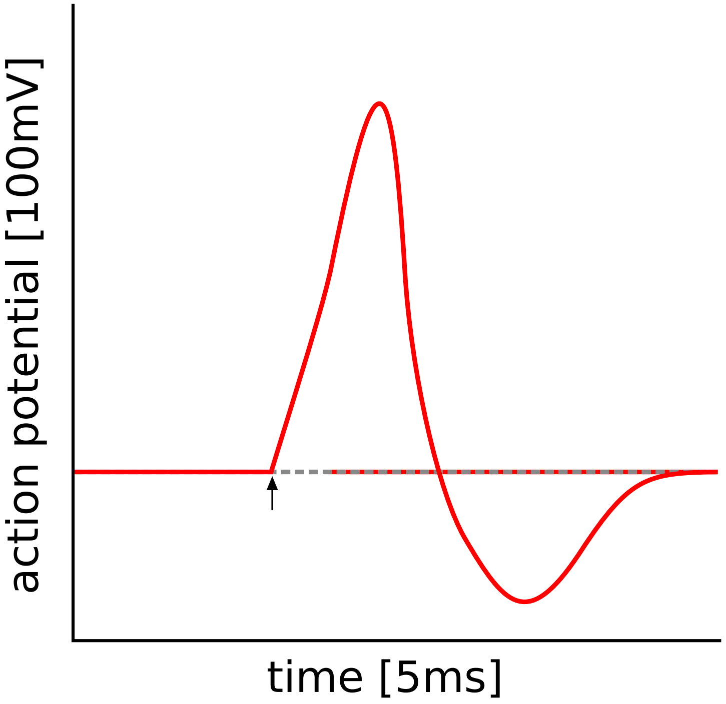


output signal: soliton along axon (spike)

[Wikipedia]

neural information processing

- neural spike
- neural firing rate $y_i(t)$
  number of spikes per time



states of the mind

- perception / thoughts / actions
- consciousness
- ...

» emerging from the interaction of billions of neurons «

neural constraints

energy : $(20-40)\%$ of body

space : dense packing

- ▷ grey matter (neurons)
- ▷ white matter (axons)

resources : proteins, ions

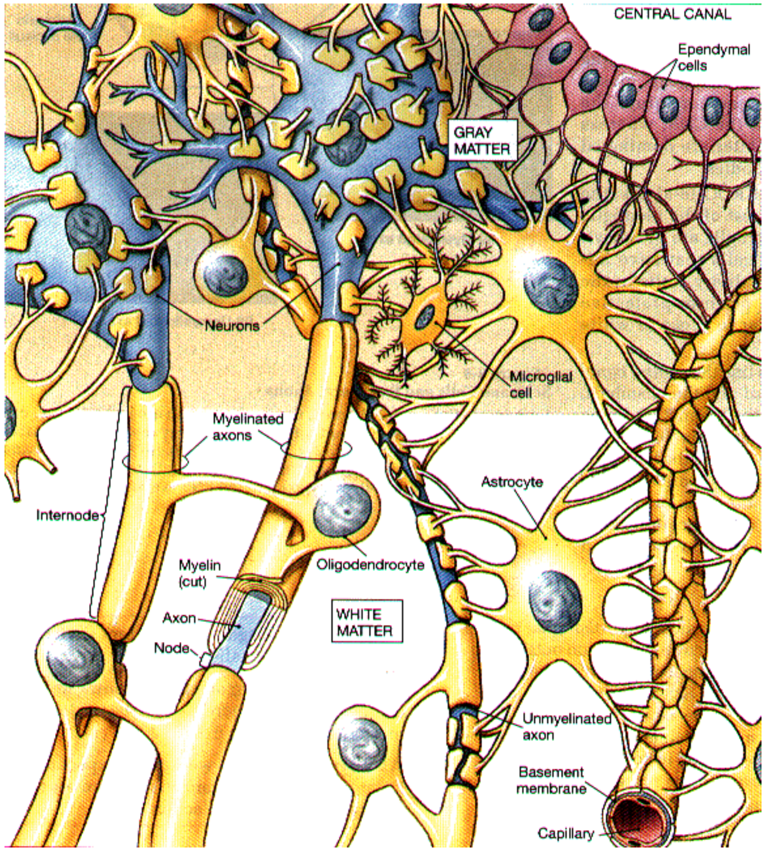
- ▷ transport from soma

connectivity : space/resource limited

- ▷ high turnover $(50-80)\%$ (daily)

computing power : neurons are slow

- ▷ massively parallel
- ▷ decisons within 100 cycles



multitude of time scales

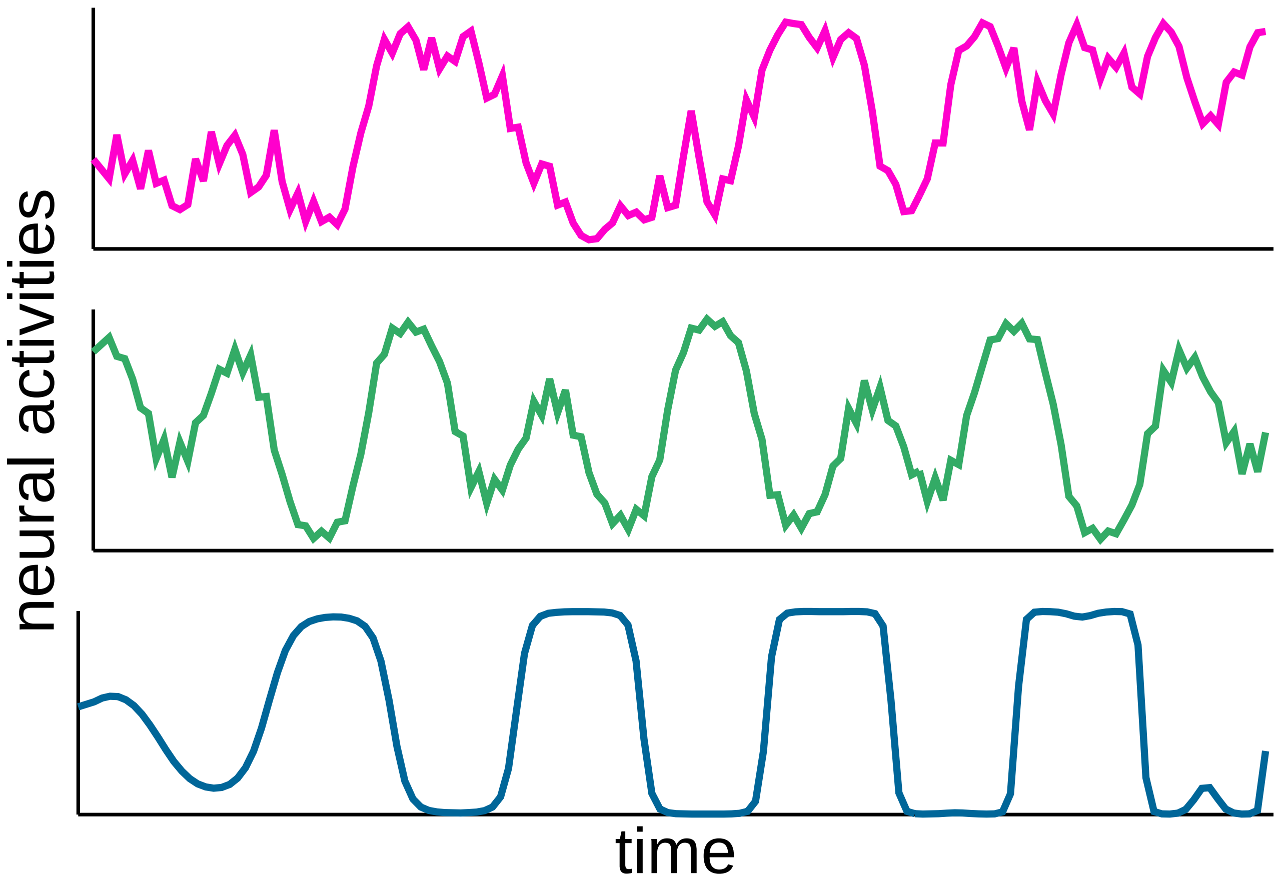
neural time scales

individual spikes	:	$(2-5)\,\mathrm{ms}$
firing rate	:	$(1-100)\,\mathrm{Hz}$
synaptic plasticity	:	ms - days
neuromodulation	:	100 ms - min

neural memory

short/long-term memory	:	modification of of synaptic strengths
working memory	:	transiently stable neural firing patters (attractors)
episodic memory	:	through the Hippocampus (subcortical)

neural working regimes

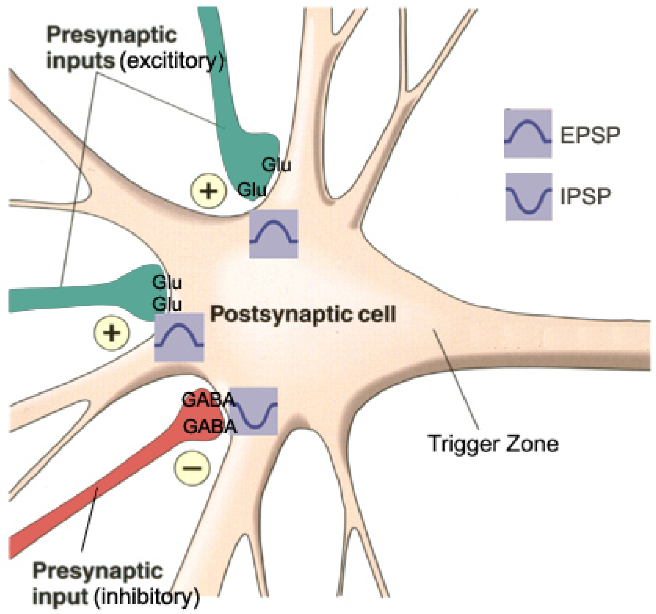


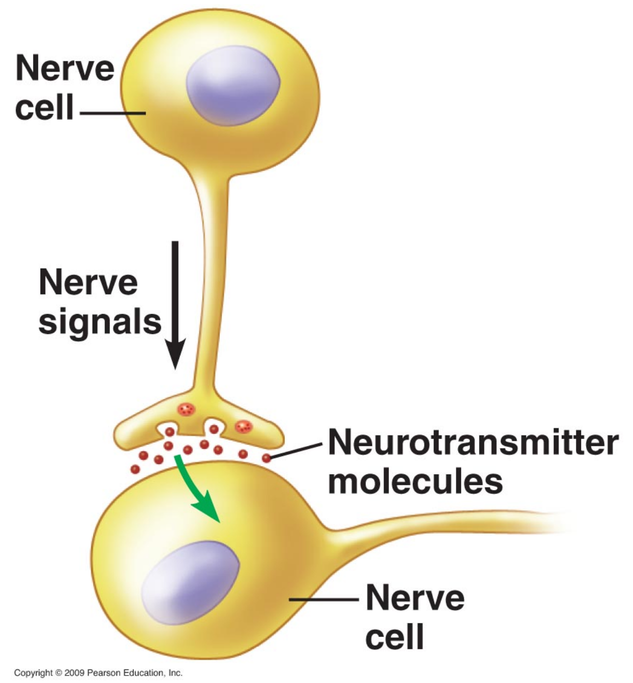
stabilization of working regimes

- ▷ slow adaption of parameter (metalearning)
- ▷ necessary for any complex dynamical system

(neuro-)transmitters and modulators

trans-synaptic information transmission is chemical





GABA: inhibitory

glutamate: excitatory

neuromodulator

modulating

synaptic plasticity

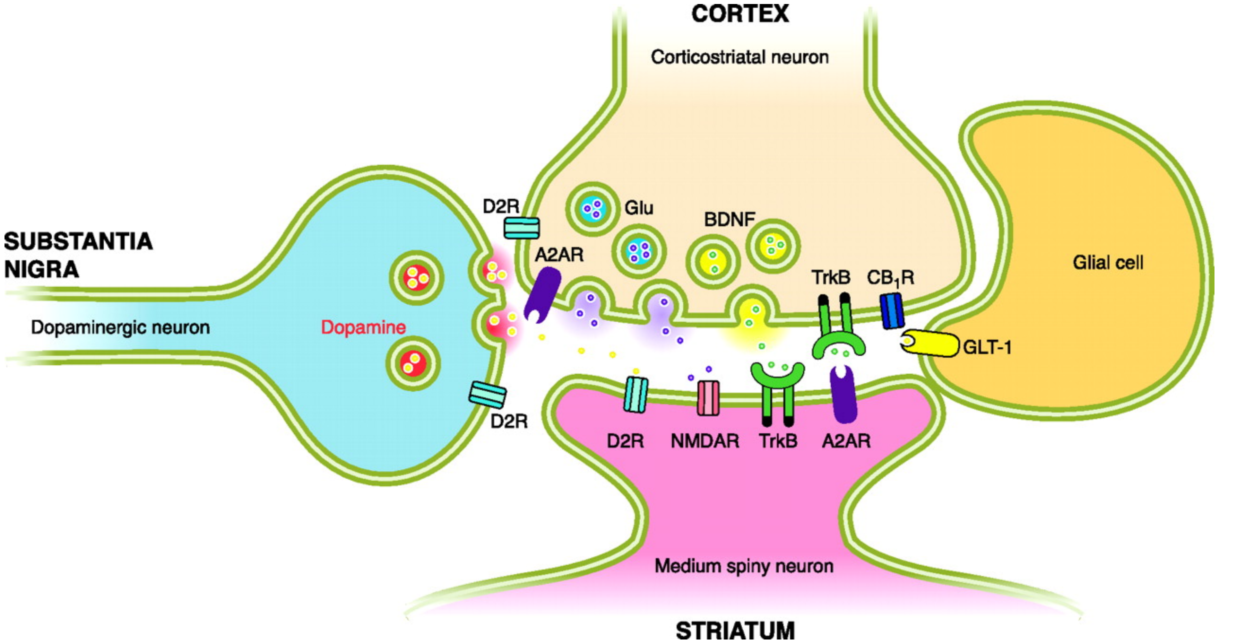
neural thresholds,
gains, . . .

⋆ norepinephrine

⋆ dopamine

⋆ serotonin

⋆ choline, oxytocin, . . .



[Physiological Reviews]

no direct cognitive
information processing - diffusive control

diffusive volume control

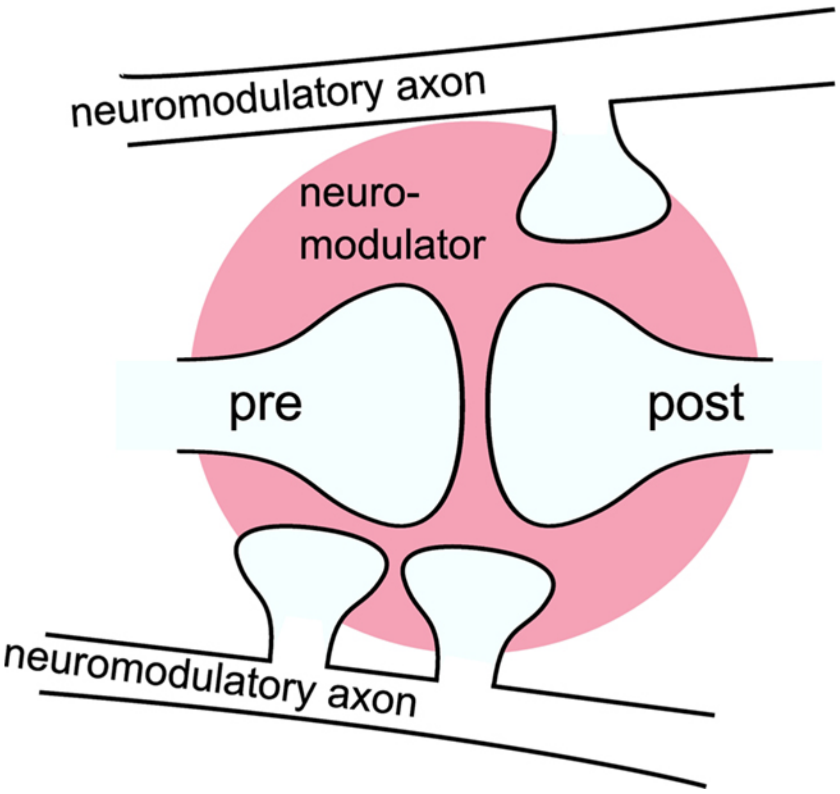
dopamine neurons

– activated by other neurons
  ‘cognitively’

– have vast projections
  ‘200.000 synapses’

– no individual target neurons
  ‘volume control’

– encoding reward, surprise, . . .

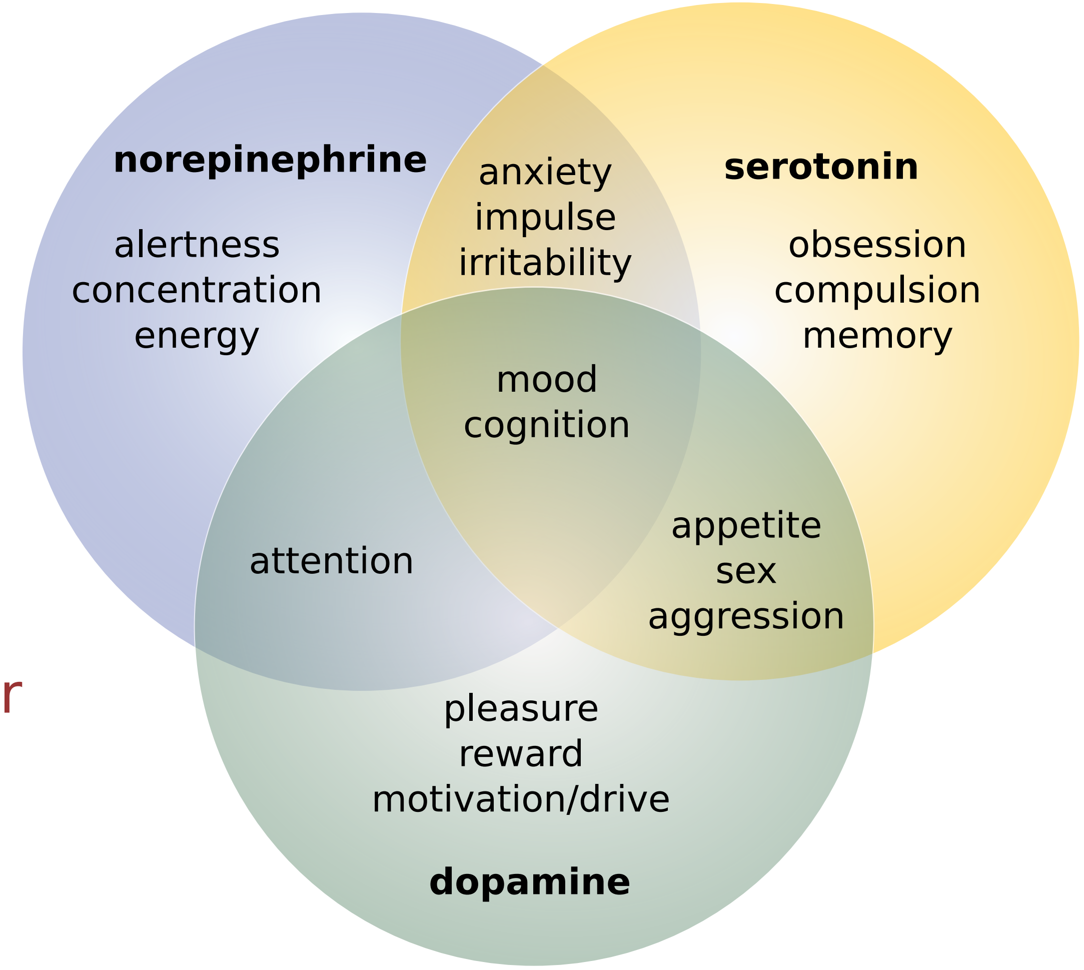


[Frontiers in Computational Neuroscience]

neuromodulation $\hat{=}$ diffusive control / signaling

▷ working regime stablization
  selection

neuromodulator & emotions

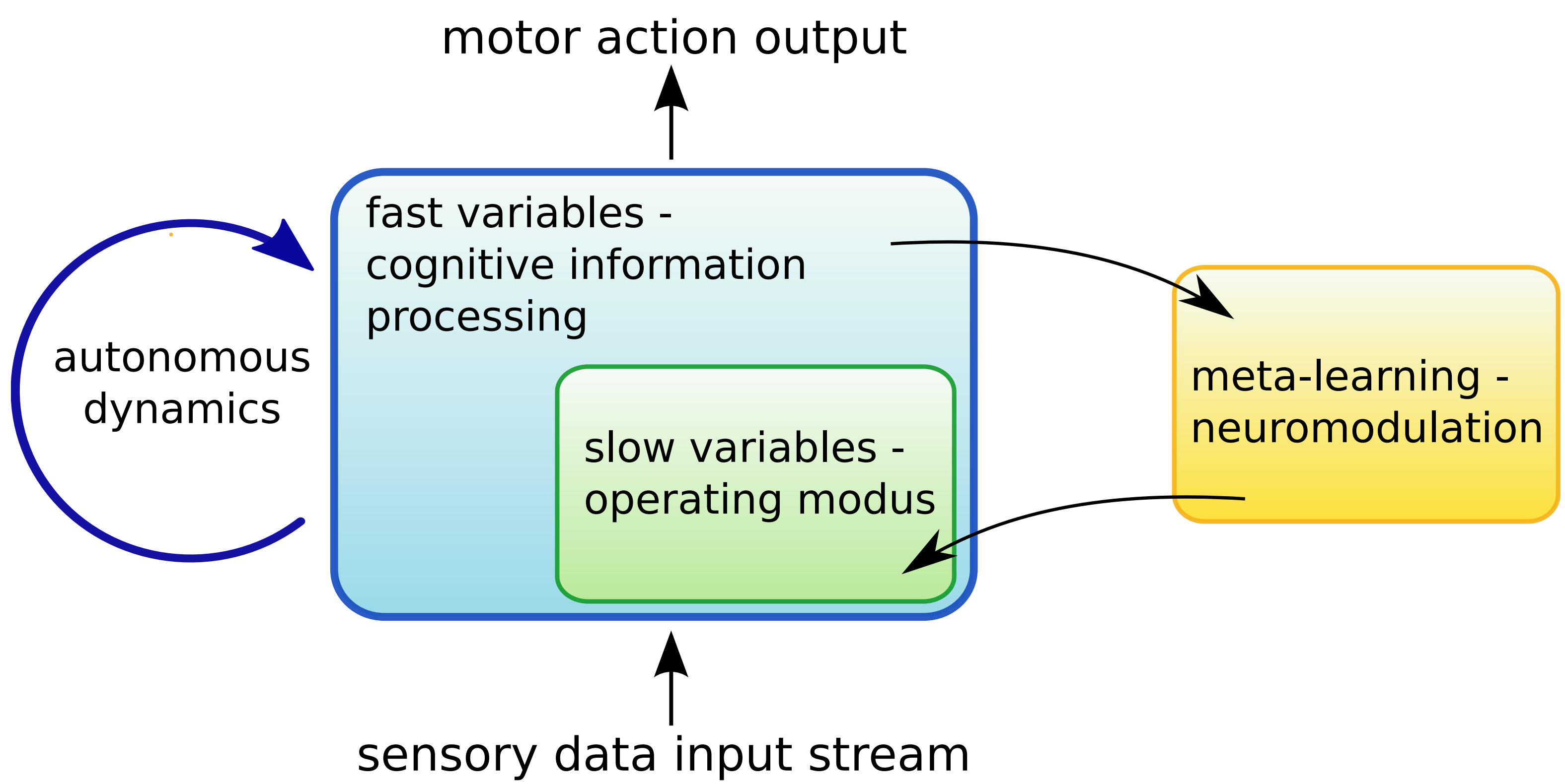


no emotion without
the concurrent release
of some neuromodulator

qualia of emotions proprioceptional?

» diffusive emotional control «

the brain as a dynamical system



modern view

- internal dynamics modulated (and not driven) by sensory input
- neuromodulation provides a very highy flexibility
  of working regimes in higher cortical areas

overview

the brain as a complex dynamical system

- slow & fast dynamical variables
- diffusive emotional control

generating functional for dynamical networks

- polyhomeostasis / synaptic flux optimization
- transient state dynamics

perspective

- complexity barriers in the sciences

the control problem

model building for complex systems

* potentially large numbers of control parameters
* high dimensional phase space

physical / biological insights

higher-level principle generating functional

equations of motion
$\dot{x}_i = \ldots$

generating equations of motion

principles

↓

generating functionals

↓

equations of motion

objective function / generating functional F

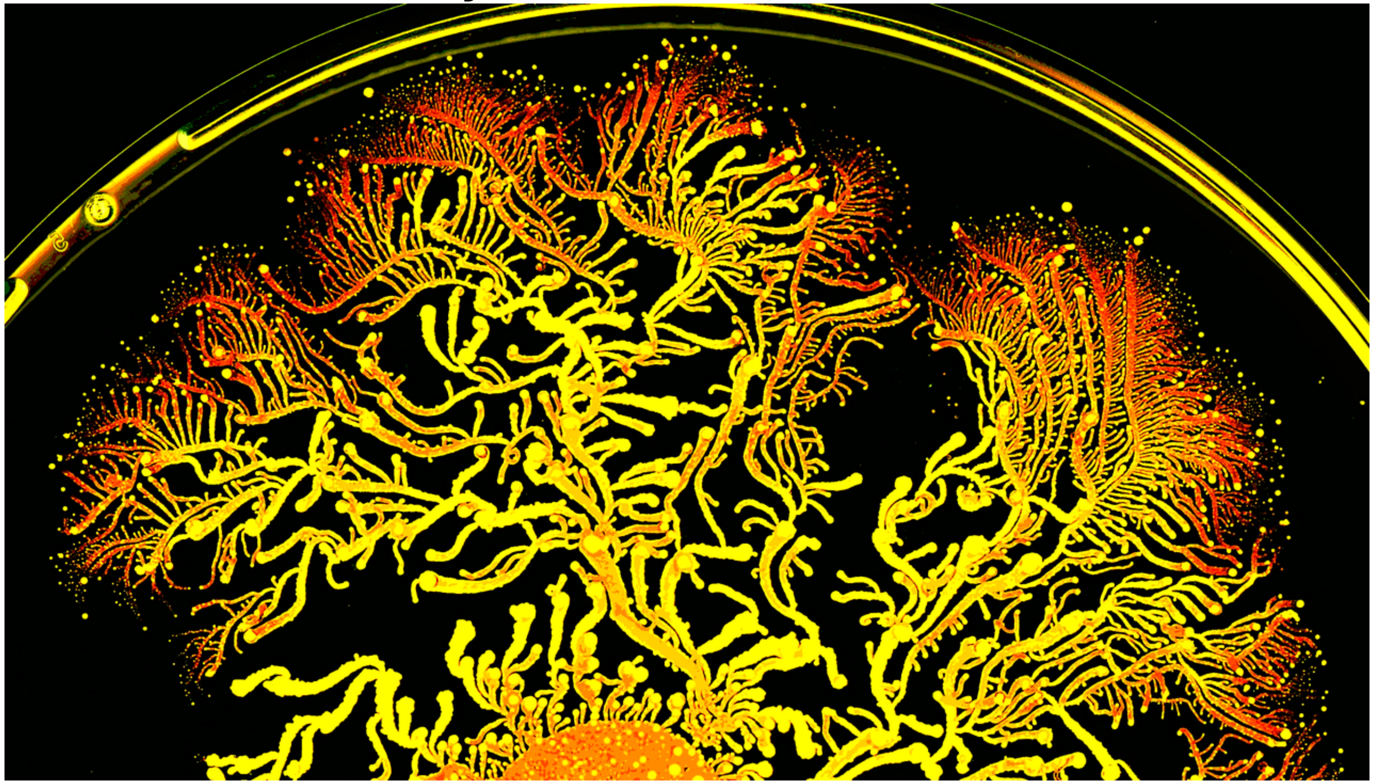
$$\dot{x} \sim -\frac{\partial F}{\partial x} \qquad \dot{\gamma} \sim -\frac{\partial F}{\partial \gamma}$$

* dimensional reduction of control problem

self-organization vs. control

classical self-organization

colony of Paenibacillus vortex bacteria



[Wikipedia]

guided self-organization

- ▷ guiding / controlling self-organizing processes
- ▷ here: using generating functionals

polyhomeostatic optimization

homeostasis

» keep in balance «

- a single scalar quantity

...
blood-sugar level
hormonal levels
body temperature
...
airplane velocity
furnace temperature
....

polyhomeostasis

- multiple scalar quantities

» keep in relative balance «

allocation & polyhomeostasis

time allocation

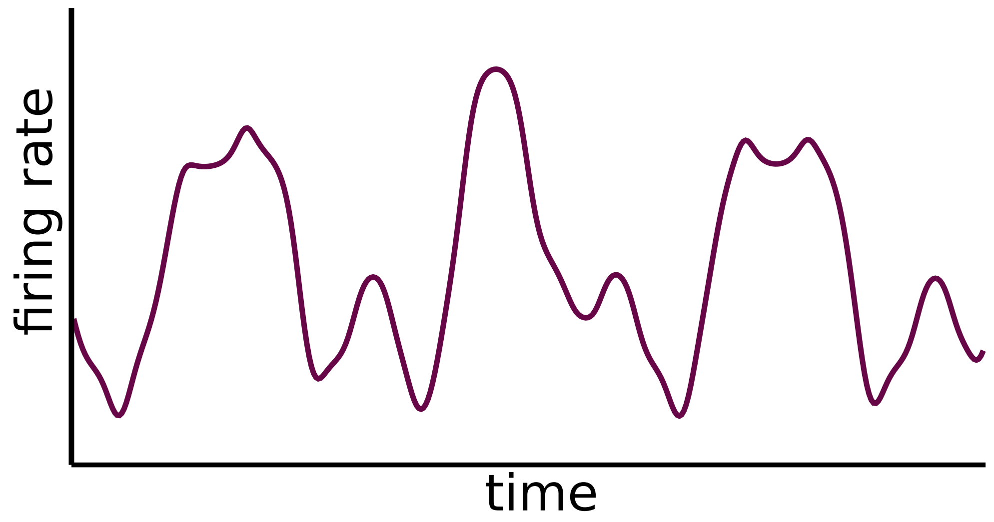
- individual target distribution functions
  - ▹ e.g. 60% working
    20% socializing
    20% eating / drinking

- dynamical process
  - ≙ time allocation
  - ≙ optimization of target distribution function

time allocation of neural activity

neural firing rate

- achieve maximal information content transmission



- firing-rate distribution $p(y) = \frac{1}{T}\int_0^T \delta(y - y(t-\tau))\, d\tau$

Shannon (information) entropy

$$H[p] \;=\; -\int dy\, p(y) \log p(y) \;\geq\; 0$$

maximal information distribution

maximal Shannon entropy $H[p]$

no constraints $\rightarrow$ $p(y) \sim \text{const.}$

given mean $\rightarrow$ $p_\mu(y) \sim \exp(-y/\mu)$, $\qquad \mu = \int y\, p(y)\, dy$

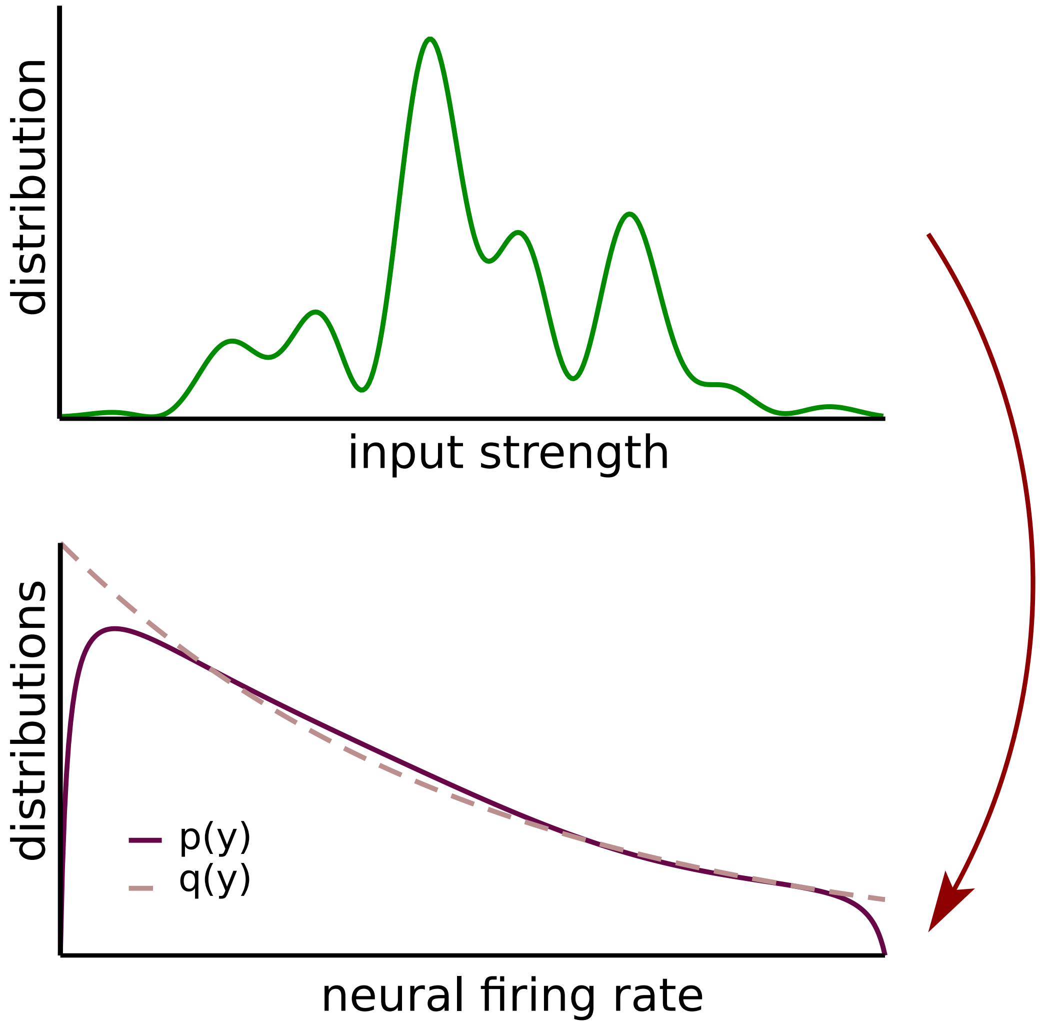
- target firing-rate distribution $\boxed{p_\mu(y)}$ (polyhomeostasis)

Kullback-Leibler divergence

$$D(p, p_\mu) \;=\; \int p(y) \log\left(\frac{p(y)}{p_\mu(y)}\right) dy \;\geq\; 0$$

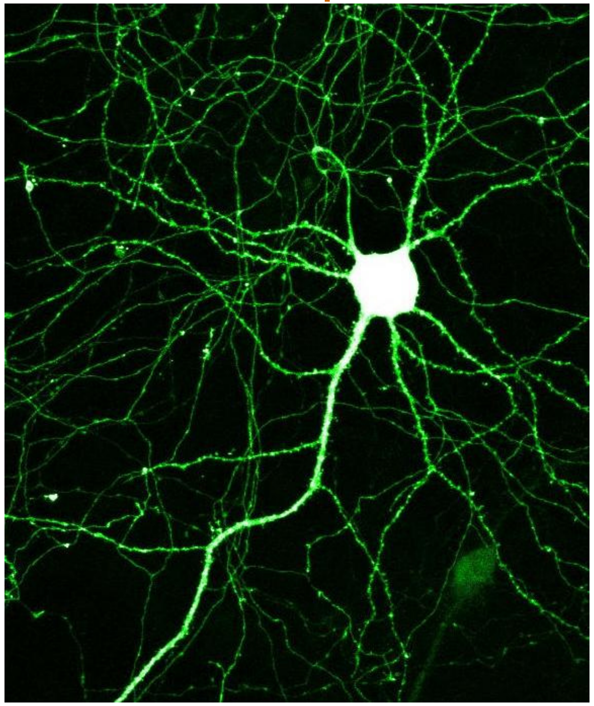
- asymmetric measure for the distance of two probability distribution functions

intrinsic plasticity



adaption of internal
neural parameters

input



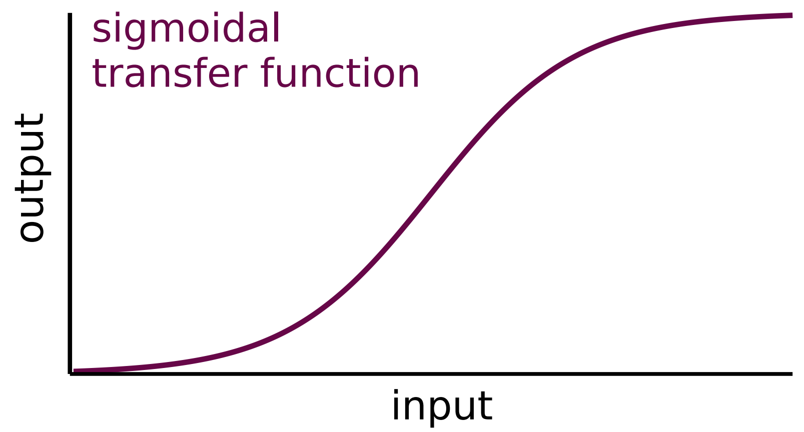
output

via non-linear neural
transfer function

intrinsic parameters

minimization of Kullback-Leibler divergence

$$D_{a,b}(p, p_\mu) = \int p(y) \log\left(\frac{p(y)}{p_\mu(y)}\right) dy \qquad y(x) = \frac{1}{e^{-a(x-b)} + 1}$$



- intrinsic parameters
  - ▷ gain a
  - ▷ threshold b
- mimimisation of D with respect to a, b

stochastic adaption rules

functional dependence on input statistics

- distributions of input / output $p(x)$ / $p(y)$

$$D \;=\; \int p(y)\log\left(\frac{p(y)}{p_\mu(y)}\right)dy \;\equiv\; \int p(x)\,d(x)\,dx$$

with

$$p(y)\,dy = p(x)\,dx, \qquad d(x) \;\equiv\; \log(p)-\log(\partial y/\partial x)-\log(p_\mu)$$

adaption rules for all input statistics

$$\left[\ \delta D \;=\; 0, \quad \forall p(x)\ \right] \;\Longrightarrow\; \delta d \;=\; 0$$

stochastic adaption rules

instantaneous adaption

$$\frac{d}{dt}a = -\epsilon_a \frac{\partial d(x)}{\partial a}$$

- average over time $\hat{=}$ average over $p(x)$
- adaption rate ϵ_a

stochastic adaption rules

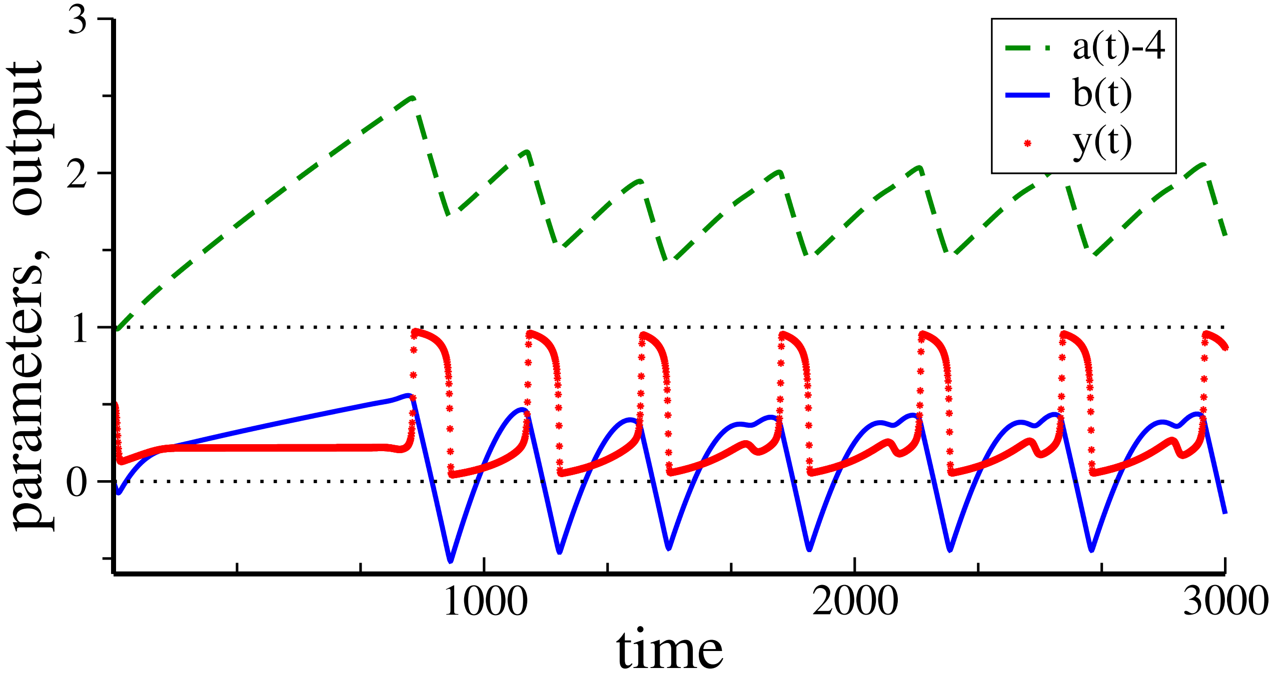
$$\frac{da}{dt} \propto (1-2y+y(1-y)/\mu)(x-b)+\frac{1}{a}$$

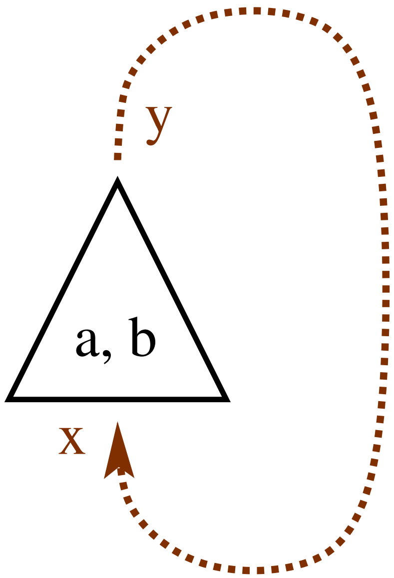
$$\frac{db}{dt} \propto (1-2y+y(1-y)/\mu)(-a)$$

[Triesch, '05]

autapse: self-coupled neuron

[Marković & Gros, PRL '10]





polyhomeostatic optimization induces
continuous, self-contained neural activity

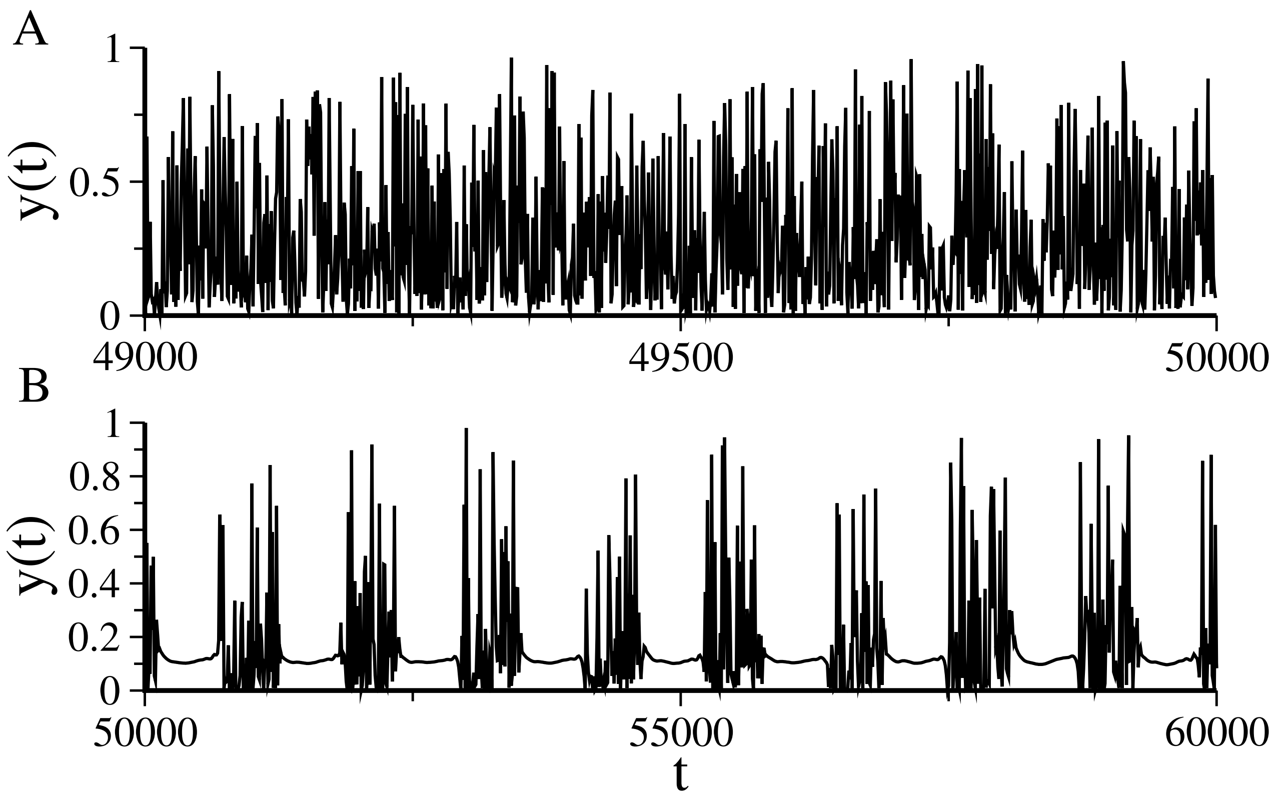
▷ limiting cycle

network of polyhomeostatic neurons

$x_i(t) \ = \ \sum_{j\neq i} w_{ij} y_j(t)$ $\qquad w_{ij} = \pm 1/\sqrt{N-1}$ randomly

$\mu = 0.28$

$\mu = 0.15$



$N = 100$

- self-organized chaos
- spontaneous intermittent bursting

} ‘guided self-organization’

transient state dynamics

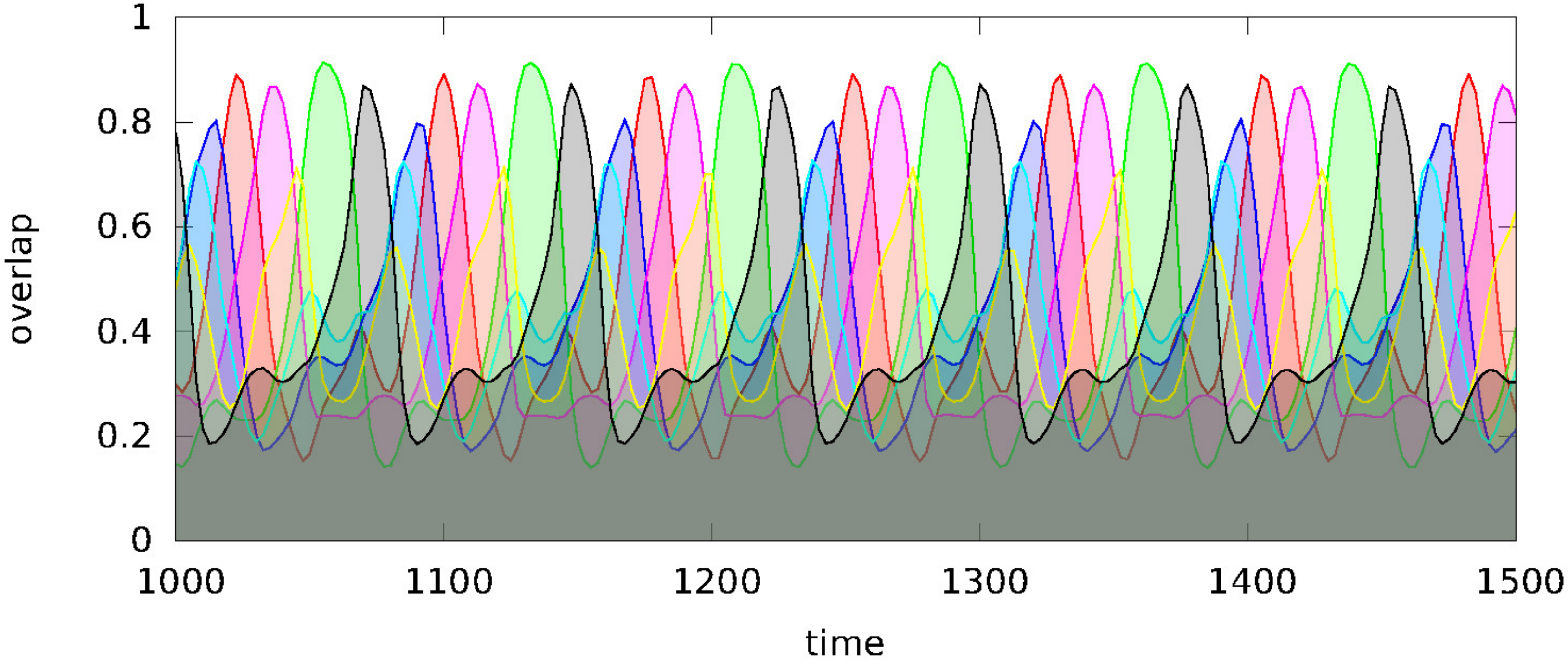
$$w_{ij} = \frac{1}{N_p} \sum_{\alpha} \xi_i^{(\alpha)} \xi_j^{(\alpha)}$$

for convenience

Hopfield patterns: $\xi_i^{(\alpha)}$

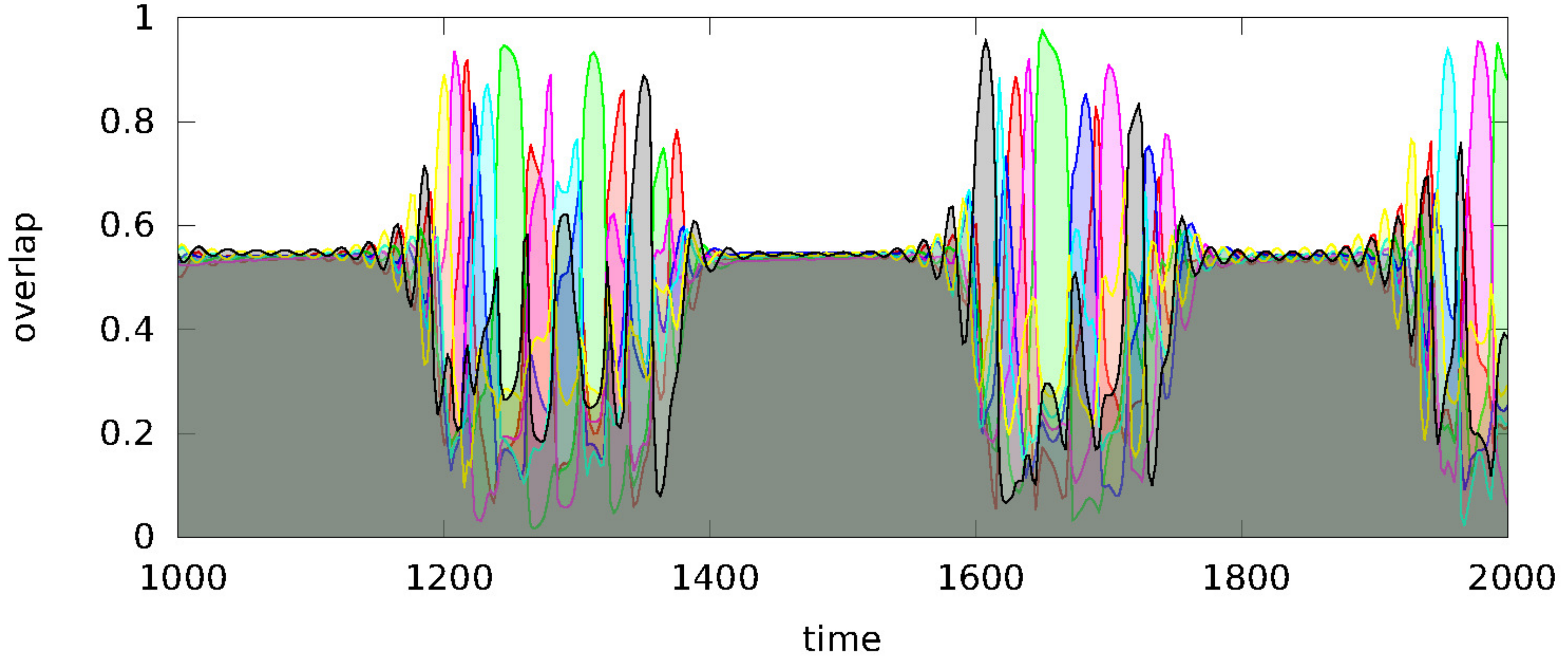
overlap firing $y_i - \xi_i^{(\alpha)}$ patterns

[Linkerhand & Gros, MMCS '13]



competing objective functions

bursting transient state dynamics



target activtiy $\mu = 0.15$

$\langle \xi_i^{(\alpha)} \rangle = 0.3$ mean activity of attractors

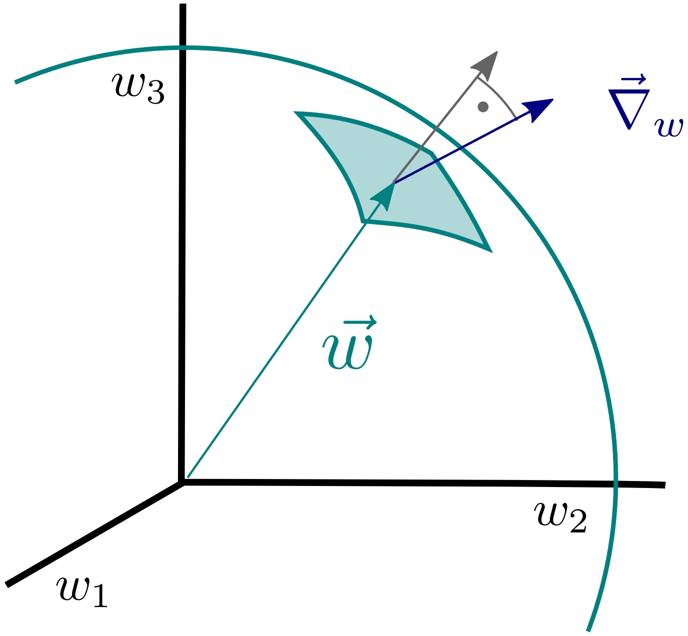
$\Rightarrow$ guiding self-organization

synaptic flux

$$y(x) = \frac{1}{e^{-a(x-b)}+1} \qquad x = \sum_j w_j\,(y_j - \bar{y}_j)$$

synaptic plasticity

$$\dot{w}_j = \ldots$$



- Hebbian learning

  $\dot{w}_j \propto y\,(y_j - \bar{y}_j)$

- avoid synaptic runaway growth

  $\sum_j (w_j)^2 \;\to\; \text{const.}$

minimisation of synpatic flux

Fisher information

$$F = \int dy\, p(y) \left(\frac{\partial}{\partial\theta} \log p_\theta(y) \right)^2$$

measures the sensibility of a probability distribution $p(y)$
with respect to a parameter θ

synaptic flux operator

$$\frac{\partial}{\partial\theta} = \sum_j w_j \frac{\partial}{\partial w_j}$$

- minimisation of Fisher information

$$\dot{w}_j = \epsilon_w G(x) H(x) \left(y_j - \bar{y}_j\right)$$

» self-limiting Hebbian learning rule «

synaptic flux optimization

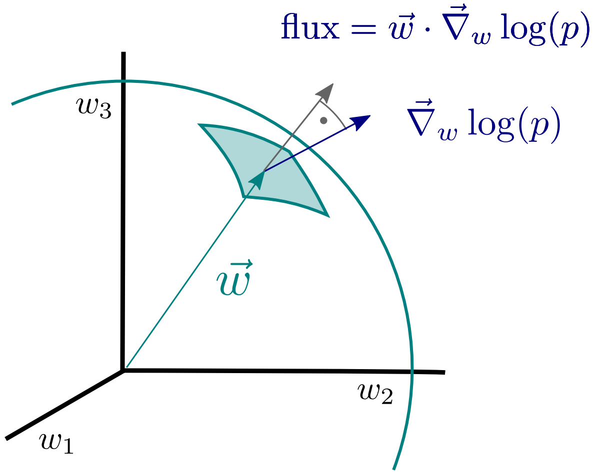
$$\dot{w}_j \;=\; \epsilon_w\, G(x)\, H(x)\, (y_j - \bar{y}_j)$$

pre-synaptic	Hebbian	$(y_j - \bar{y}_j)$
post-synaptic	Hebbian	$H(x) = (2y-1) + 2x(1-y)y$
post-synaptic	self-limiting	$G(x) = 2 + x(1-2y)$

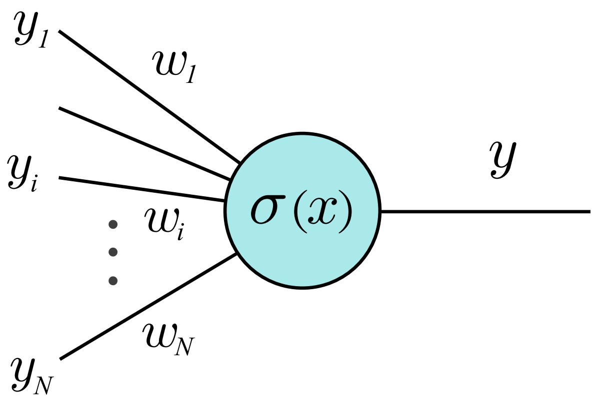
$$\dot{w}_j \;\propto\; \begin{cases} -(2+x)\,(y_j - \bar{y}_j) & (y=0) \\ (2-x)\,(y_j - \bar{y}_j) & (y=1) \end{cases}$$

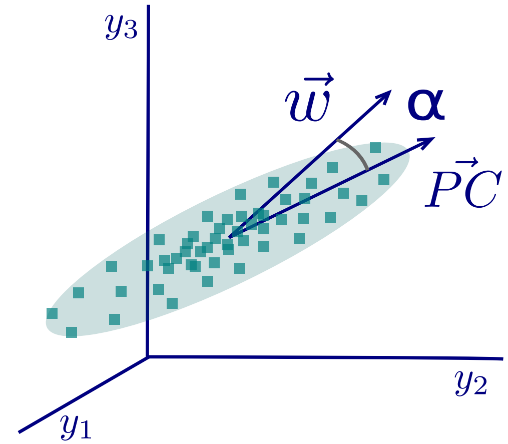
miminisation of
Fisher information for synaptic flux

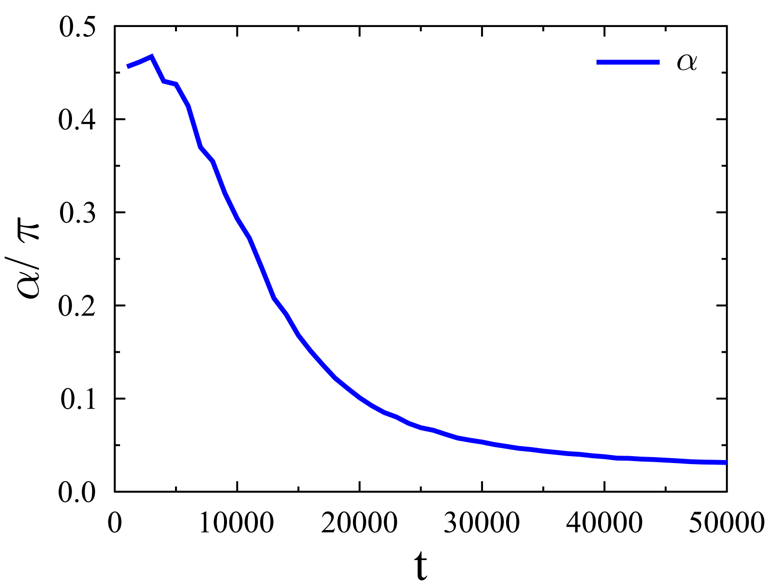
- ▹ Hebbian
- ▹ self-limiting

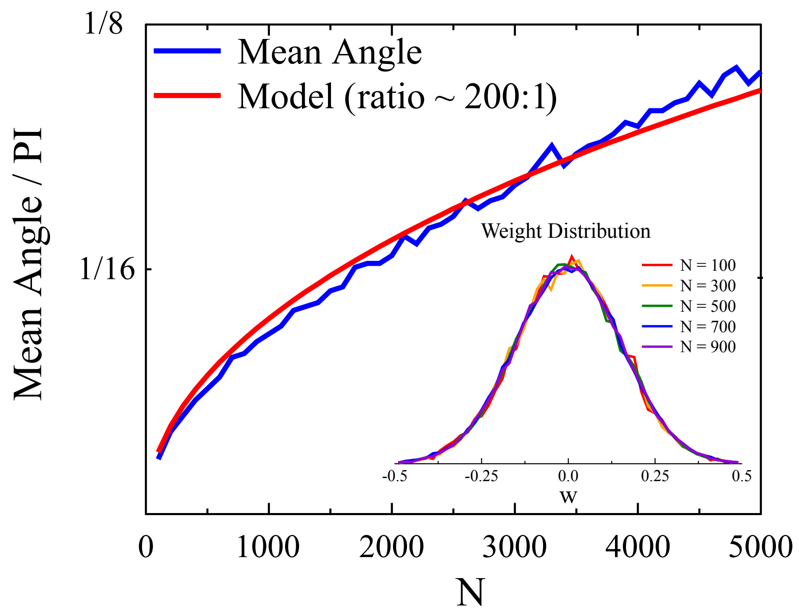


principal component analysis

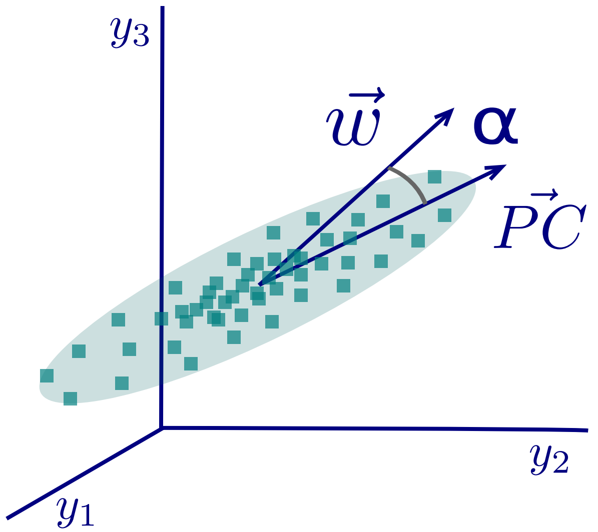


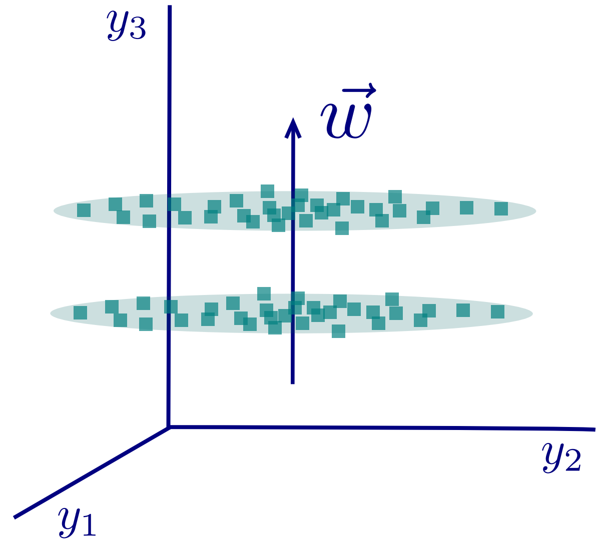


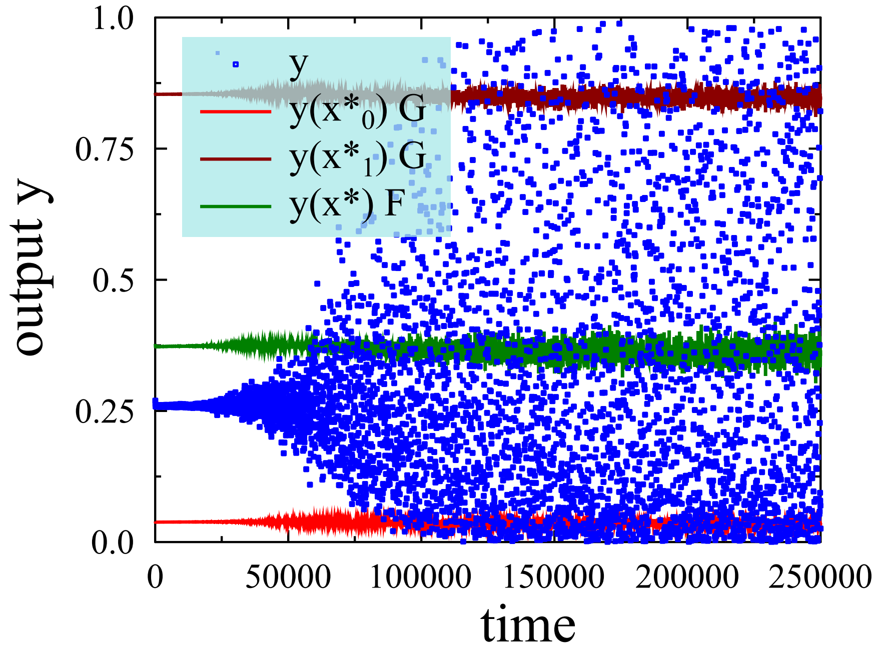


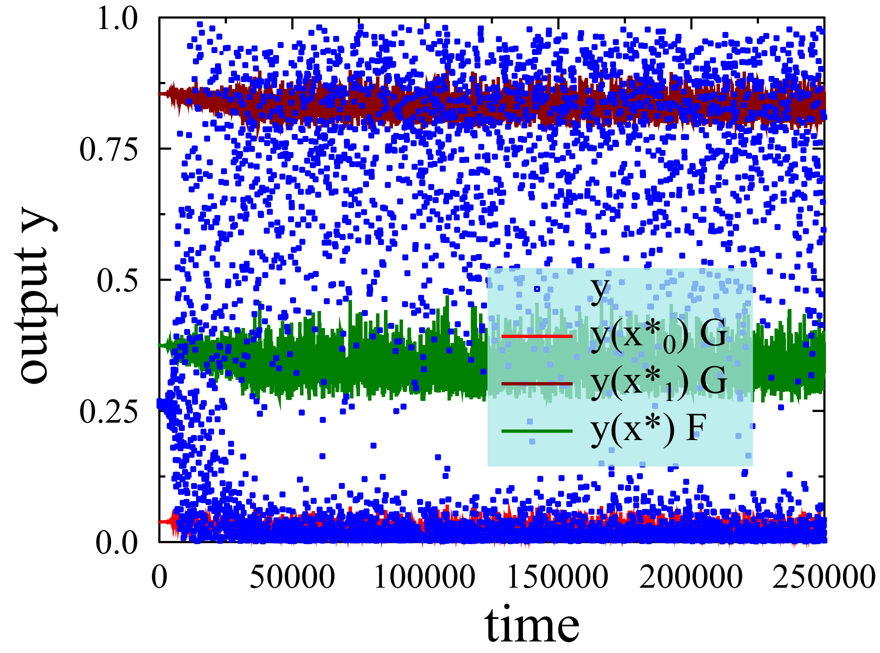


binary classification









overview

the brain as a complex dynamical system

- slow & fast dynamical variables
- diffusive emotional control

generating functional for dynamical networks

- polyhomeostasis / synaptic flux optimization
- transient state dynamics

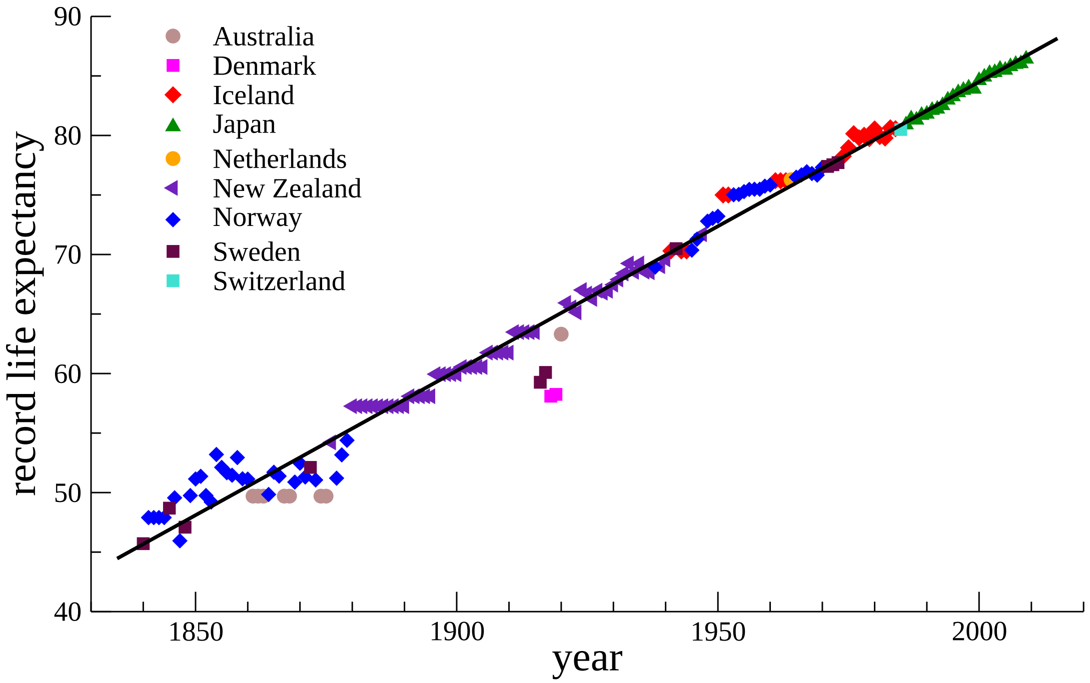
perspective

- complexity barriers in the sciences

record life expectancy

fast or slow?

- 2.4 years / decade
- revolutions:
  – hygiene
  – immunology
  – antibiotics
  – technological medicine
  – . . .



complexity barrier

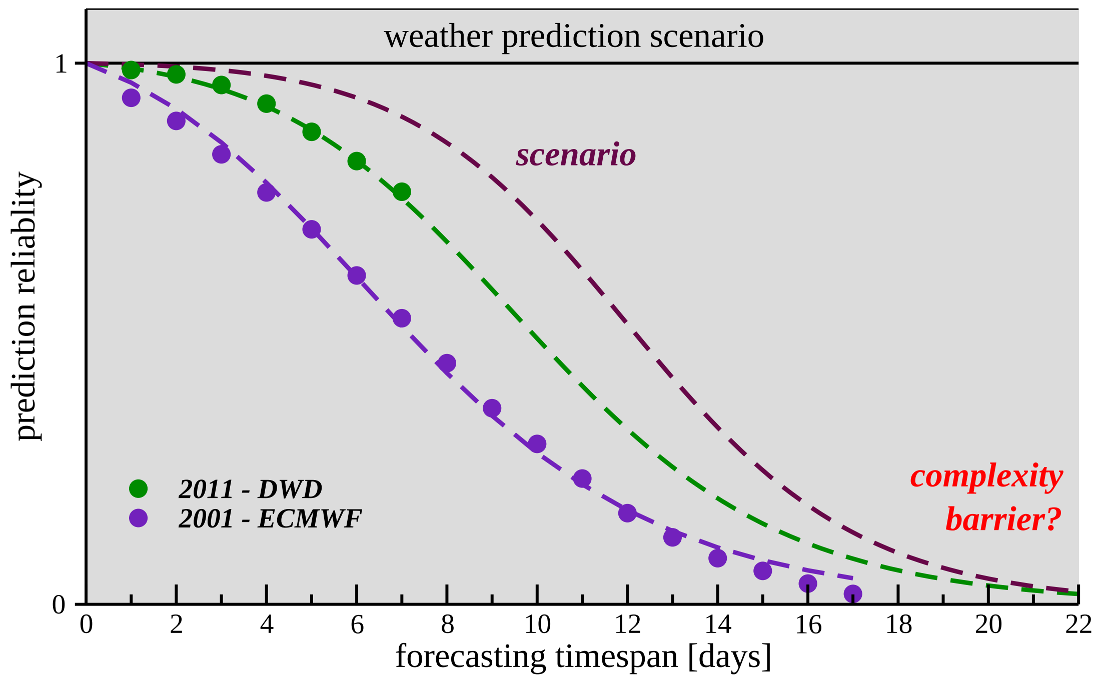
> devoting exponentially growing resources
> to medical research and healthcare . . .

. . . leads to a linear increase in life expectancy

weather forecast

computer resources
increase exponentiall

chaotic components



C. Gros, *Pushing the complexity barrier: diminishing returns in the sciences*
Complex Systems **21**, 183 (2012)

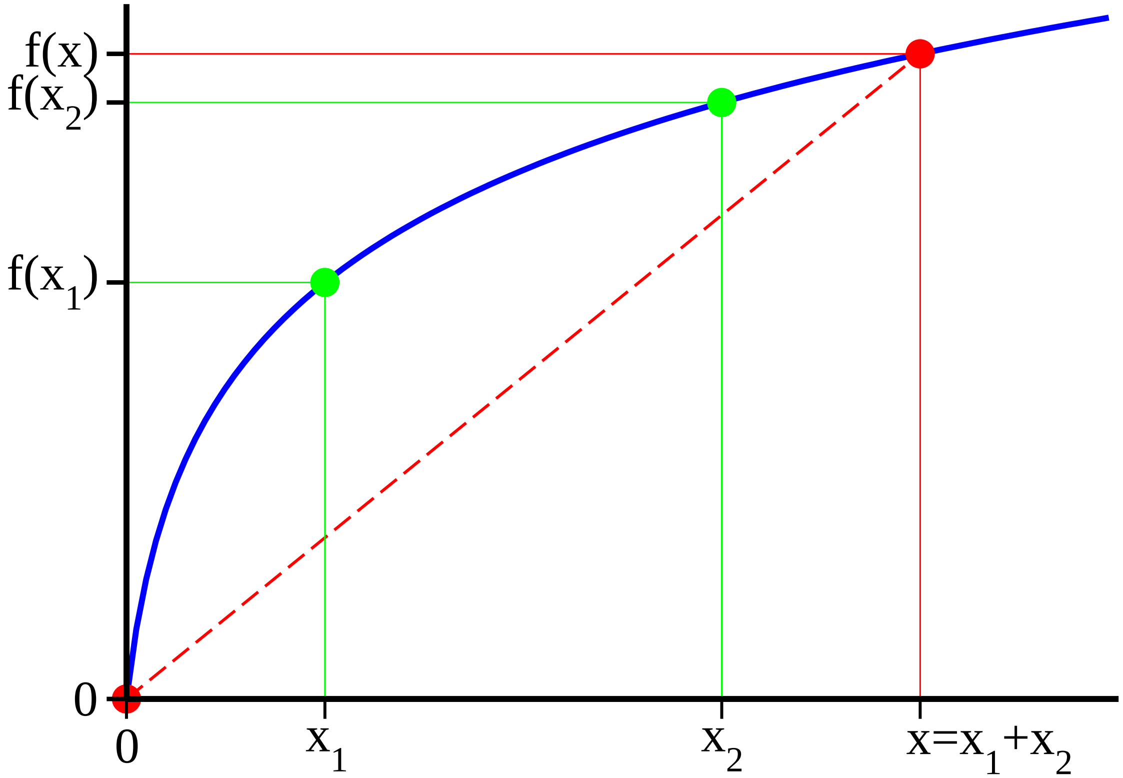
C. Gros, *Forschungsförderung quo vadis –*
Effizienz und Komplexitätsbarrieren in den Wissenschaften
Forschung & Lehre, April 2013

small is beautiful

concave functions

$$f(x_1) + f(x_2) > f(x)$$

$$\forall \quad x_1 + x_2 = x$$



science funding

» smaller science projects are generically more efficient «

(law of diminishing returns)

conclusions

the brain as a complex dynamical system

- slow & fast dynamical variables
- diffusive emotional control

generating functional for dynamical networks

- polyhomeostasis / synaptic flux optimization
- transient state dynamics

perspective

- complexity barriers in the sciences