University of Oldenburg June 20, 2019

Dynamics and control of multilayer networks

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SFB 910 Control of self-organizing nonlinear systems
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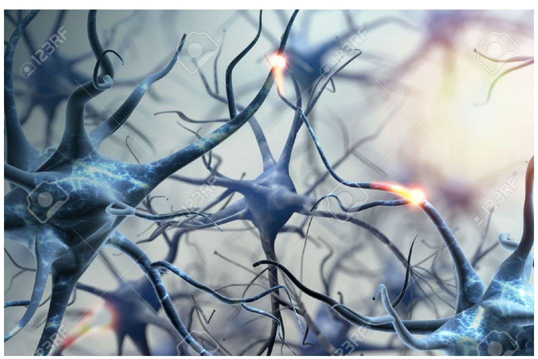
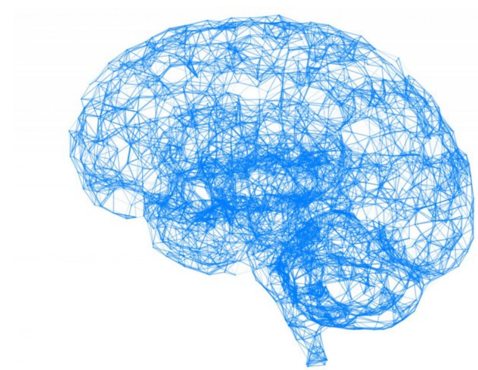
Multilayer networks

Why multilayer?

Better representation of real-world systems





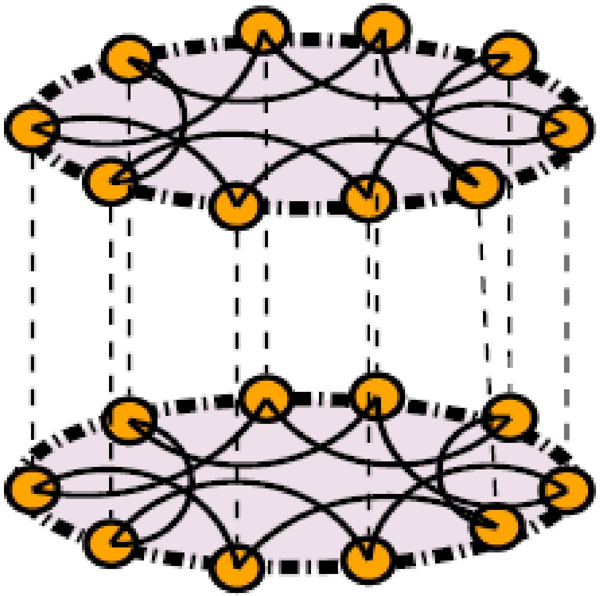
Reviews:

S. Boccaletti et al.,The structure and dynamics of multilayer networks, Physics Reports 544, 1 (2014)

M. Kivelä, A. Arenas et al., Multilayer networks, Journal of Complex Networks 2, 3, 203 (2014)

What is a multilayer network?

A set of nodes interacting in layers,
each reflecting a distinct type of interaction.

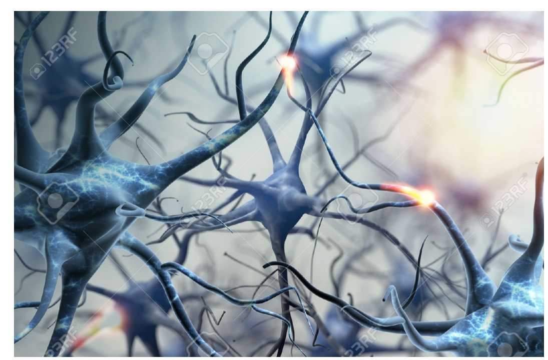


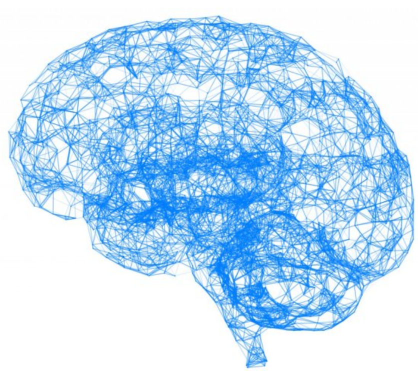
Examples

- **Social networks**: friendships in Facebook: family, friends, coworkers

- **Transportation networks**: air, train and bus transportation networks

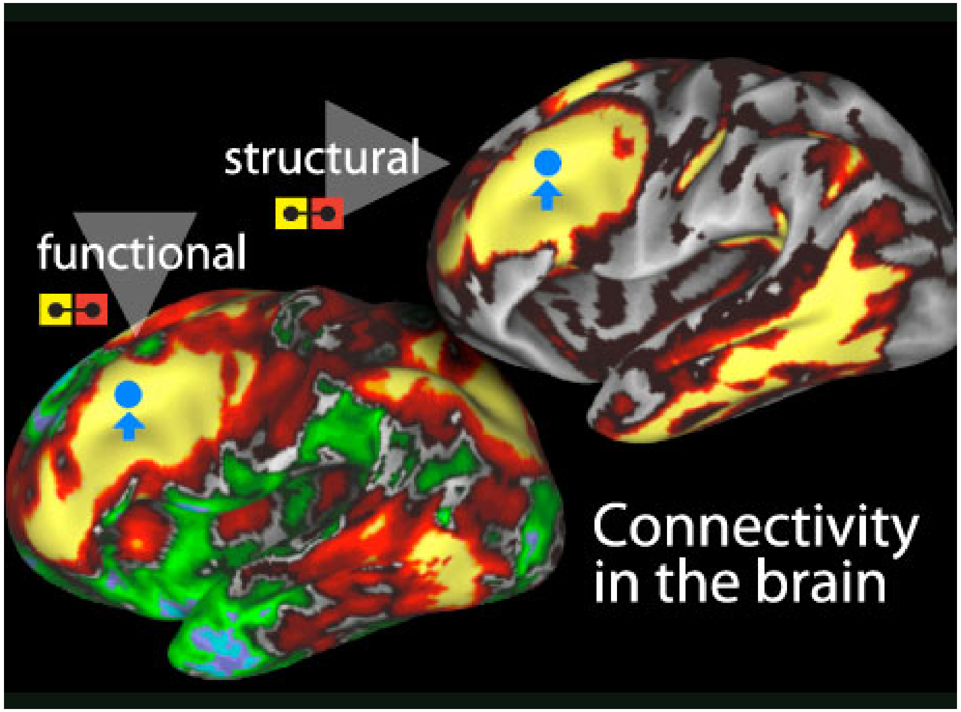
- **Neural networks**: chemical link or ionic channel

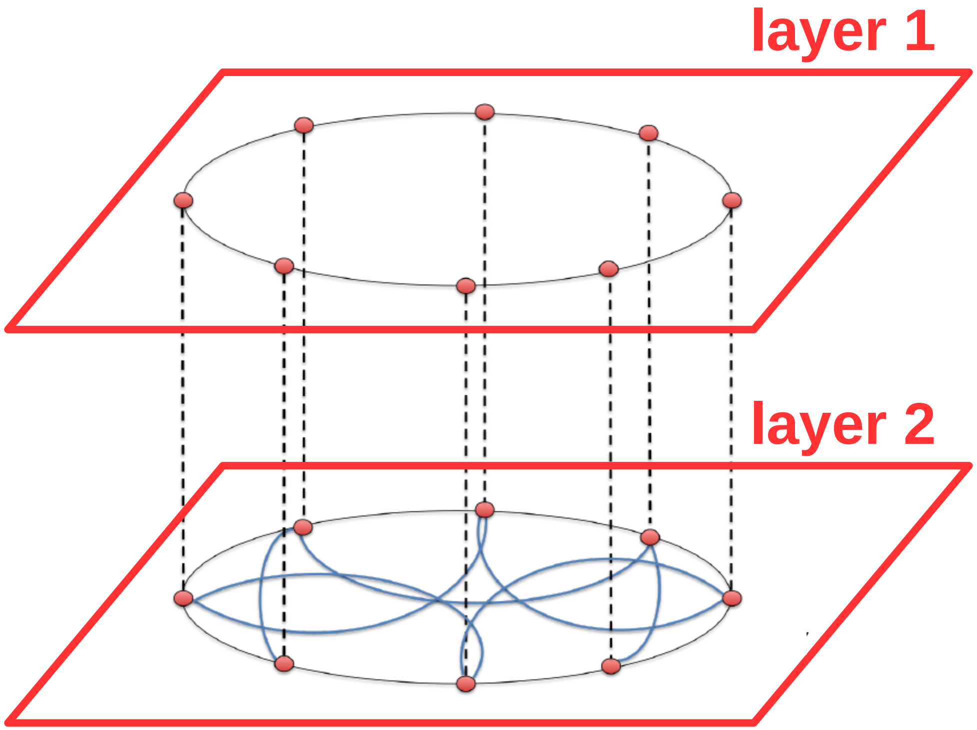




- **Brain networks**: different regions can be seen connected by functional and structural neural networks

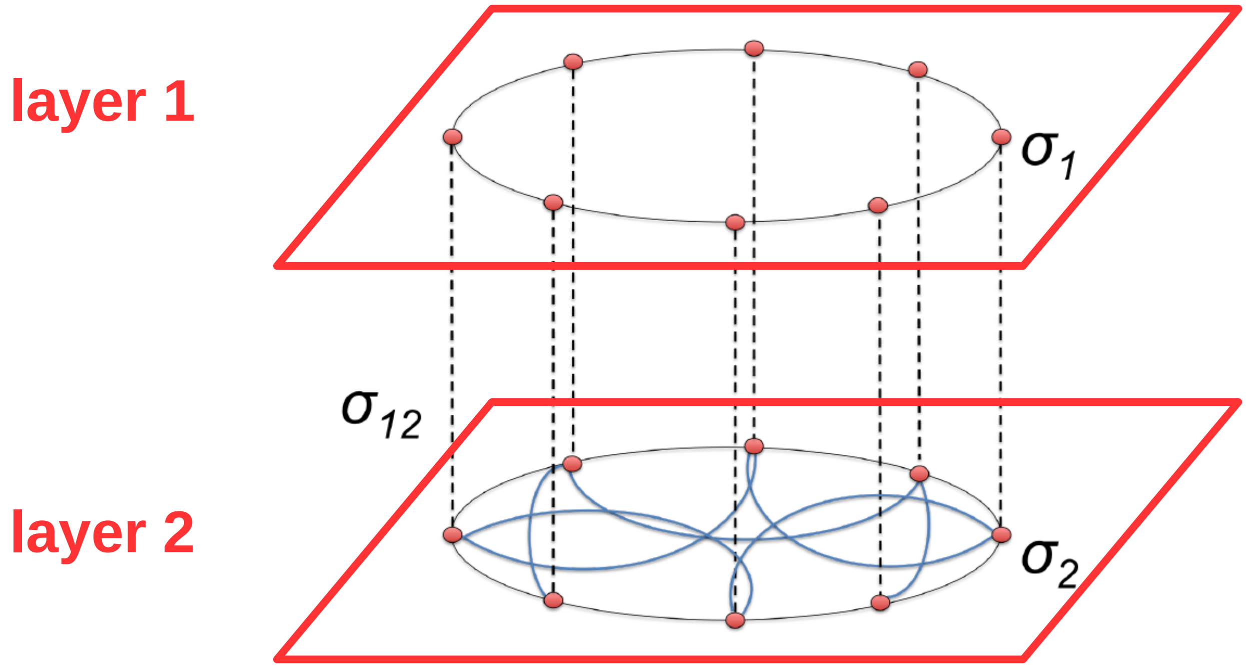
Multilayer modeling of brain networks

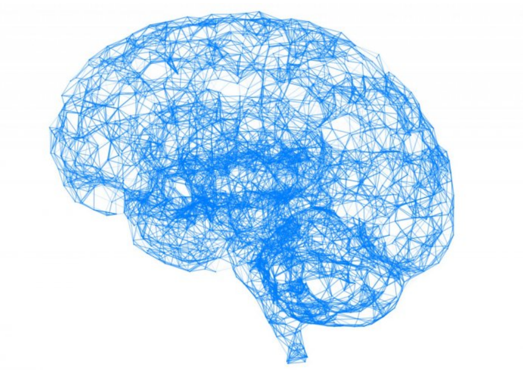




M. De Domenico, Multilayer modeling and analysis of human brain networks, Gigascience 6, 1 (2017)

Control by multiplexing

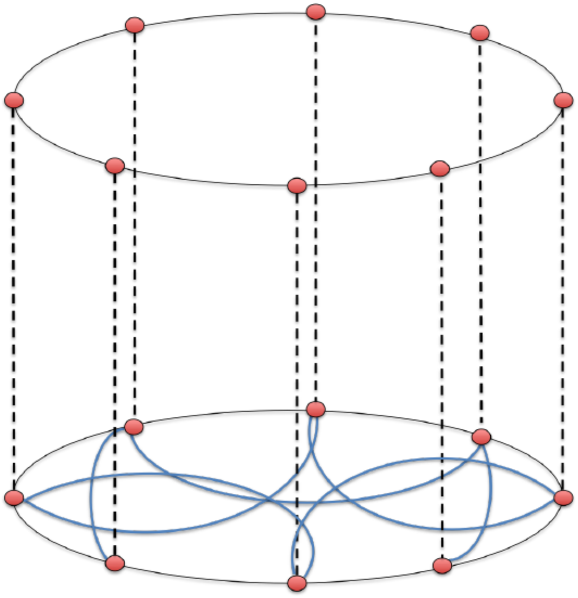




Controlling **one layer** **by manipulating the parameters of** **the other** **layer**

Strong and weak multiplexing

Multiplex network



weak multiplexing

$\sigma_{12} < \sigma_1, \sigma_{12} < \sigma_2$

strong multiplexing

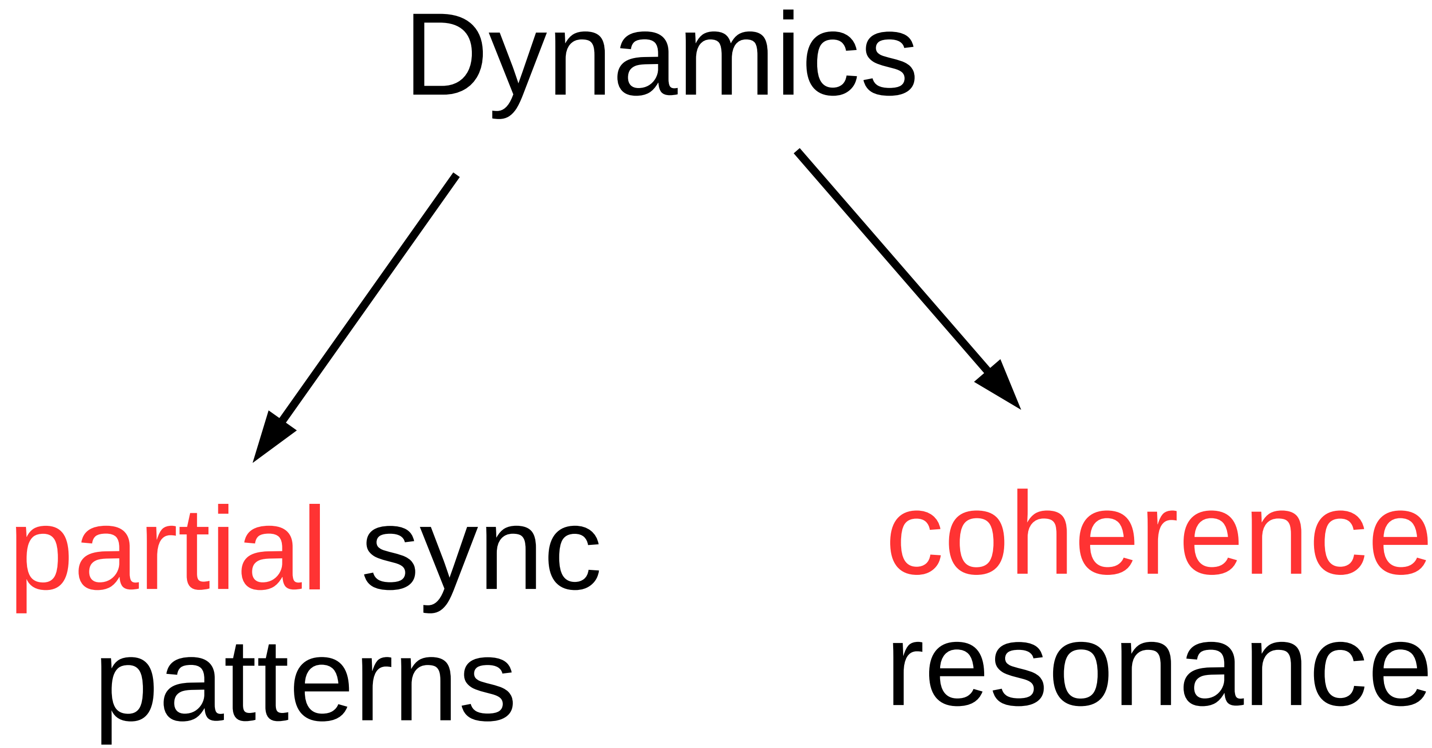
$\sigma_{12} \geq \sigma_1, \sigma_{12} \geq \sigma_2$

Strong multiplexing:

S. Ghosh, A. Kumar, A. Zakharova, S. Jalan, Birth and death of chimera: interplay of delay and multiplexing, EPL 115, 60005 (2016)

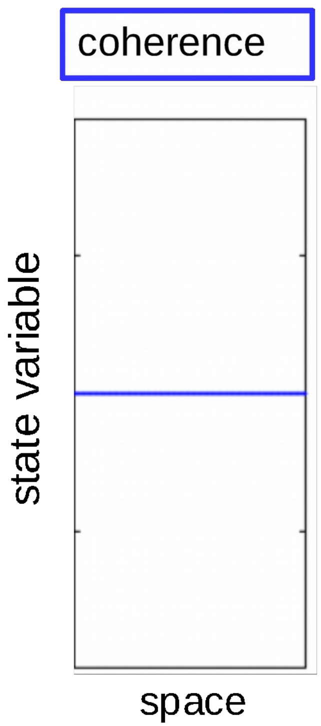
S. Ghosh, A. Zakharova, S. Jalan, Non-identical multiplexing promotes chimera states, Chaos, Solitons and Fractals 106, 56-60 (2018)

Can weak multiplexing have a strong impact on the dynamics?

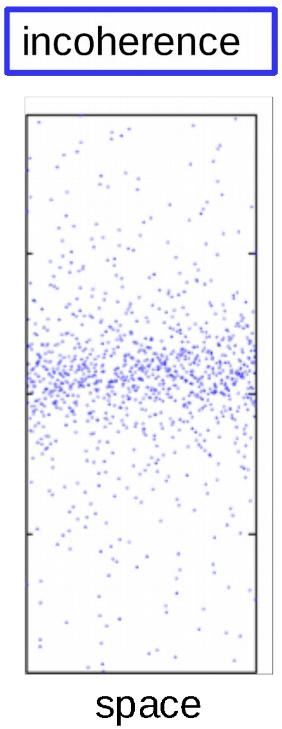

Dynamics
partial sync patterns
coherence resonance

Partial synchronization patterns

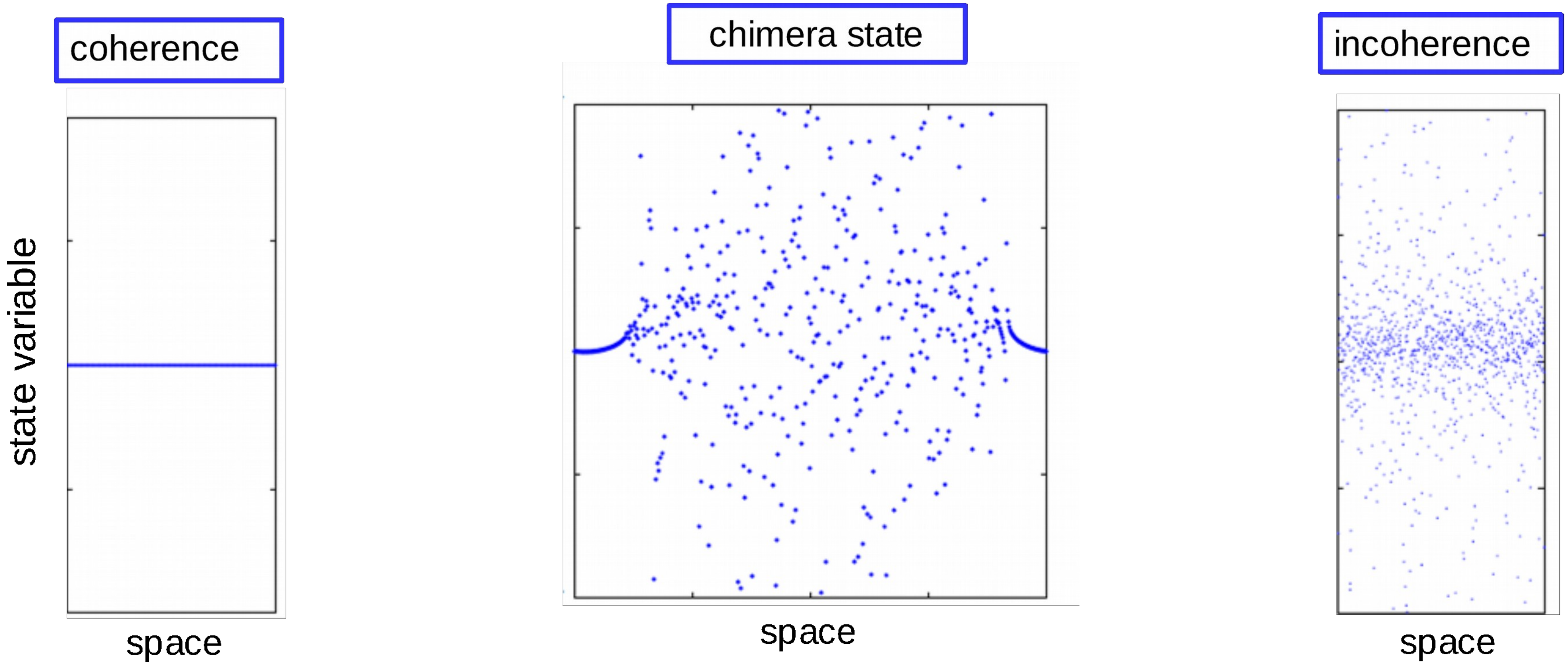
Transition from coherence/sync to incoherence/desync



?



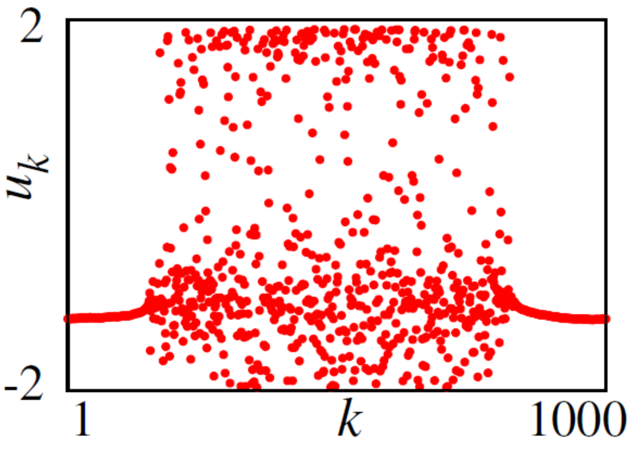
Transition from coherence/sync to incoherence/desync



Chimera state – spatial coexistence of coherent/synchronized and incoherent/desynchronized domains in a dynamical network

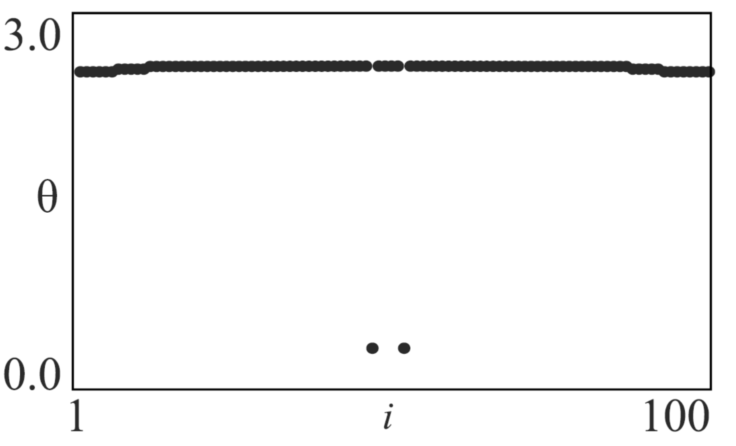
Transition from coherence/sync to incoherence/desync

chimera states



Localized in space incoherent domain

solitary states



Randomly distributed solitary nodes

I. Omelchenko, O. Omel'chenko, P. Hövel, E. Schöll, When nonlocal coupling between oscillators becomes stronger: patched synchrony or multichimera states, *Phys. Rev. Lett.* 110, 224101 (2013)

P. Jaros, S. Brezetsky, R. Levchenko, D. Dudkowski, T. Kapitaniak, and Y. Maistrenko, Solitary states for coupled oscillators with inertia, Chaos 28, 011103 (2018)

Partial sync patterns in a multiplex network of coupled neurons

Previous studies on one-layer netwrok

Network of nonlocally coupled FitzHugh-Nagumo systems

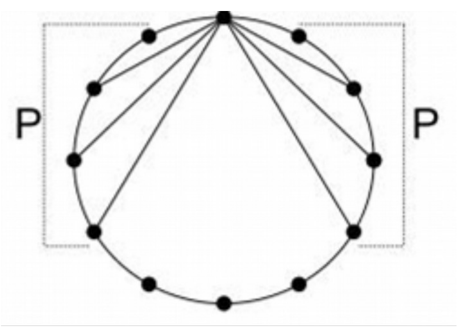
$$\varepsilon \dot{u}_i = u_i - \frac{u_i^3}{3} - v_i + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} \left[b_{uu}(u_j - u_i) + b_{uv}(v_j - v_i) \right],$$

$$\dot{v}_i = u_i + a_i + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} \left[b_{vu}(u_j - u_i) + b_{vv}(v_j - v_i) \right]$$

$$\begin{pmatrix} b_{uu} & b_{uv} \\ b_{vu} & b_{vv} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$$

$|a_i| < 1$
oscillatory

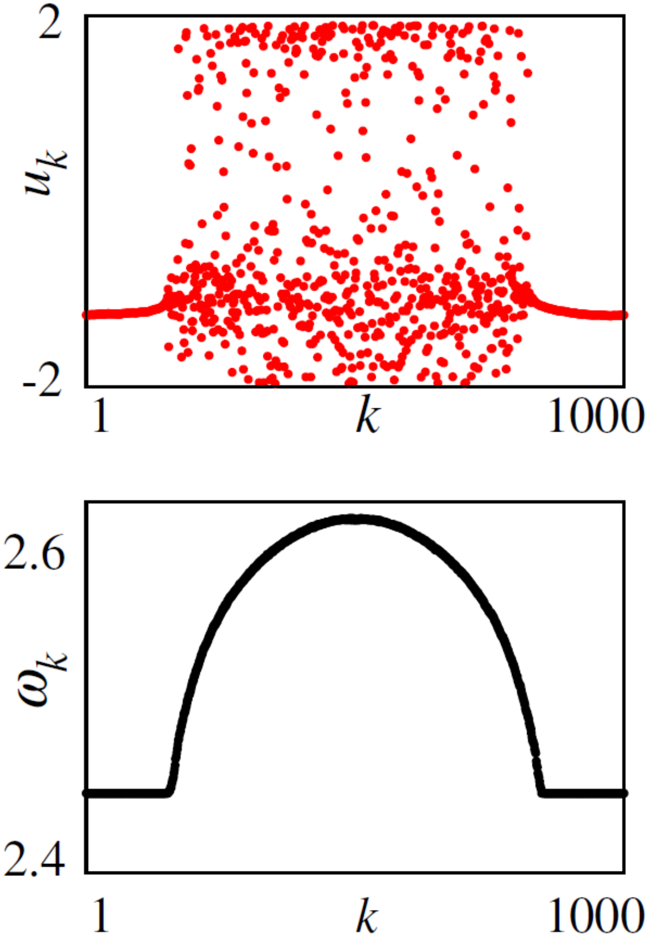
$$\phi = \frac{\pi}{2} - 0.1$$

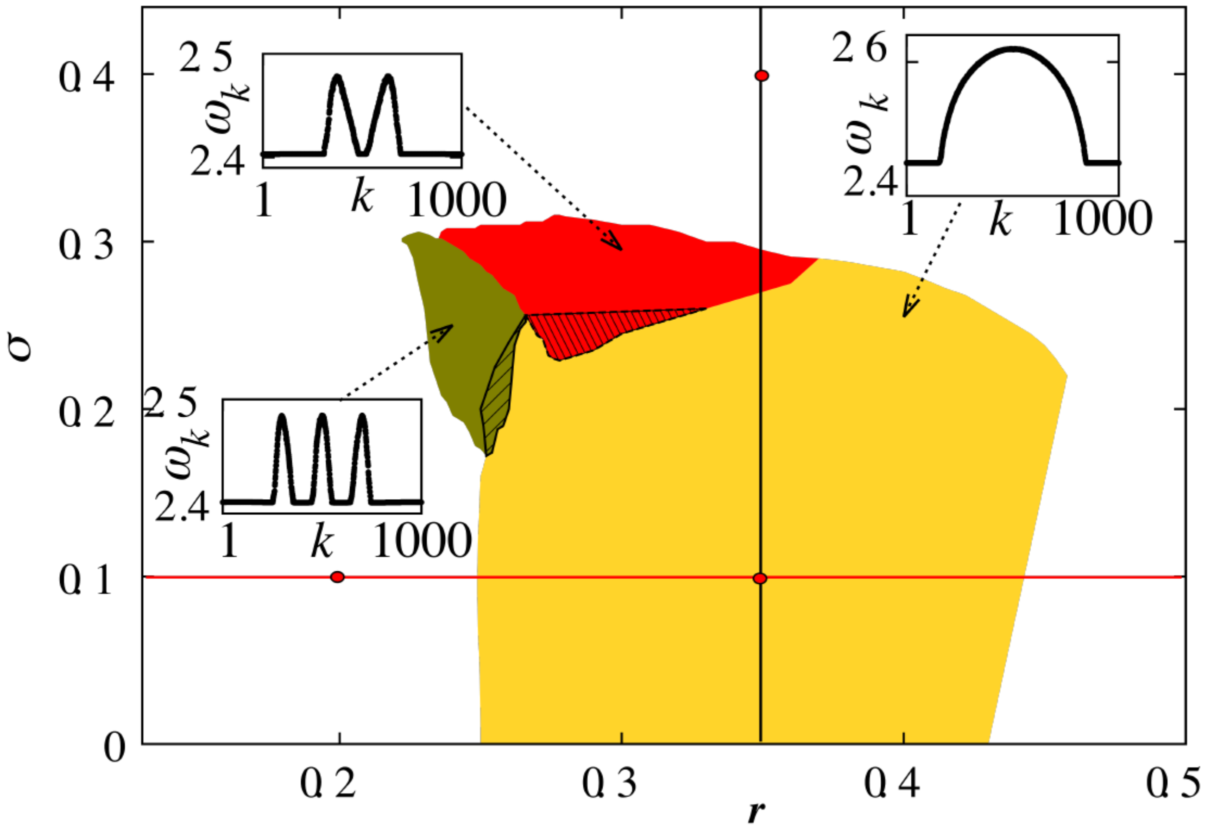


nonlocal

I. Omelchenko, O. Omel'chenko, P. Hövel, E. Schöll, *Phys. Rev. Lett.* 110, 224101 (2013)

Chimera states in one-layer network

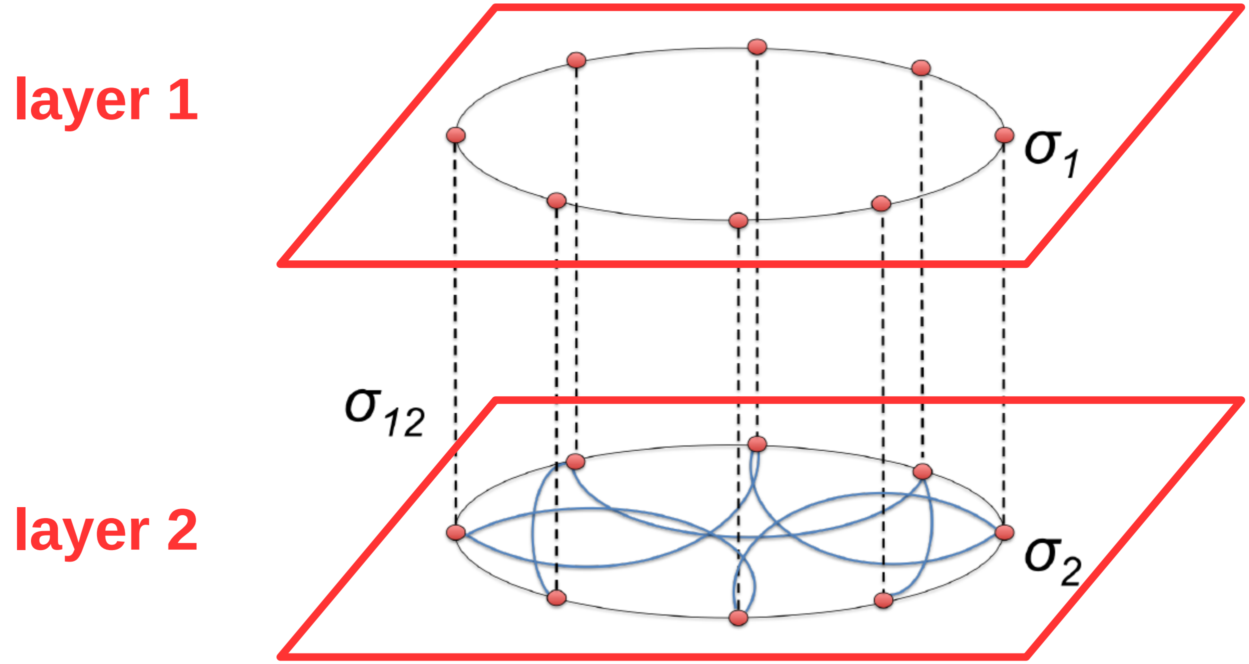




I. Omelchenko, O. Omel'chenko, P. Hövel, E. Schöll, *Phys. Rev. Lett.* 110, 224101 (2013)

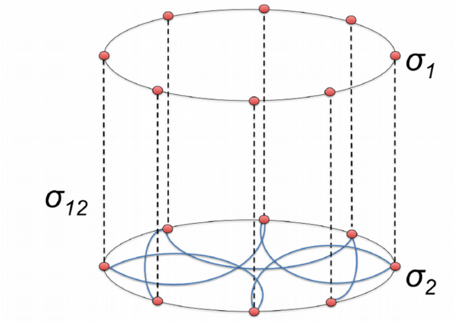
Multiplex network

Can we control the dynamics in the presence of weak multiplexing?



Can we control one layer by manipulating the parameters of the other layer?

Multiplex network



Layer 1

$$\varepsilon \frac{du_{1i}}{dt} = u_{1i} - \frac{u_{1i}^3}{3} - v_{1i} + \frac{\sigma_1}{2R_1} \sum_{j=i-R_1}^{i+R_1} [b_{uu}(u_{1j} - u_{1i}) + b_{uv}(v_{1j} - v_{1i})] + \sigma_{12}(u_{2i} - u_{1i}),$$

$$\frac{dv_{1i}}{dt} = u_{1i} + a_i + \frac{\sigma_1}{2R_1} \sum_{j=i-R_1}^{i+R_1} [b_{vu}(u_{1j} - u_{1i}) + b_{vv}(v_{1j} - v_{1i})],$$

Layer 2

$$\varepsilon \frac{du_{2i}}{dt} = u_{2i} - \frac{u_{2i}^3}{3} - v_{2i} + \frac{\sigma_2}{2R_2} \sum_{j=i-R_2}^{i+R_2} [b_{uu}(u_{2j} - u_{2i}) + b_{uv}(v_{2j} - v_{2i})] + \sigma_{12}(u_{1i} - u_{2i}),$$

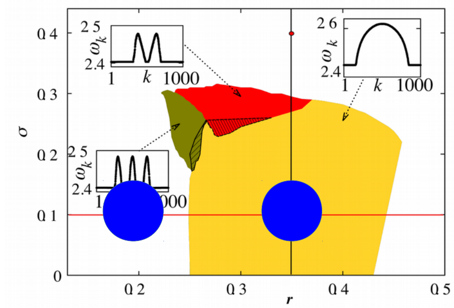
$$\frac{dv_{2i}}{dt} = u_{2i} + a_i + \frac{\sigma_2}{2R_2} \sum_{j=i-R_2}^{i+R_2} [b_{vu}(u_{2j} - u_{2i}) + b_{vv}(v_{2j} - v_{2i})],$$

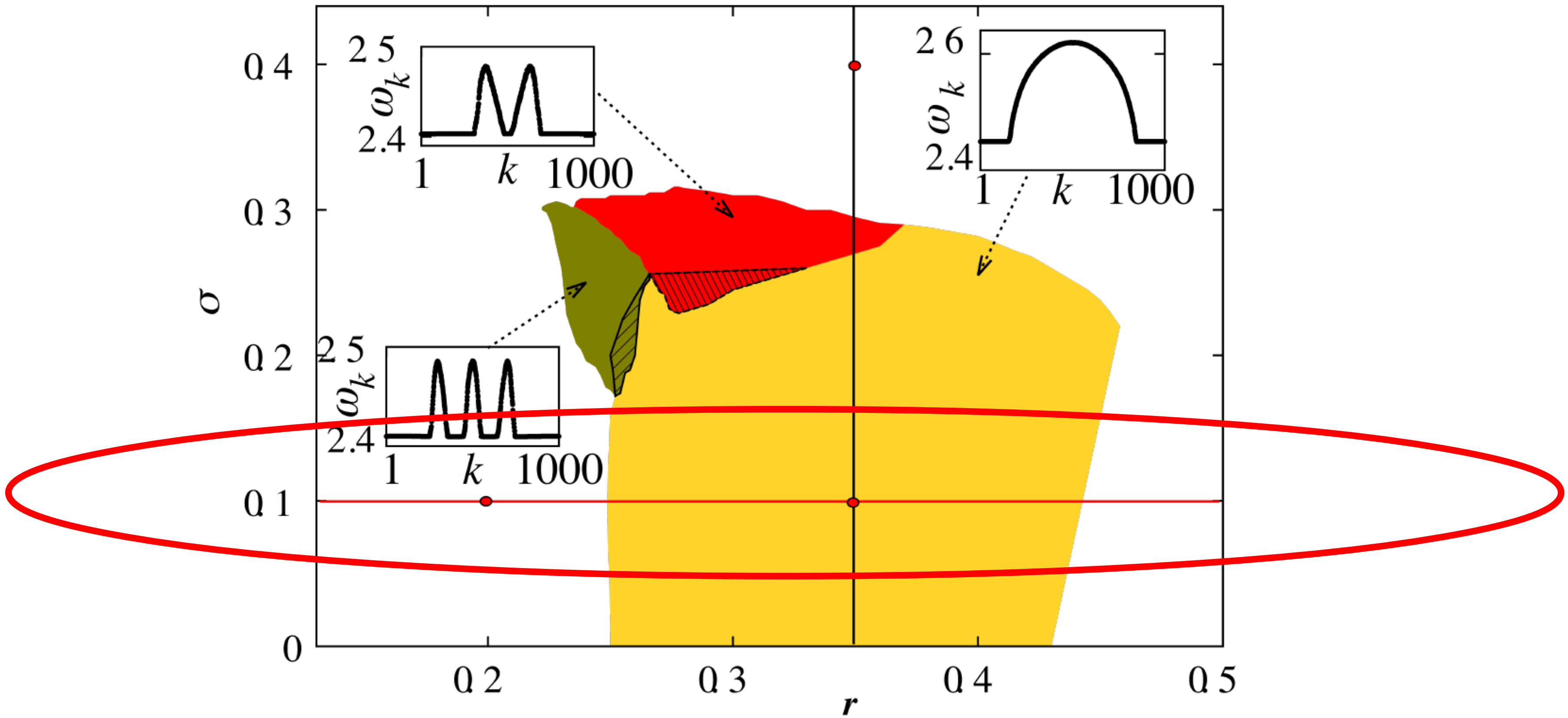
M. Mikhaylenko, L. Ramlow, S. Jalan, A. Zakharova, Weak multiplexing in neural networks: Switching between chimera and solitary states, Chaos 29, 023122 (2019)

Multiplex network

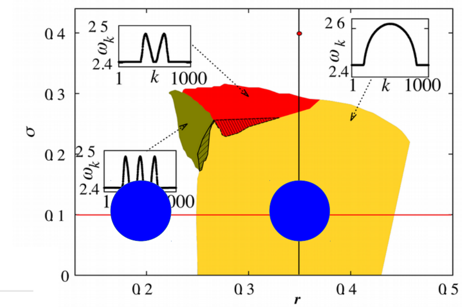
Case one: coupling range mismatch

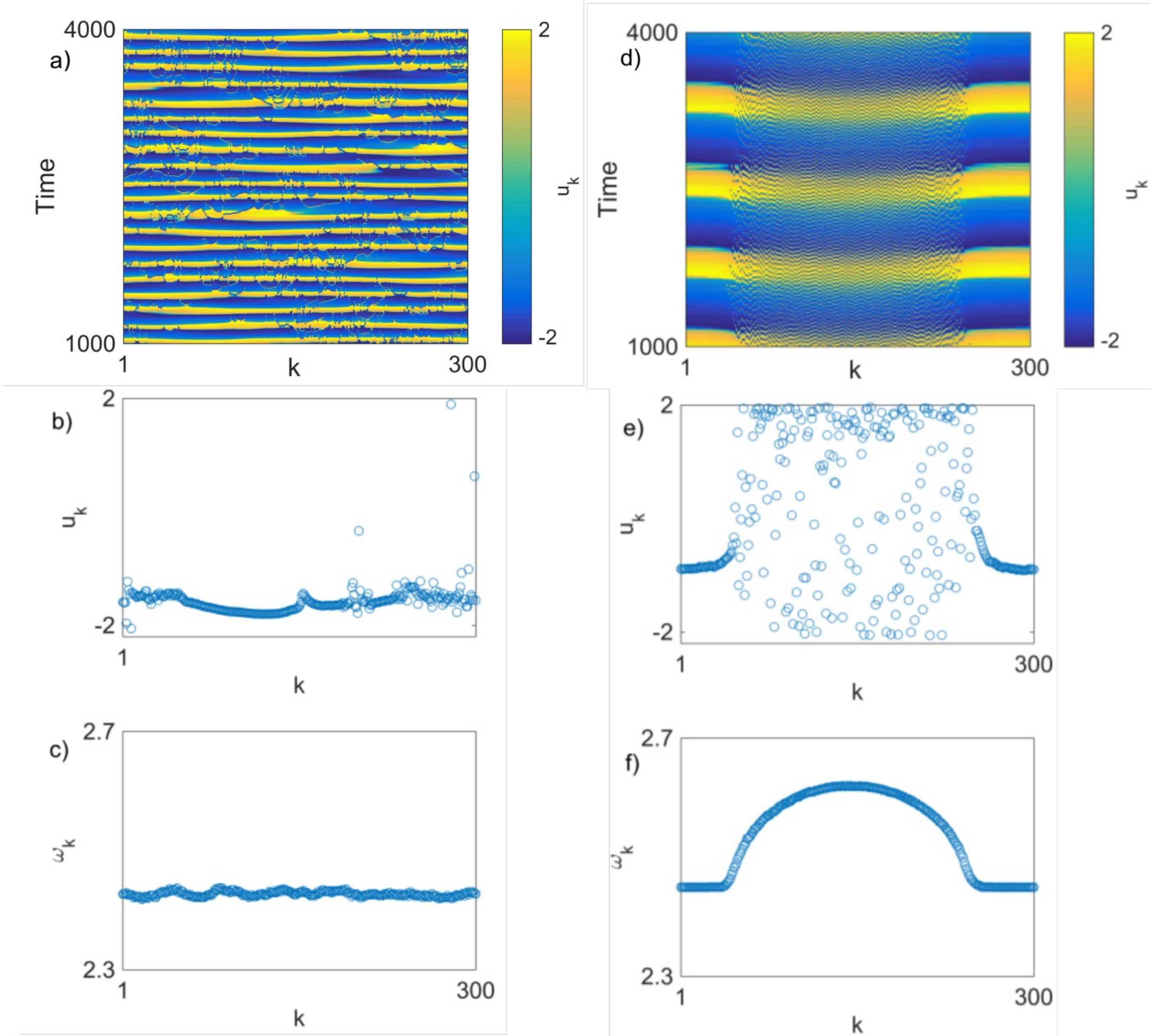
Chimeras in isolated layers: different coupling range



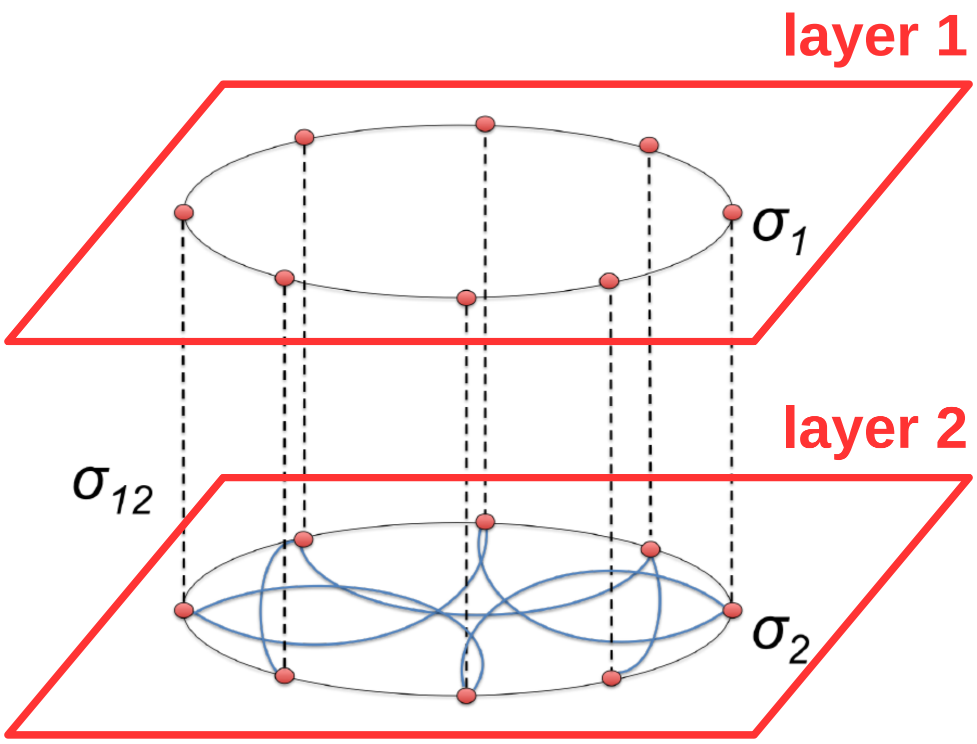


Isolated layers: different coupling range





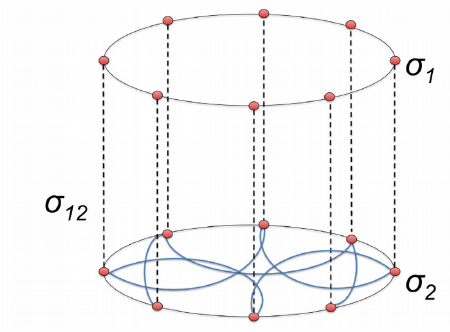
Multiplex network



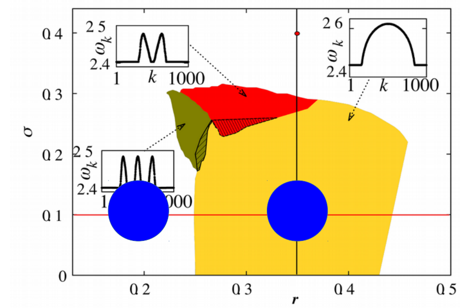
weak multiplexing
$\sigma_{12} = 0.01$

$\sigma_1 = \sigma_2 = 0.1$

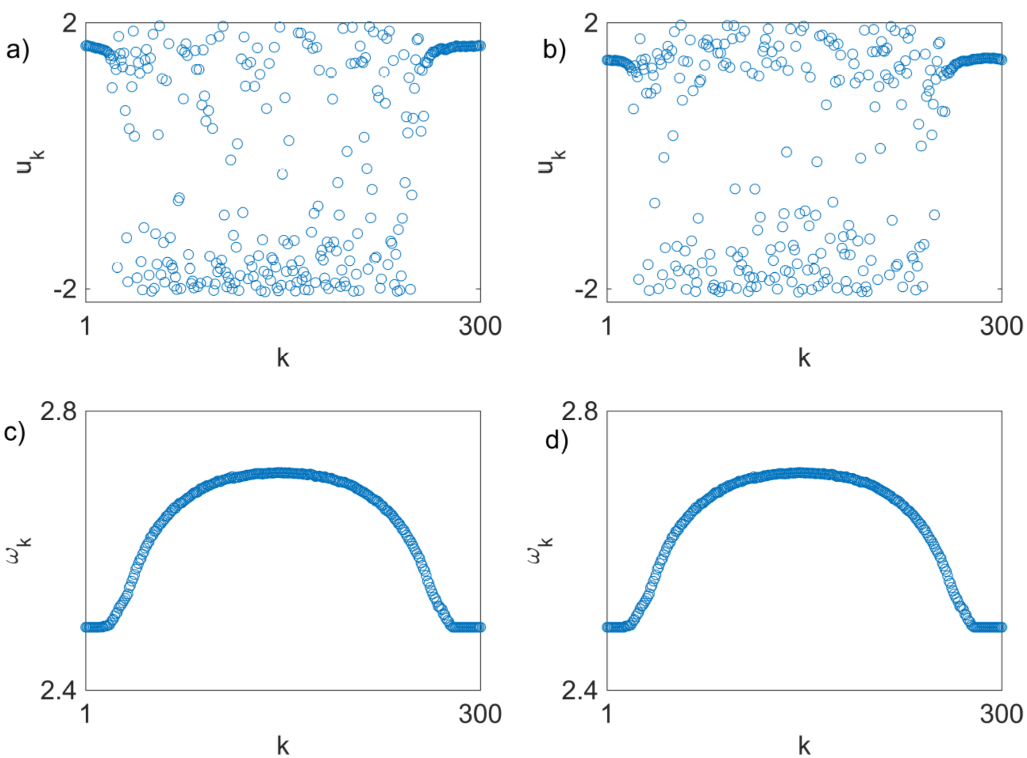
$r_1 = 0.2, r_2 = 0.35$



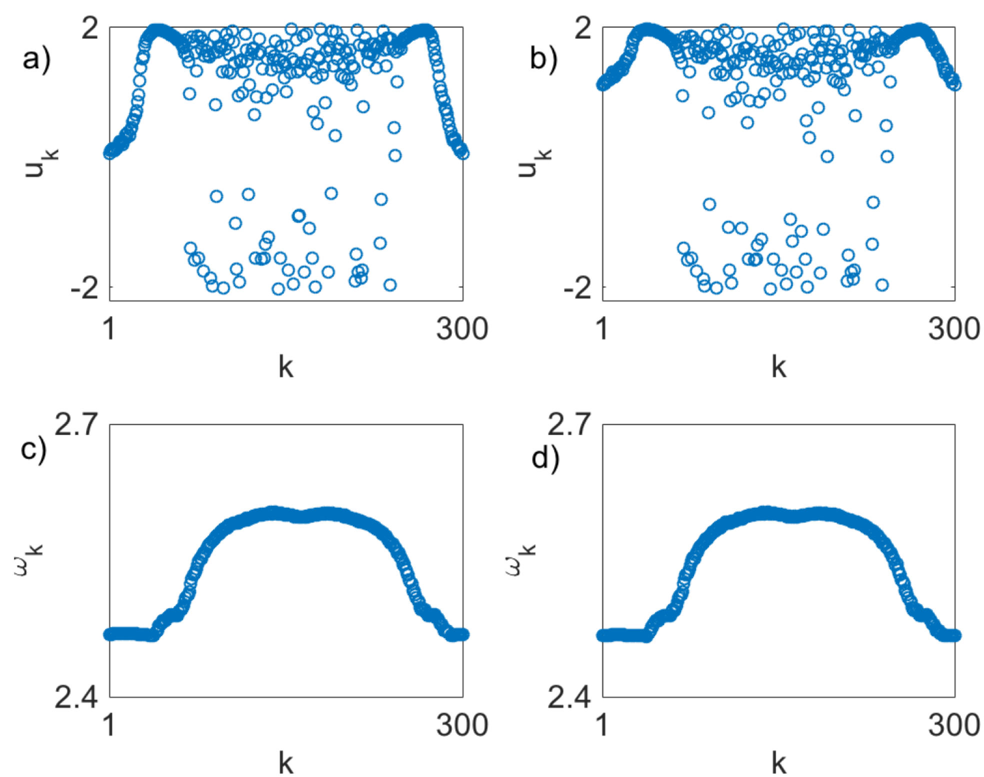
Multiplex network



weak multiplexing
$\sigma_{12} = 0.01$



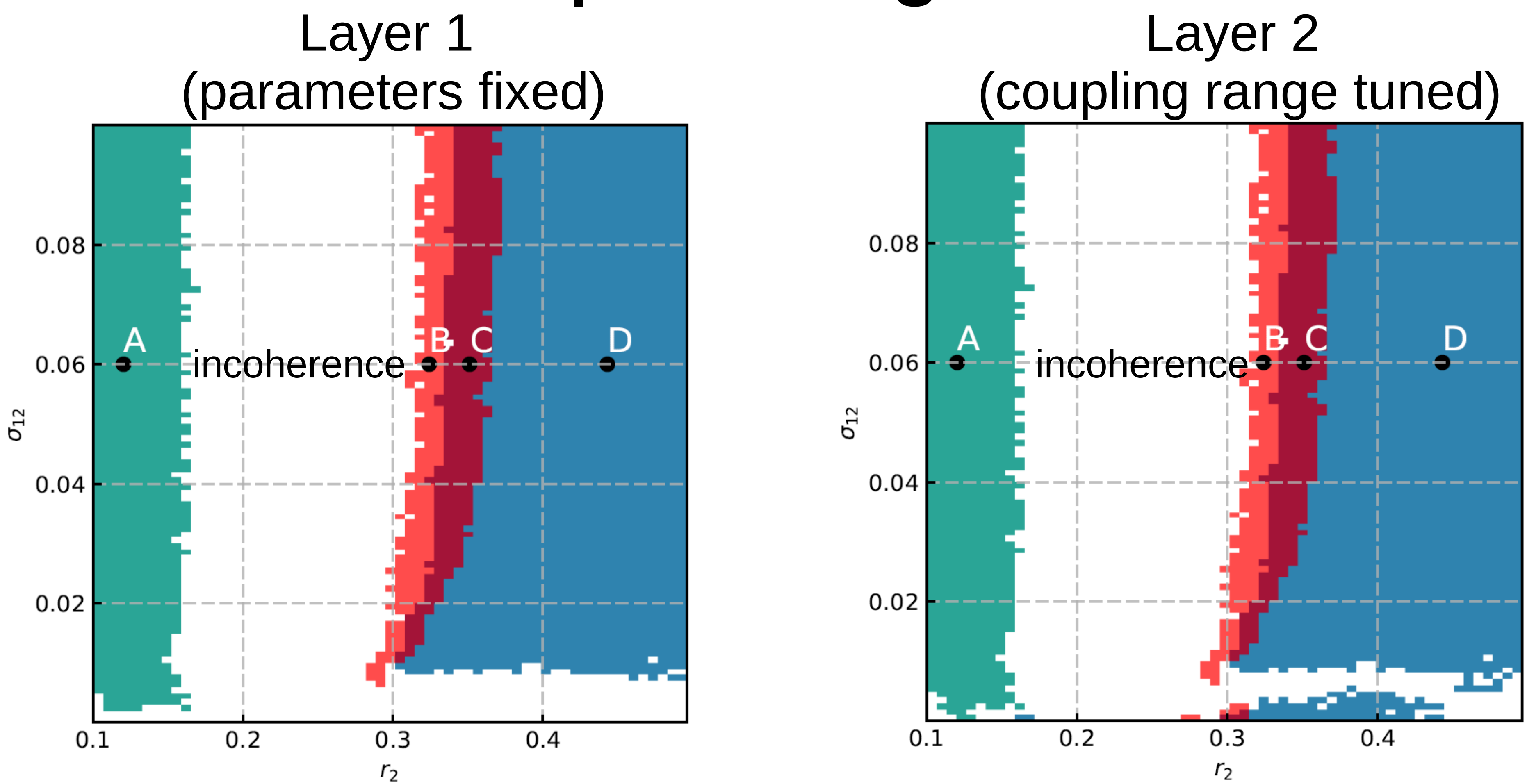
strong multiplexing
$\sigma_{12} = 0.1$



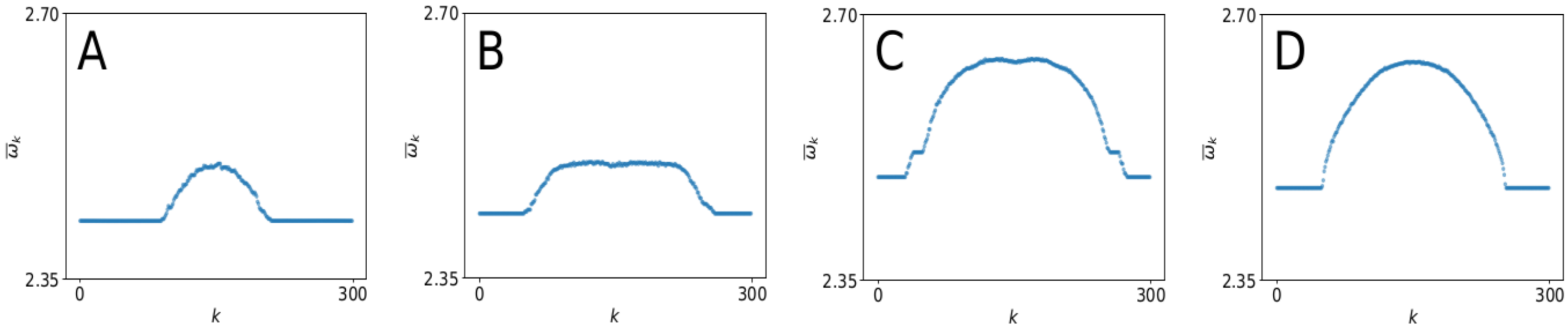
Weak multiplexing induces chimeras

M. Mikhaylenko, L. Ramlow, S. Jalan, A. Zakharova, Weak multiplexing in neural networks: Switching between chimera and solitary states, Chaos 29, 023122 (2019)

Maps of regimes



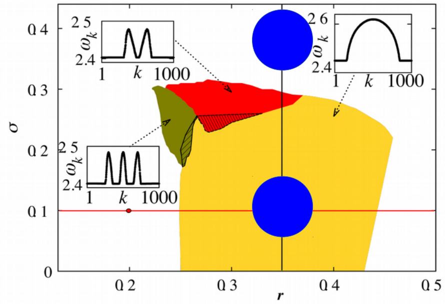
We can induce chimeras with different profiles in layer 1 by multiplexing it with layer 2.

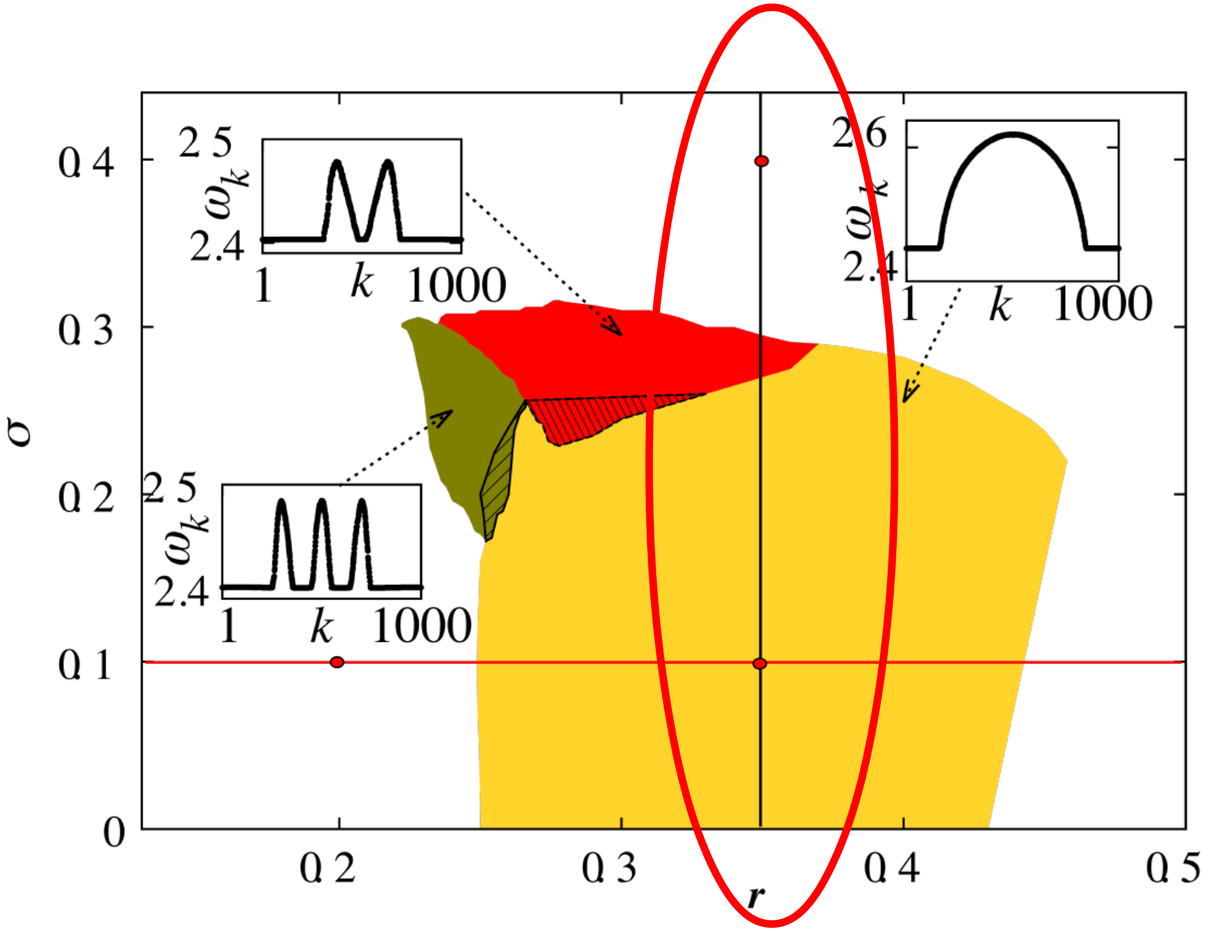


Multiplex network

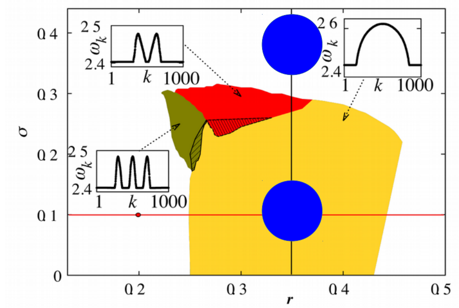
Case two: coupling strength mismatch

Chimeras in isolated layers: different coupling strength

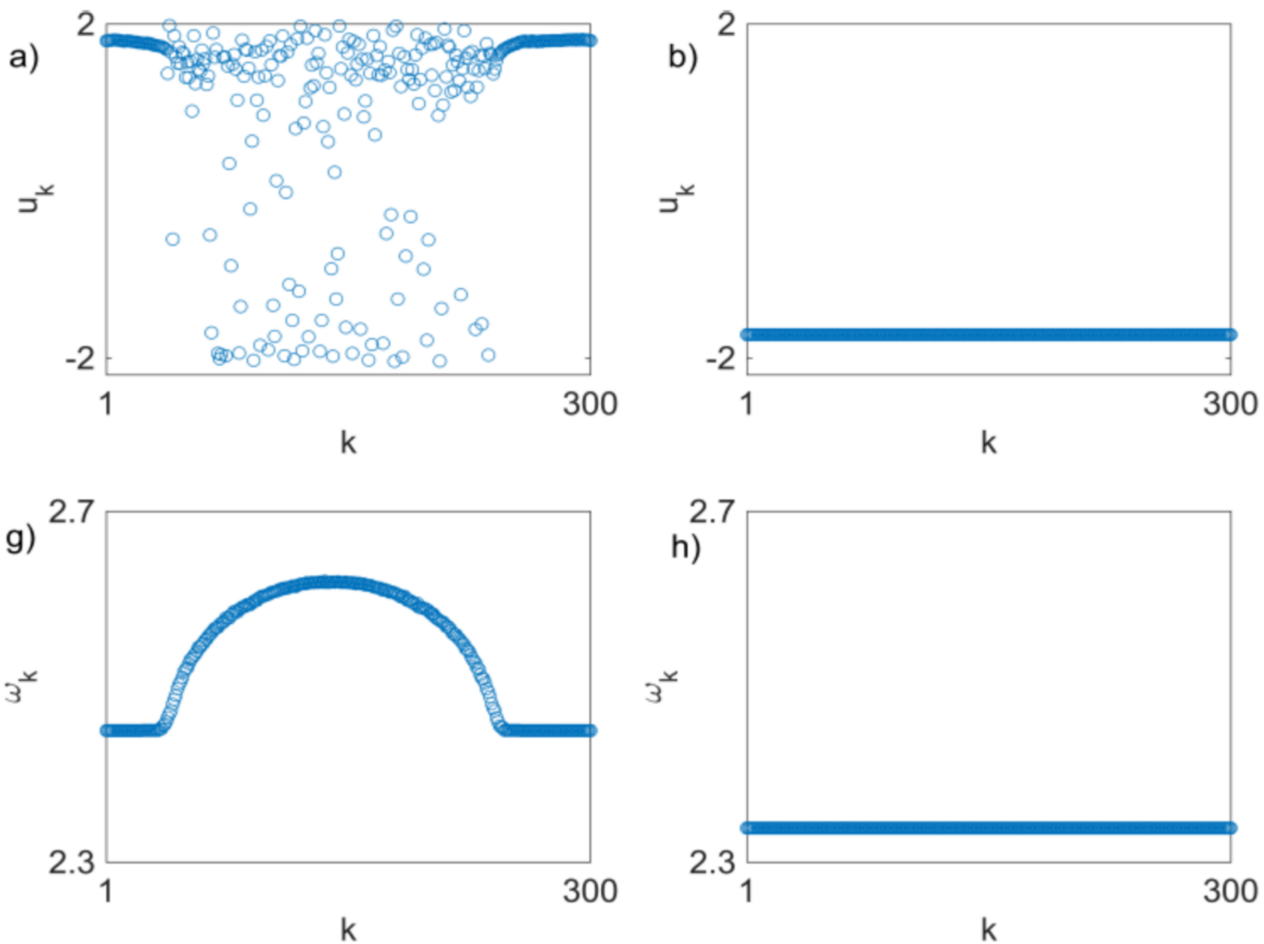




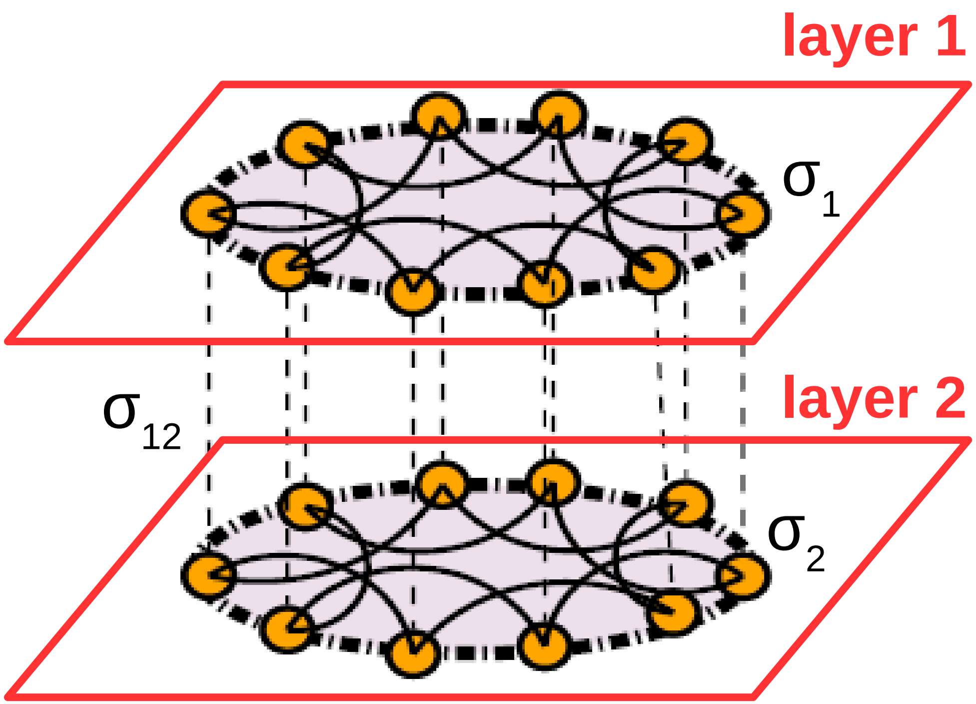
Isolated layers: different coupling strength



layer 1 **layer 2**



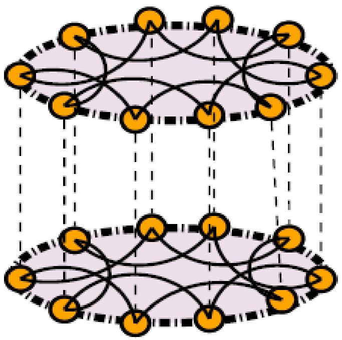
Multiplex network



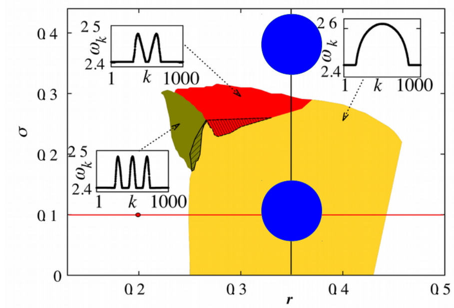
weak multiplexing
$\sigma_{12} = 0.01$

$r_1 = r_2 = 0.35$

$\sigma_1 = 0.1, \sigma_2 = 0.4$

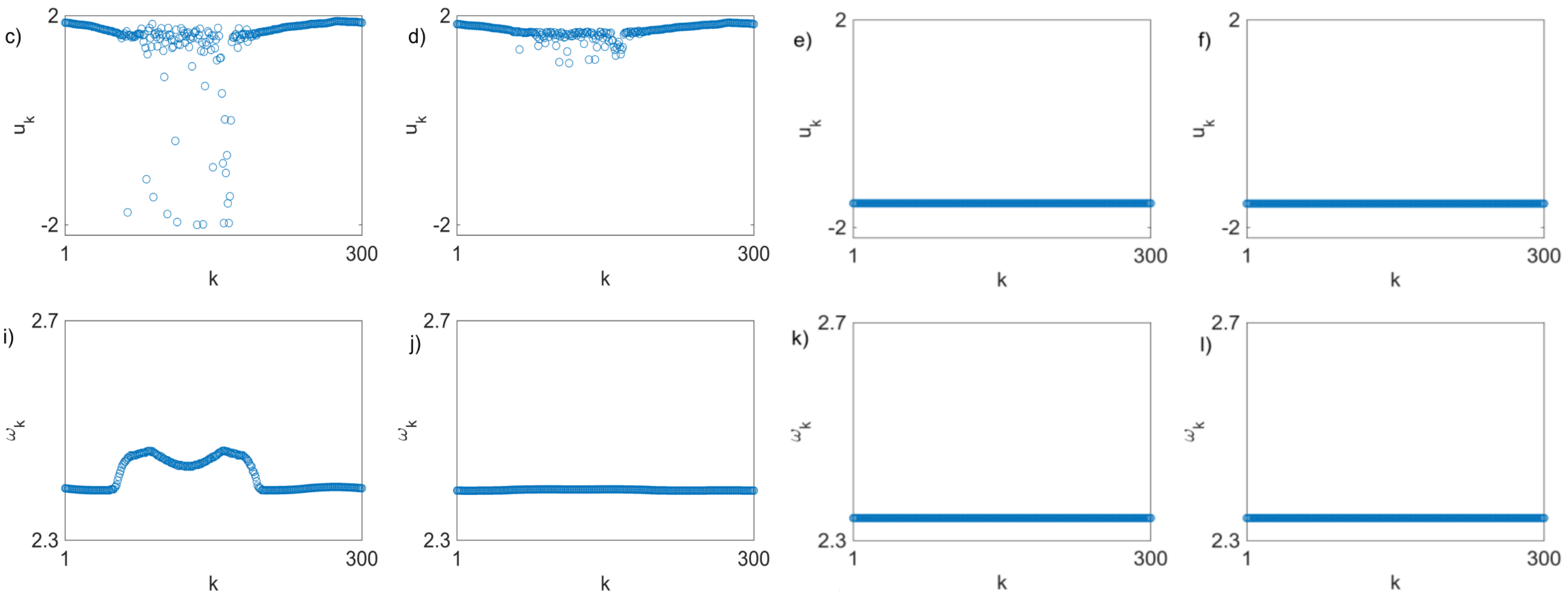


Multiplex network



weak multiplexing
$\sigma_{12} = 0.01$

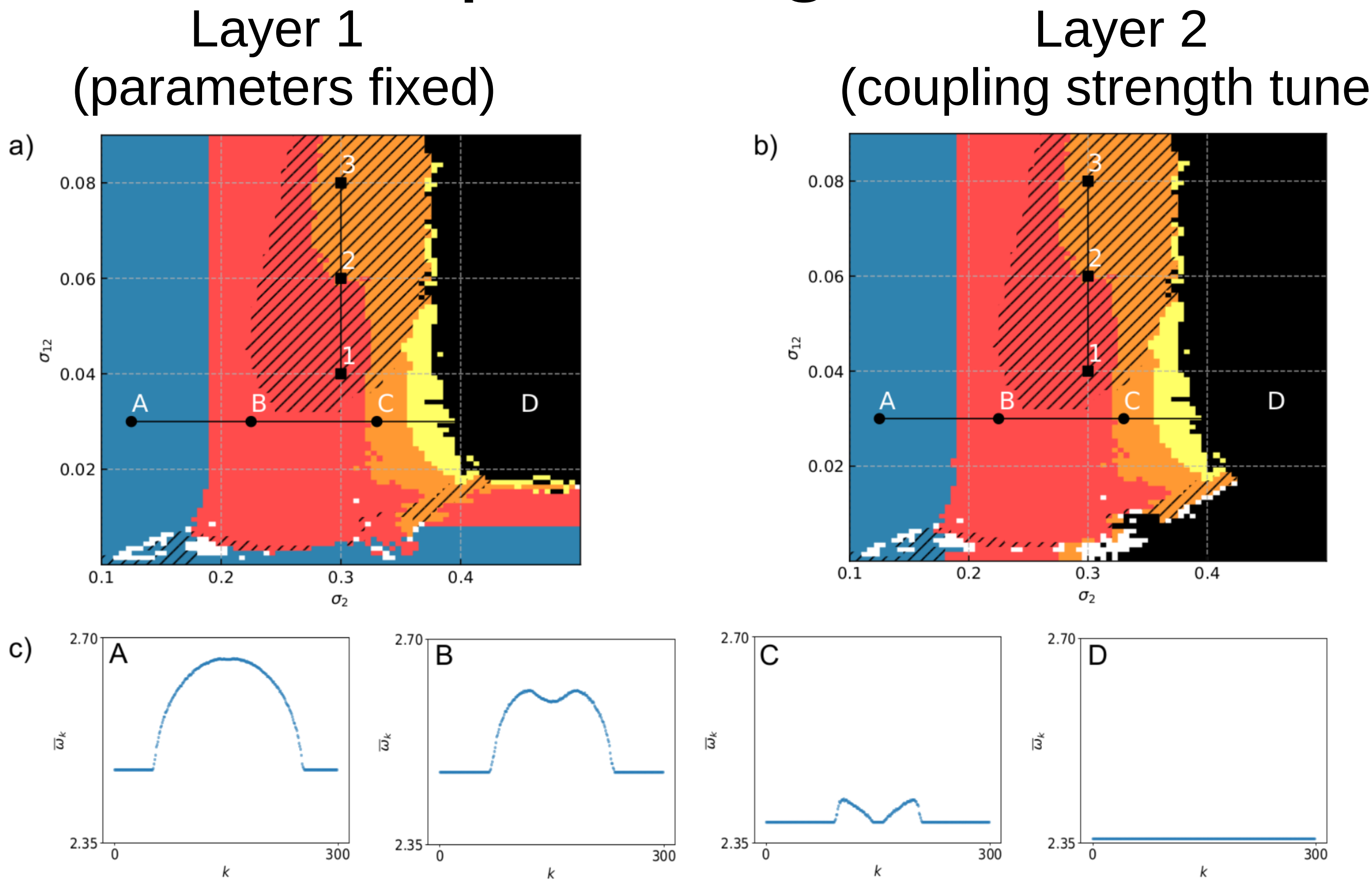
weak multiplexing
$\sigma_{12} = 0.05$



Weak multiplexing suppresses chimeras

M. Mikhaylenko, L. Ramlow, S. Jalan, A. Zakharova, Weak multiplexing in neural networks: Switching between chimera and solitary states, Chaos 29, 023122 (2019)

Maps of regimes



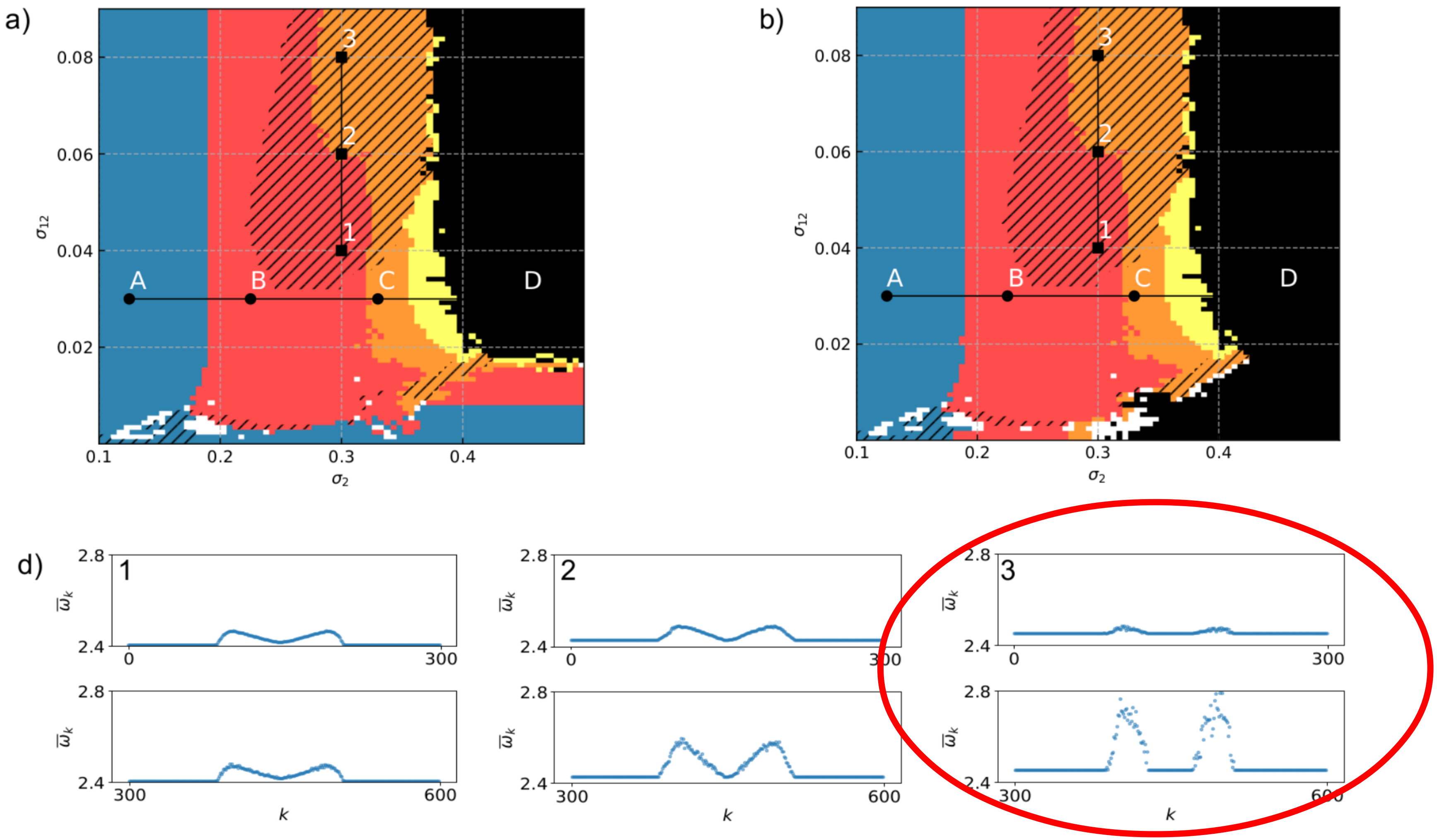
We can induce in-phase sync and two-headed chimeras in layer 1 by multiplexing it with layer 2. We can make the layers behave differently.

M. Mikhaylenko, L. Ramlow, S. Jalan, A. Zakharova, Weak multiplexing in neural networks: Switching between chimera and solitary states, Chaos 29, 023122 (2019)

Maps of regimes

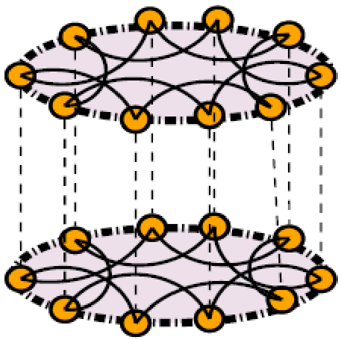
Layer 1 (parameters fixed)

Layer 2 (coupling strength tuned)

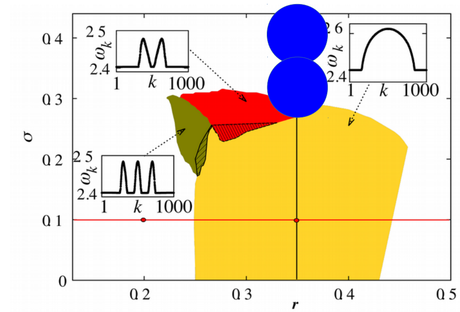


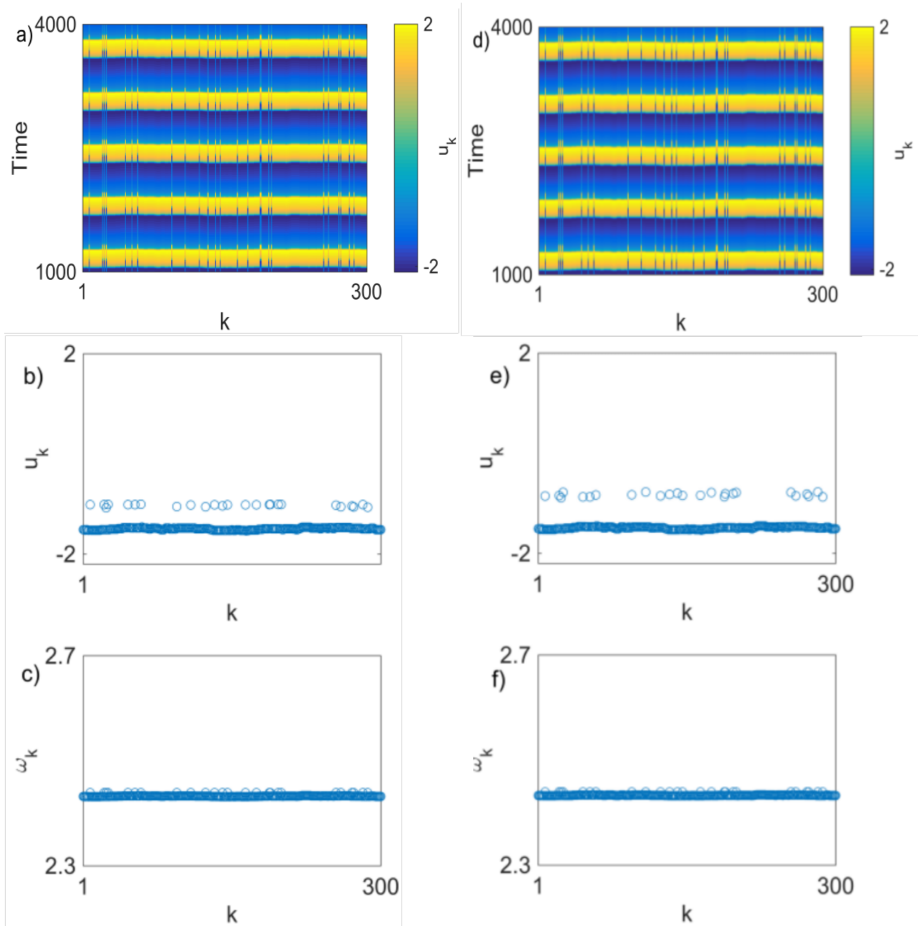
The two-headed chimeras are better pronounced in layer 2 for the range of parameters that correspond to no chimera (in-phase synchronization) for this layer in isolation.

M. Mikhaylenko, L. Ramlow, S. Jalan, A. Zakharova, Weak multiplexing in neural networks: Switching between chimera and solitary states, Chaos 29, 023122 (2019)



Multiplex network: solitary states





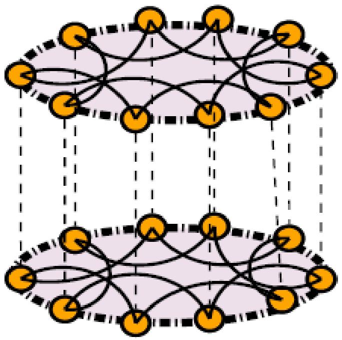
Small coupling strength mismatch

and

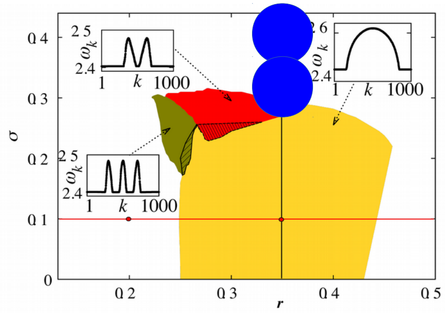
weak multiplexing $\sigma_{12} = 0.05$

Weak multiplexing induces solitary states in both layers

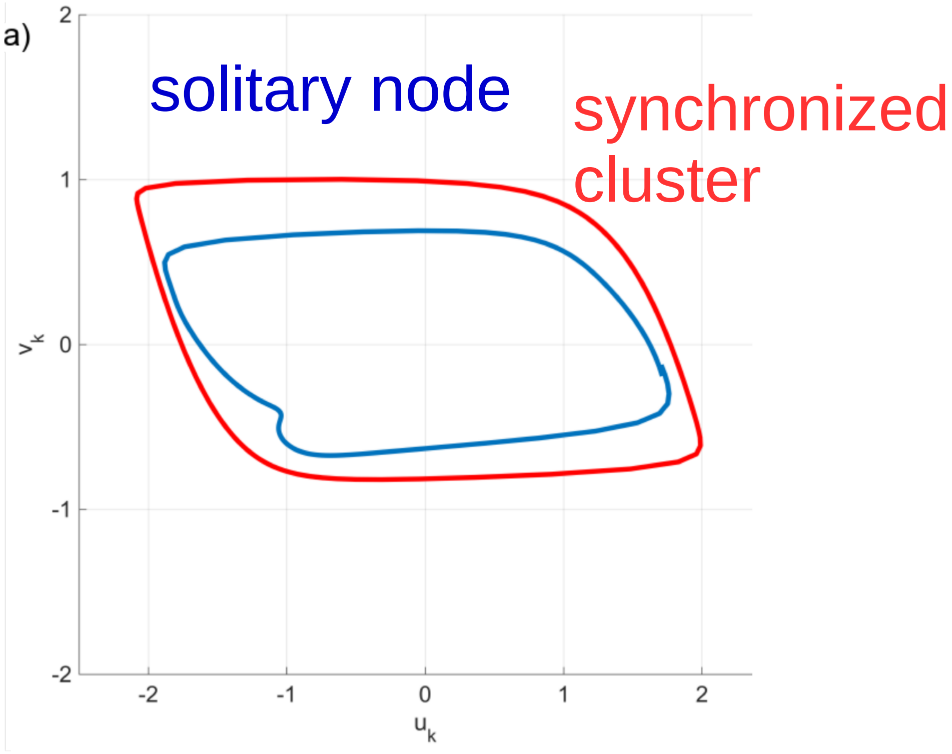
M. Mikhaylenko, L. Ramlow, S. Jalan, A. Zakharova, Weak multiplexing in neural networks: Switching between chimera and solitary states, Chaos 29, 023122 (2019)

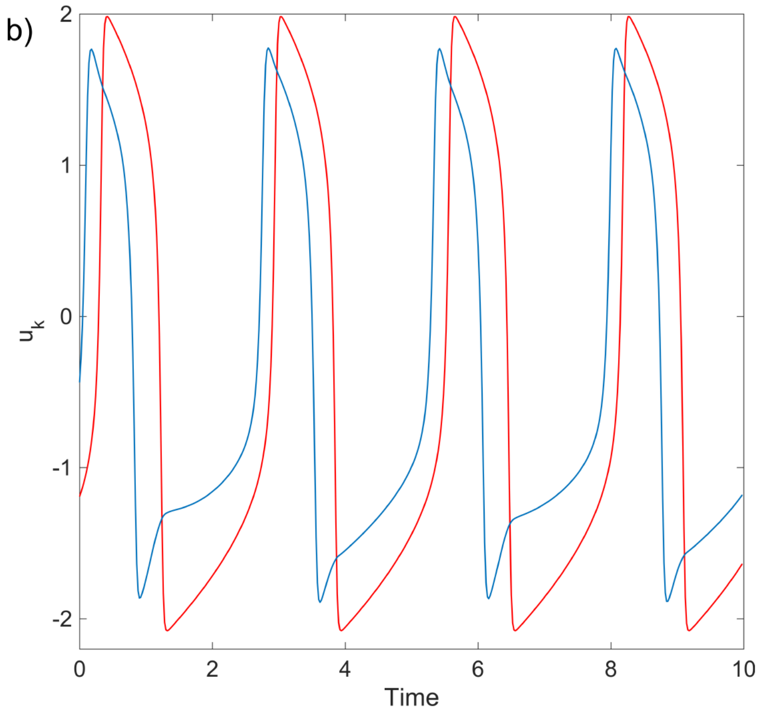


Multiplex network: solitary states



weak multiplexing
$\sigma_{12} = 0.05$





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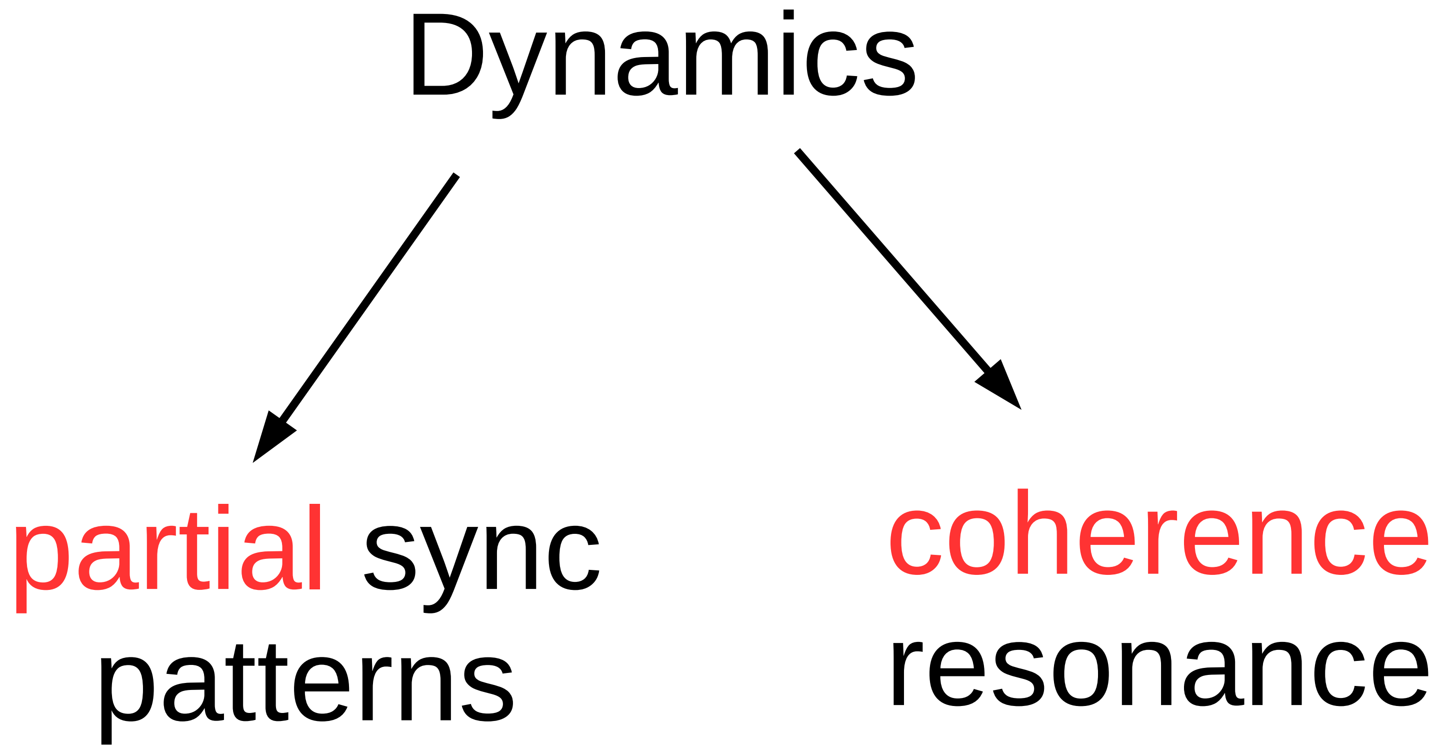
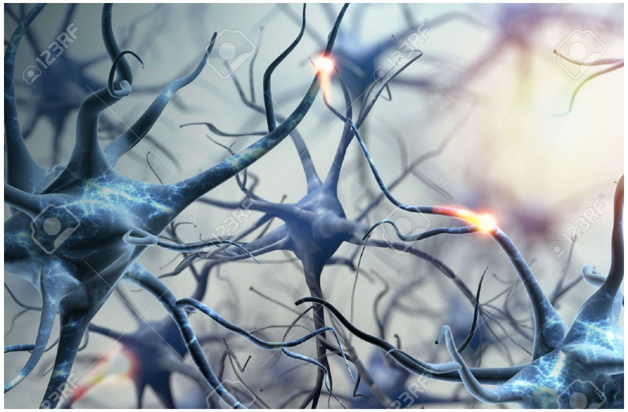

Dynamics
partial sync
patterns
coherence
resonance

Coherence resonance

Coherence resonance

The best temporal regularity of the noise-induced oscillations occurs for an intermediate value of noise intensity

- discovered by Haken et al. in 1993
- named ***coherence resonance*** by Pikovsky and Kurths in 1997
- analytical treatment by Lindner and Schimansky-Geier in 1999



constructive role of noise, counter-intuitive phenomenon

Model: FitzHugh-Nagumo system in excitable regime

$$\varepsilon \dot{u} = u - \frac{u^3}{3} - v,$$

$$\dot{v} = u + a + \sqrt{2D}\,\xi(t)$$

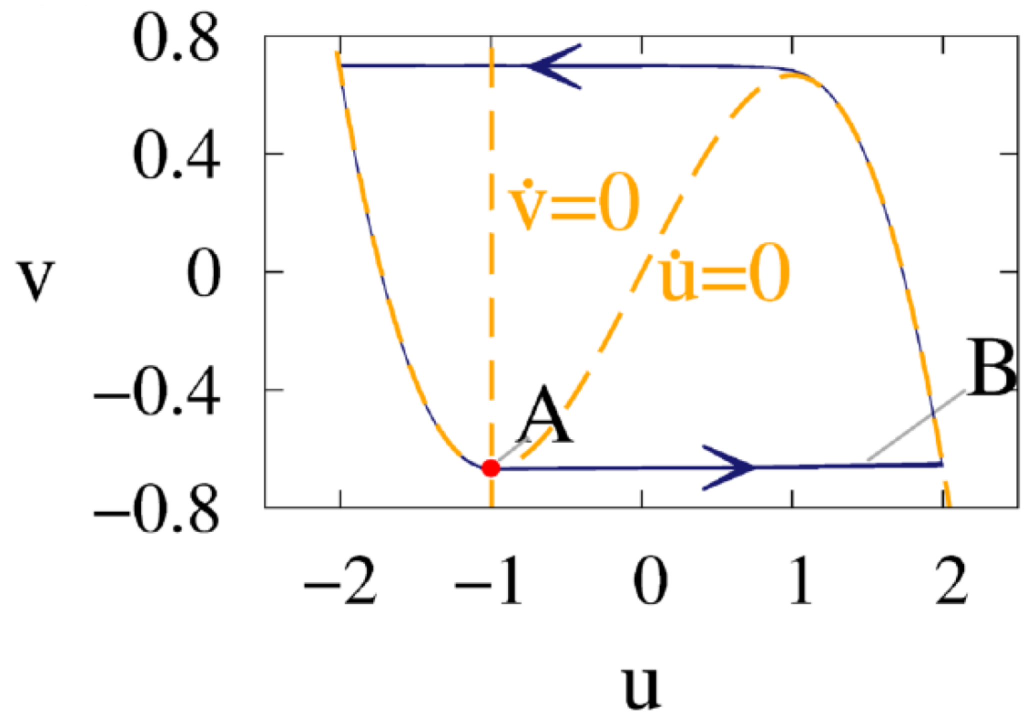
single node dynamics

u – activator

v – inhibitor

$|a_i| < 1$ oscillatory

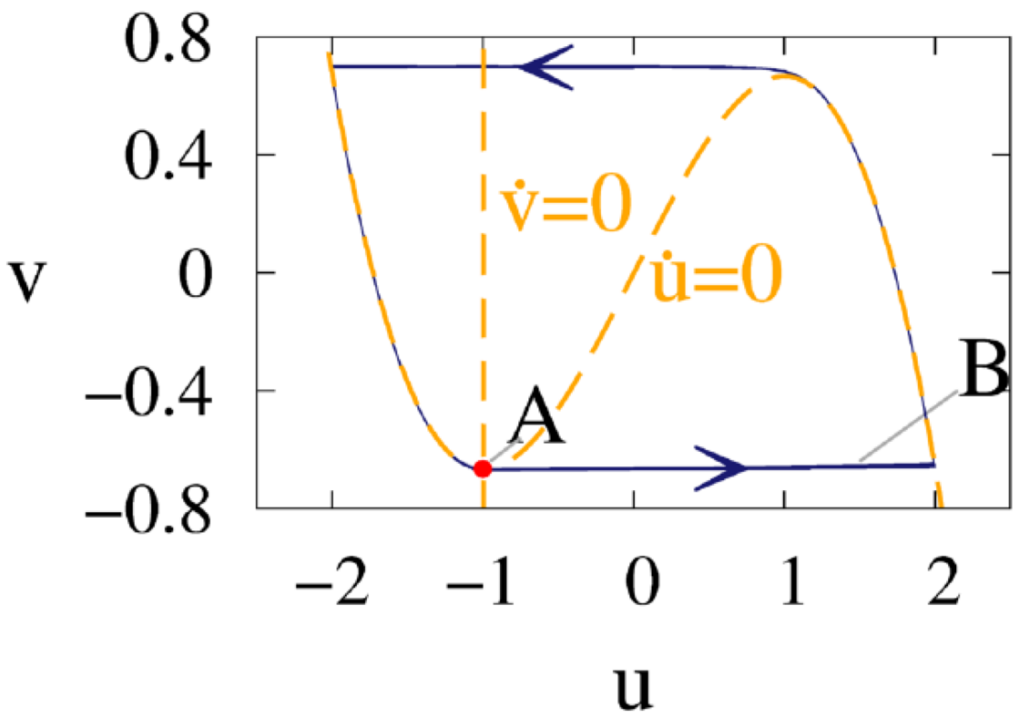
$|a_i| > 1$ excitable

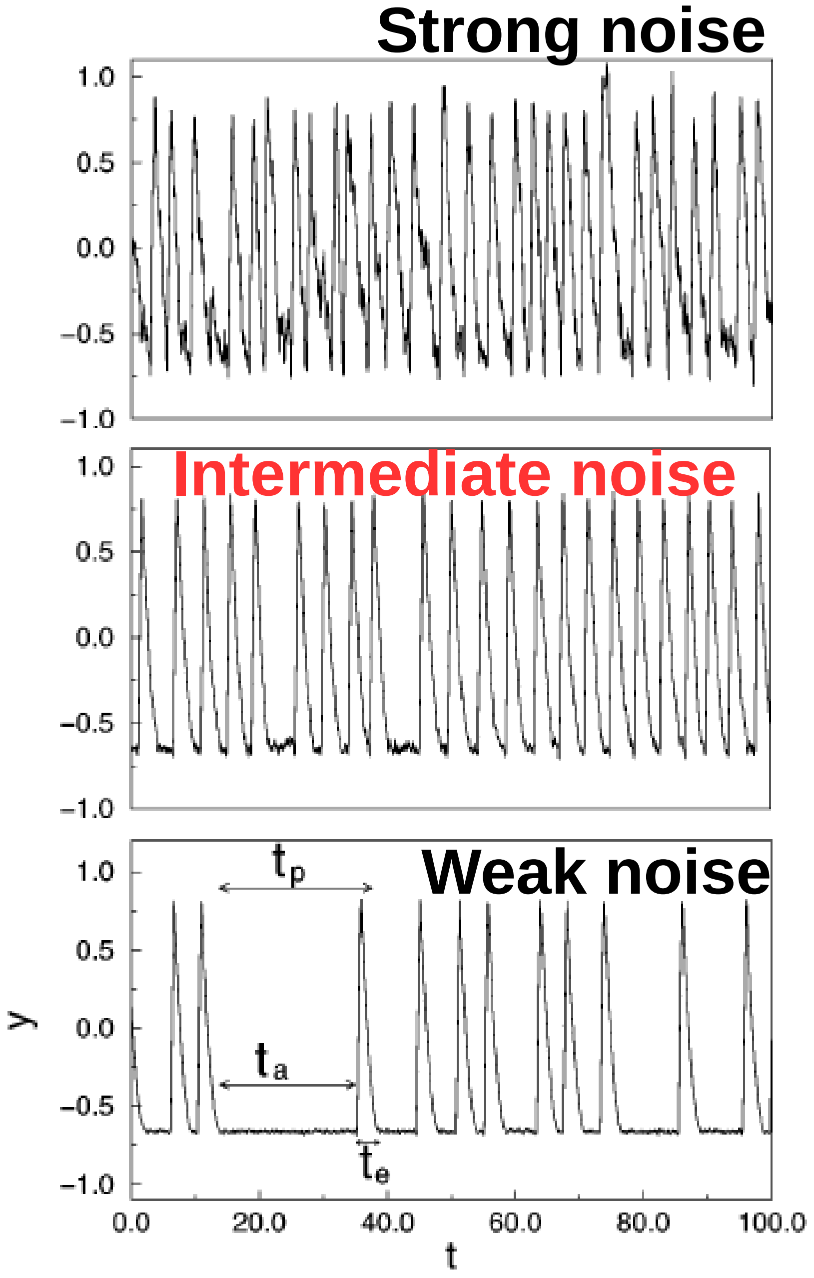


System parameters: $\varepsilon = 0.01$, a=1.001, $D = 0.0001$

Coherence resonance

$$\varepsilon \dot{u} = u - \frac{u^3}{3} - v,$$
$$\dot{v} = u + a + \sqrt{2D}\xi(t)$$





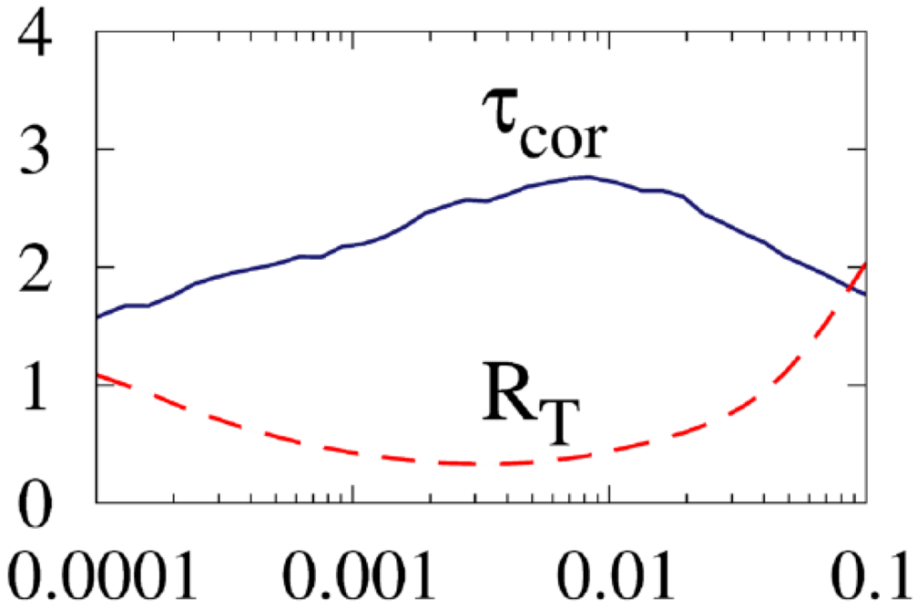
Model: FitzHugh-Nagumo system in excitable regime

$$\varepsilon \dot{u} = u - \frac{u^3}{3} - v,$$

$$\dot{v} = u + a + \sqrt{2D}\xi(t)$$

$|a_i| > 1$
excitable

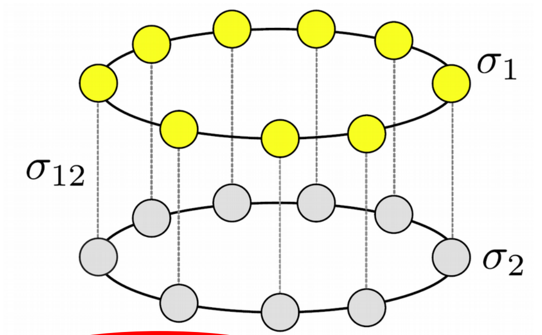
Coherence resonance



System parameters: $\varepsilon = 0.01$, a=1.001, $D = 0.0001$

Can we control coherence resonance by weak multiplexing?

Multiplex network of excitable FHN neurons



$$\varepsilon \frac{du_{1i}}{dt} = u_{1i} - \frac{u_{1i}^3}{3} - v_{1i} + \frac{\sigma_1}{2} \sum_{j=i-1}^{i+1} (u_{1j} - u_{1i}) + \sigma_{12}(u_{2i} - u_{1i}),$$

$$\frac{dv_{1i}}{dt} = u_{1i} + a + \sqrt{2D_1}\xi_i(t),$$

$$\varepsilon \frac{du_{2i}}{dt} = u_{2i} - \frac{u_{2i}^3}{3} - v_{2i} + \frac{\sigma_2}{2} \sum_{j=i-1}^{i+1} (u_{2j} - u_{2i}) + \sigma_{12}(u_{1i} - u_{2i}),$$

$$\frac{dv_{2i}}{dt} = u_{2i} + a + \sqrt{2D_2}\eta_i(t),$$

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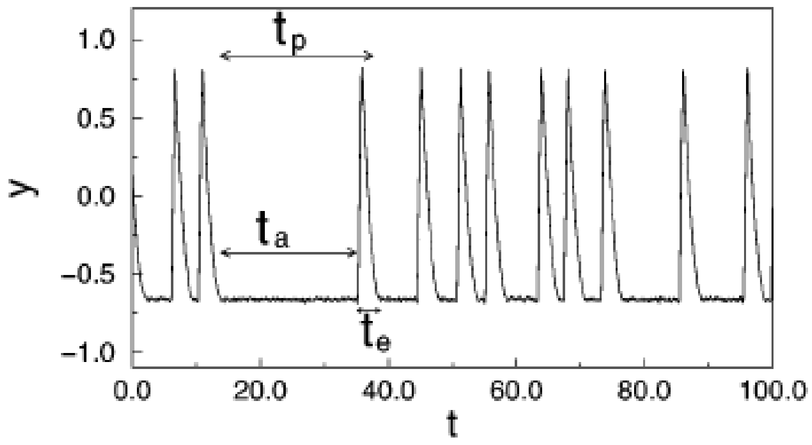
Coherence resonance: measures

Normalized standard deviation of the interspike interval

single node

$$R_T = \frac{\sqrt{\langle t_{ISI}^2 \rangle - \langle t_{ISI} \rangle^2}}{\langle t_{ISI} \rangle}$$



network

$$R_T = \frac{\sqrt{\langle \overline{t_{ISI}^2} \rangle - \langle \overline{t_{ISI}} \rangle^2}}{\langle \overline{t_{ISI}} \rangle}$$

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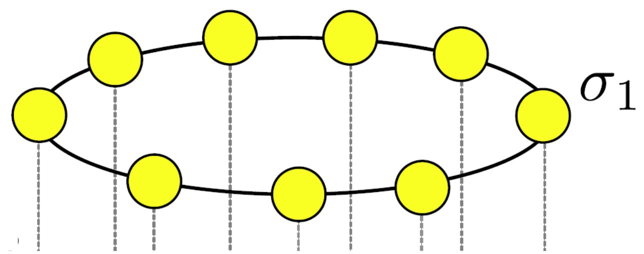
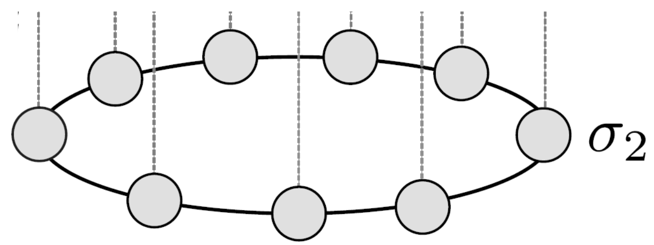
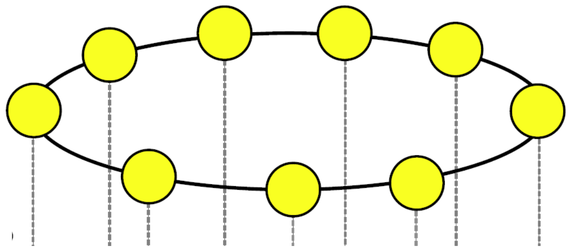
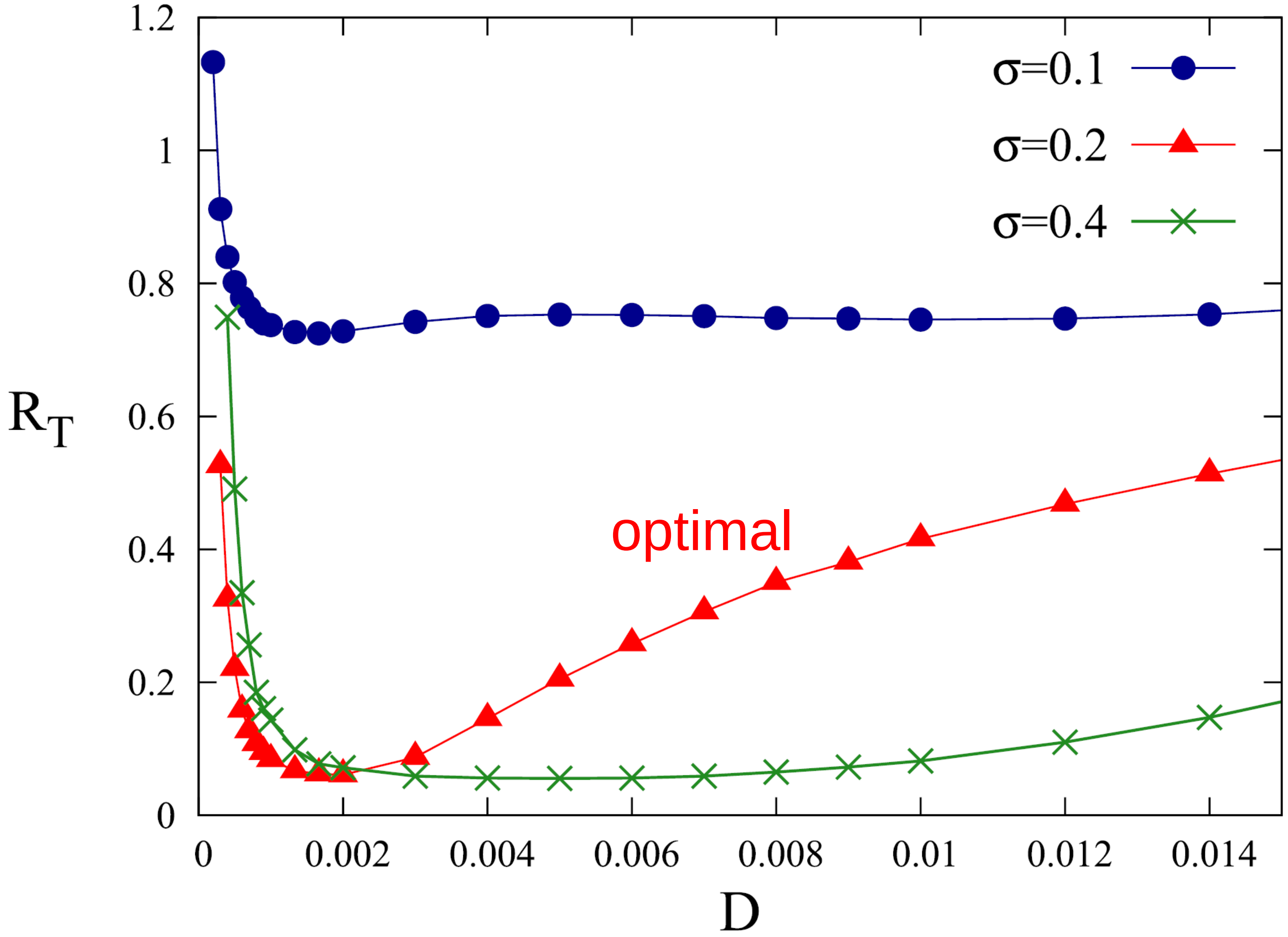


Dynamics of isolated layers

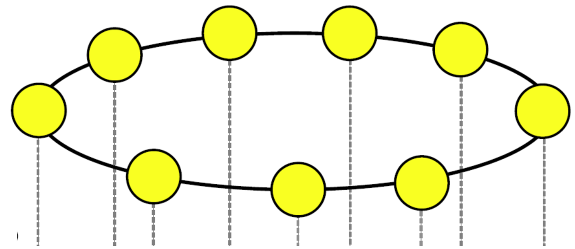


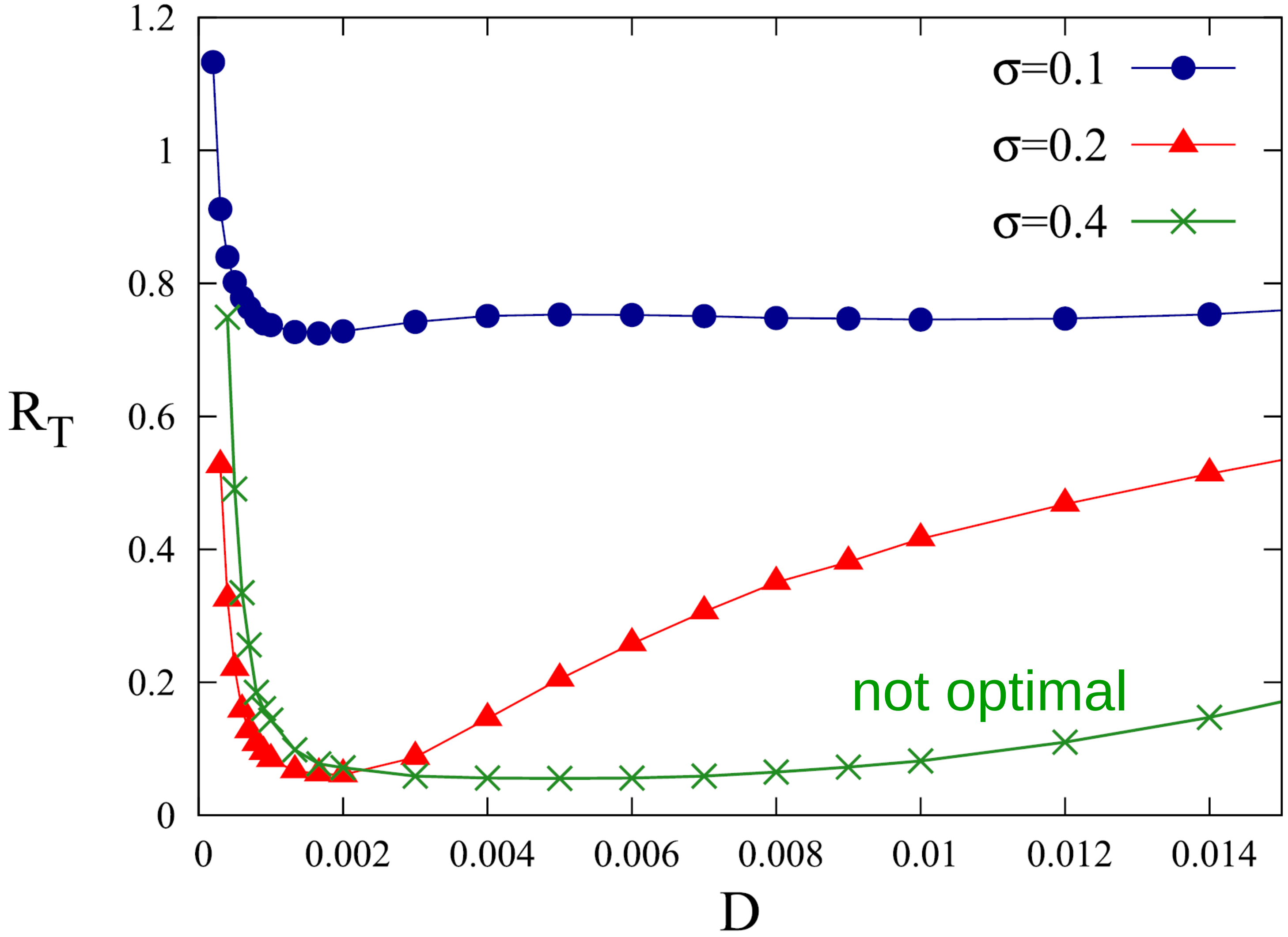
Isolated ring network



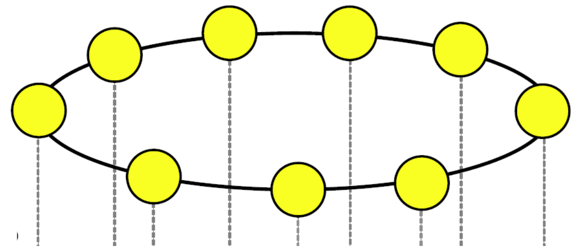


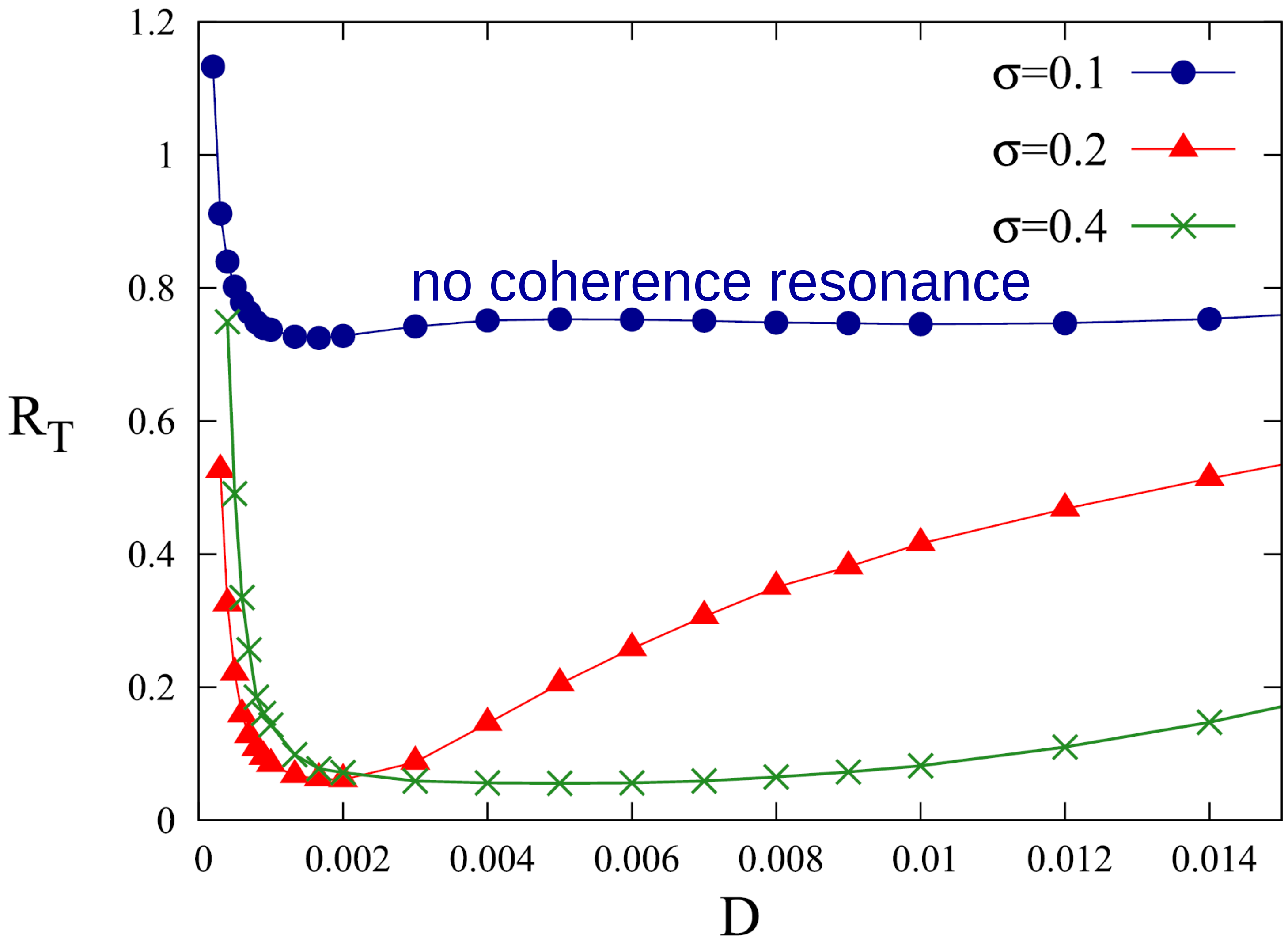
Isolated ring network



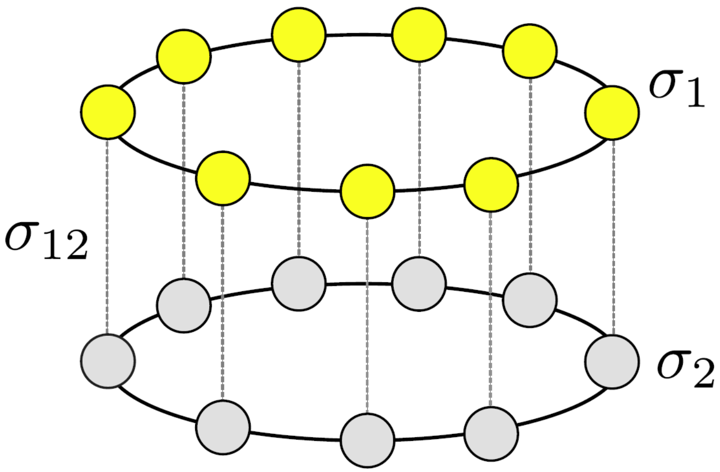


Isolated ring network

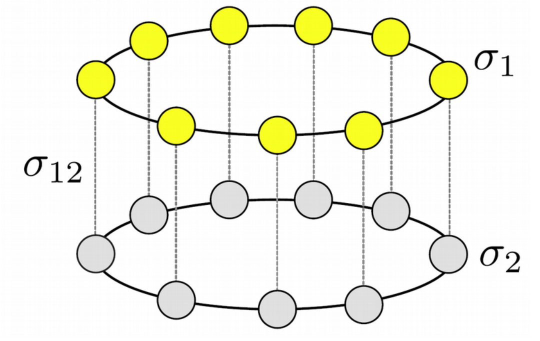


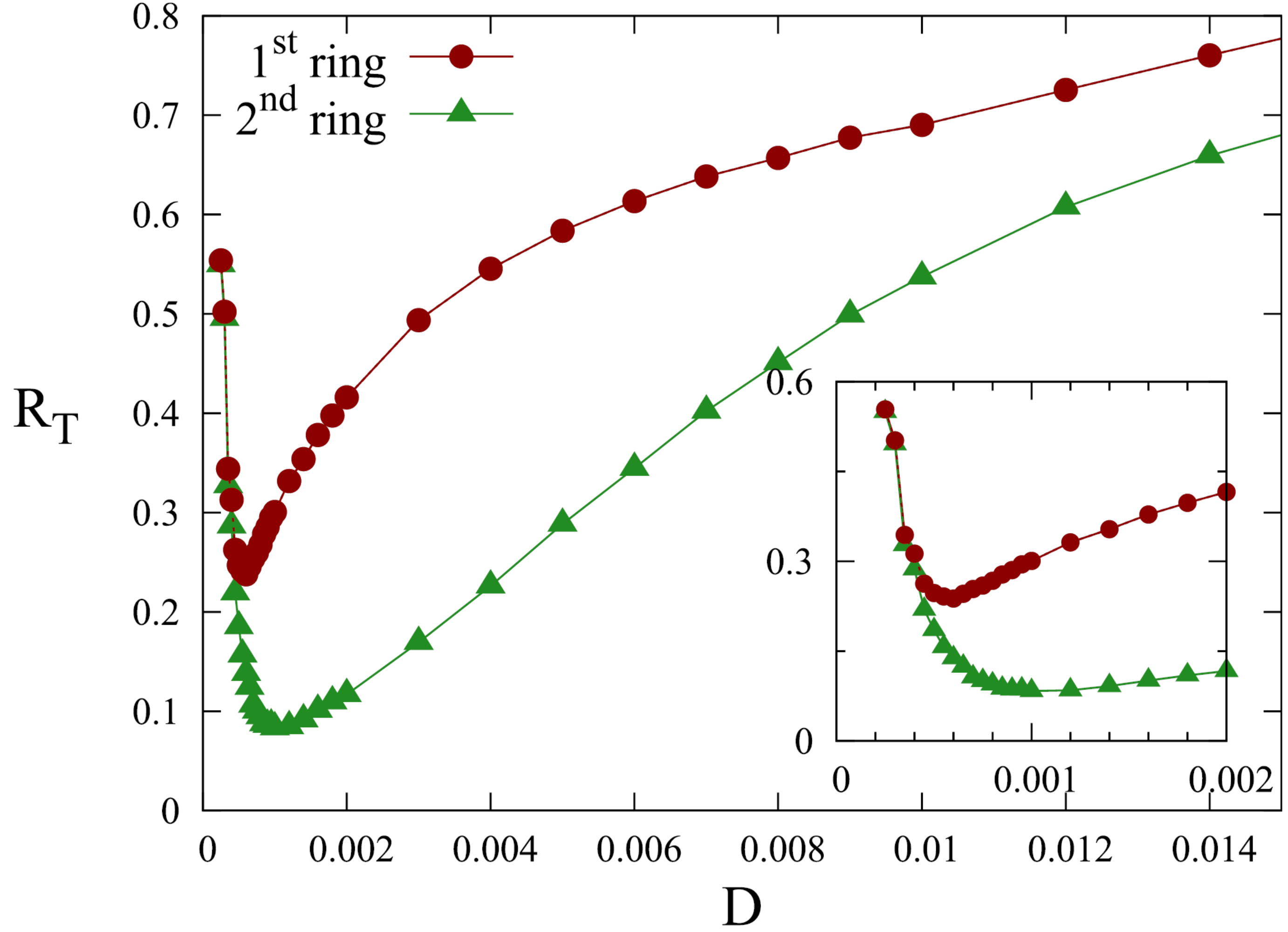


Multiplex network: coupling strength mismatch



Coupling strength mismatch





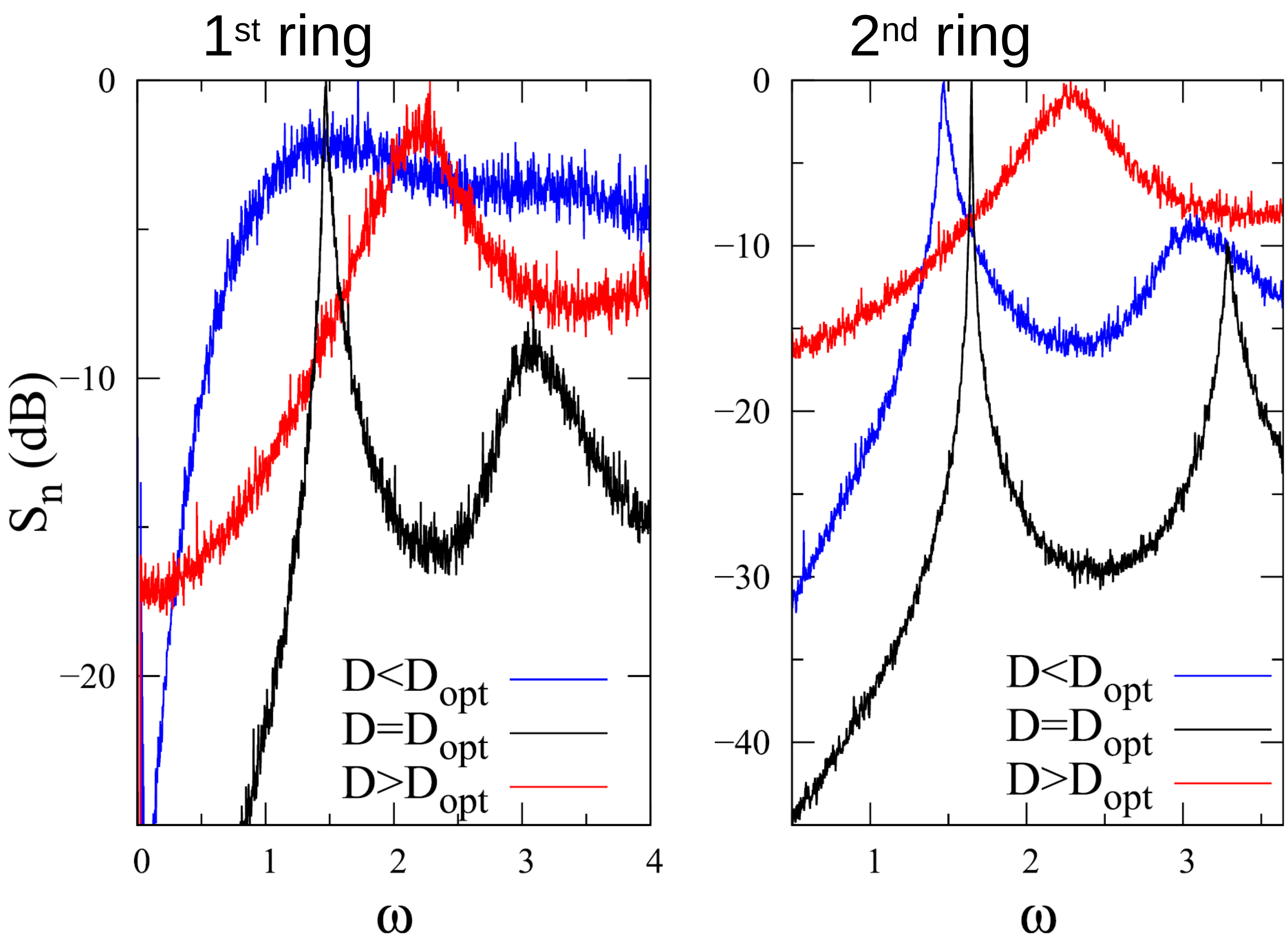
weak multiplexing
σ_{12} = 0.04

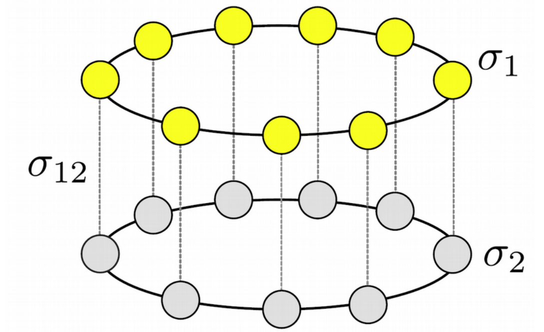
σ_1 = 0.1 (no CR in isolation)

σ_2 = 0.2 (optimal)

- Weak multiplexing induces coherence resonance

Coupling strength mismatch





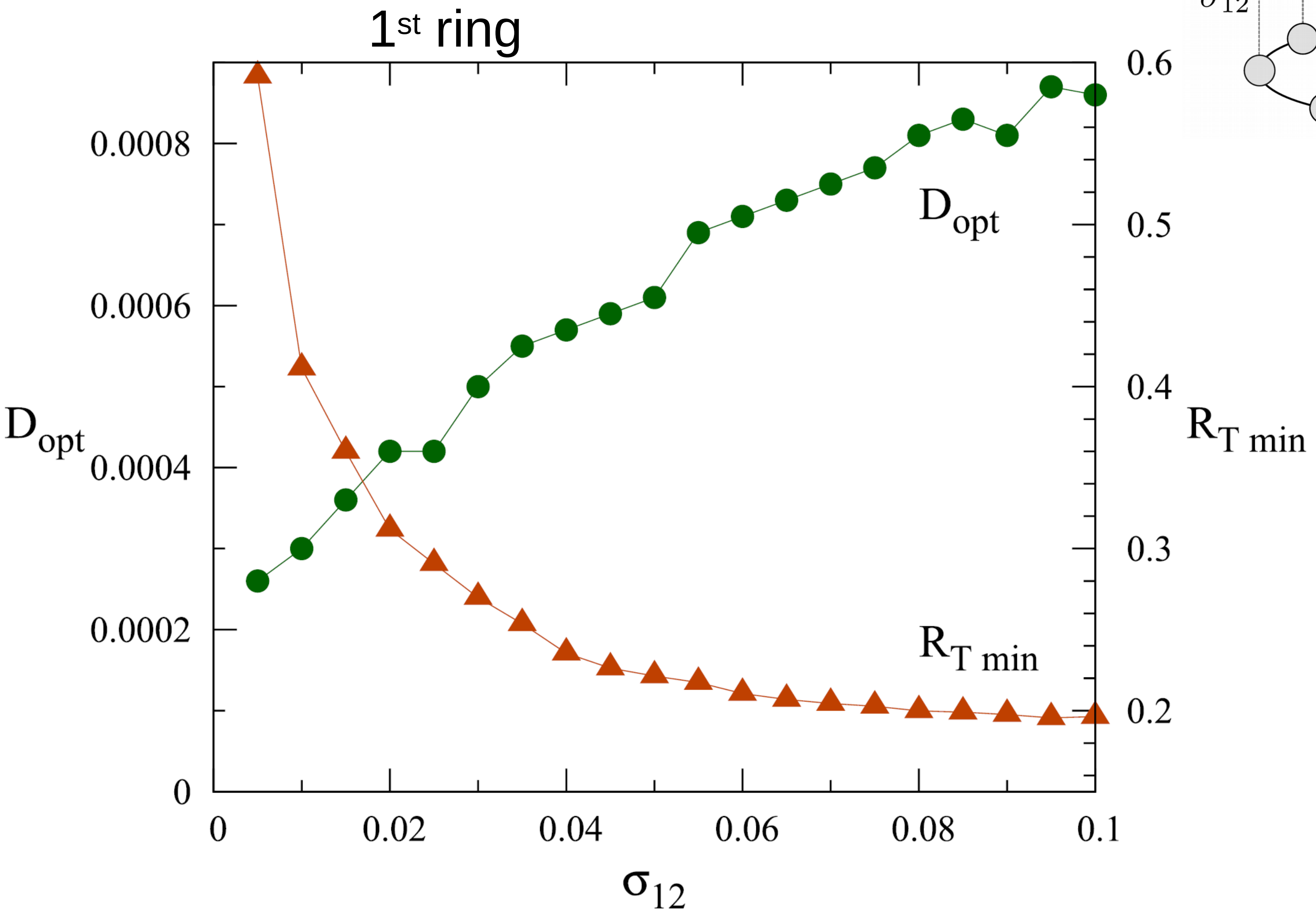
weak multiplexing $\sigma_{12} = 0.04$

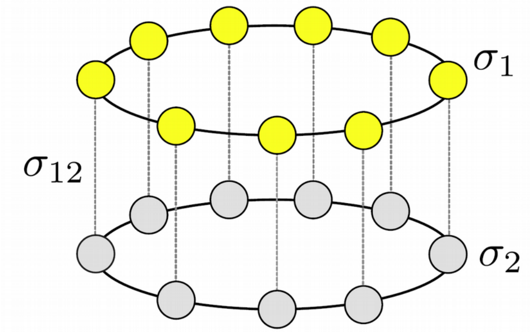
$\sigma_1 = 0.1$ (no CR in isolation)

$\sigma_2 = 0.2$ (optimal)

- Coherence resonance is better pronounced in the 2nd ring

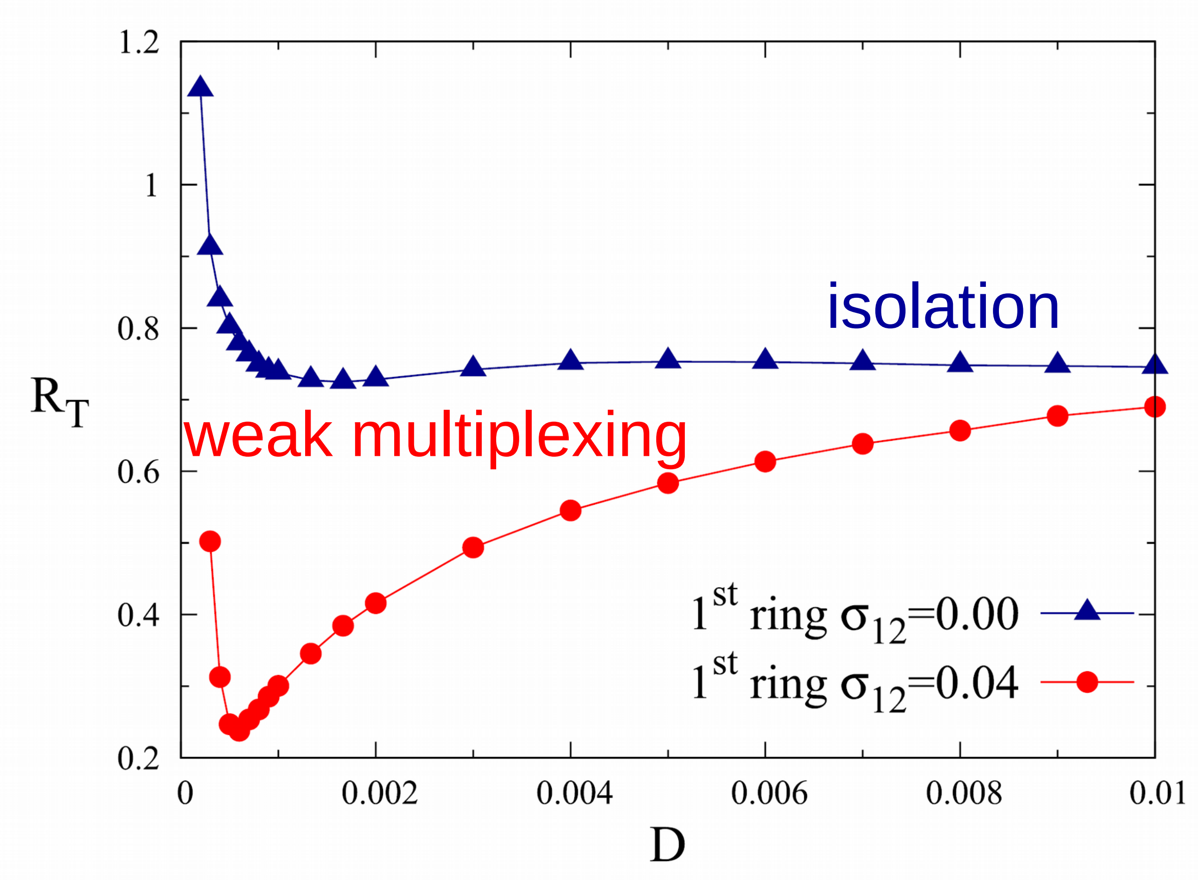
Coupling strength mismatch

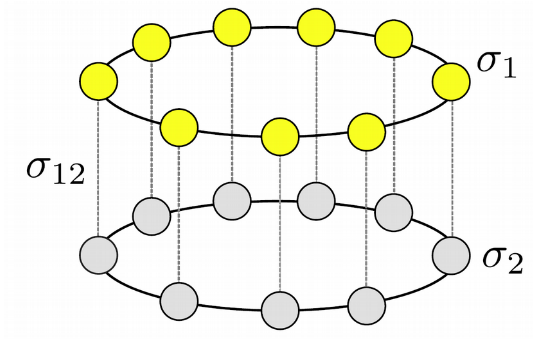




- Stronger multiplexing increases the coherence of oscillations in the 1st ring

Coupling strength mismatch





- Weak multiplexing induces coherence resonance

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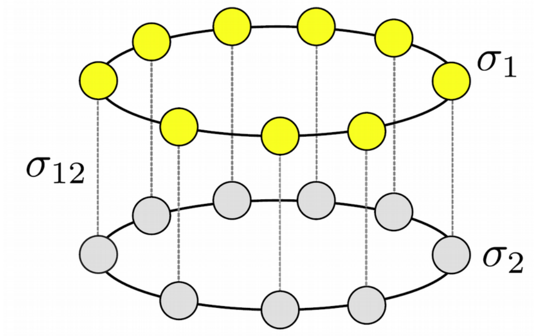
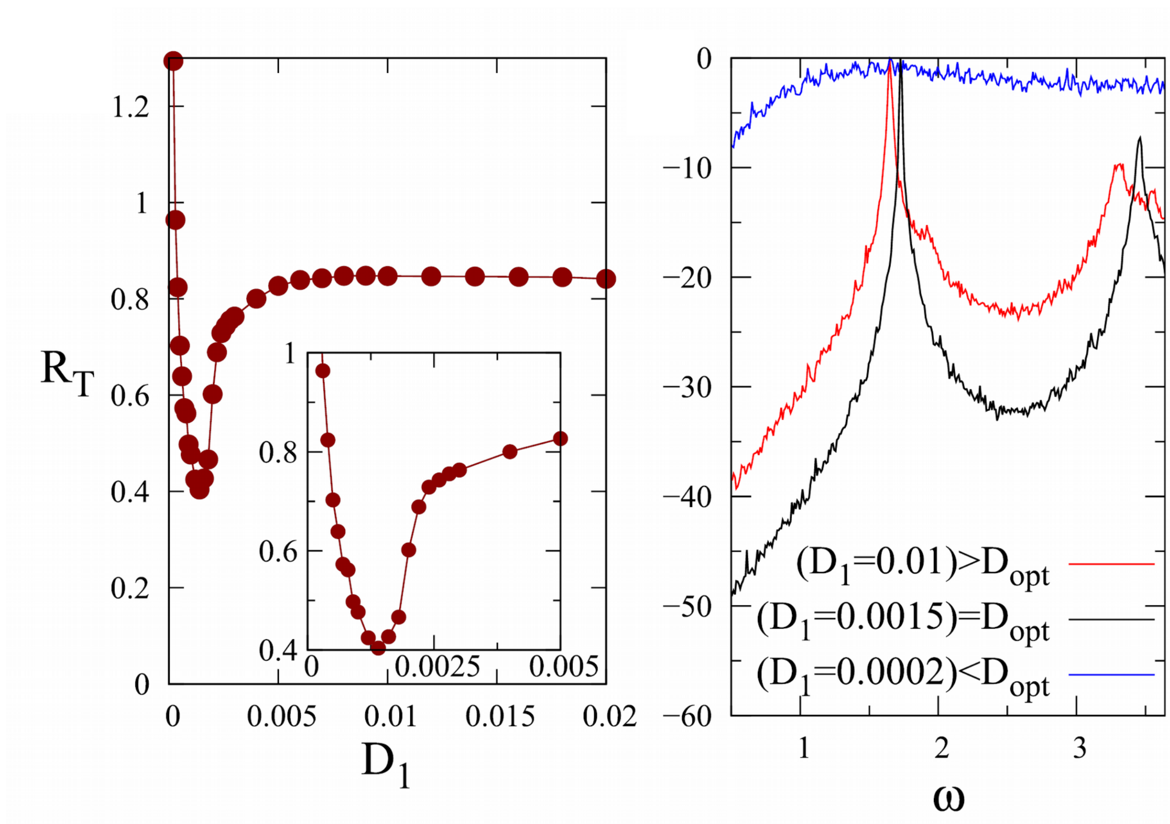
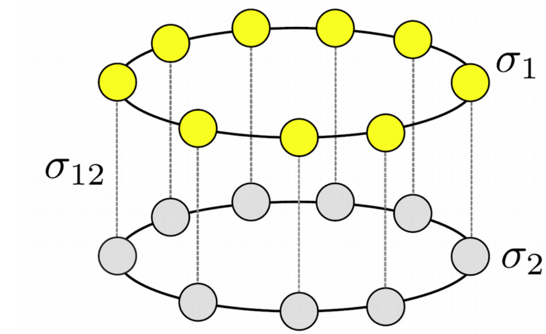


Deterministic layer
multiplexed with a noisy layer

Deterministic layer multiplexed with a noisy layer

2nd ring





weak multiplexing

$\sigma_{12} = 0.01$

$\sigma_1 = \sigma_2 = 0.1$

$D_2 = 0$

- Weak multiplexing induces coherence resonance in the deterministic layer

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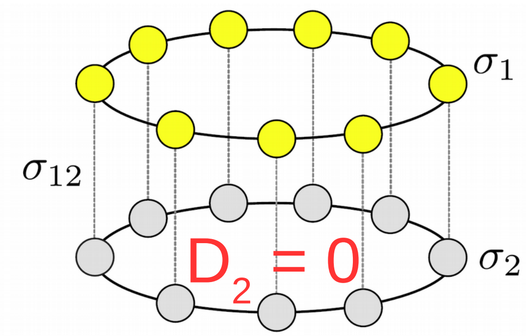
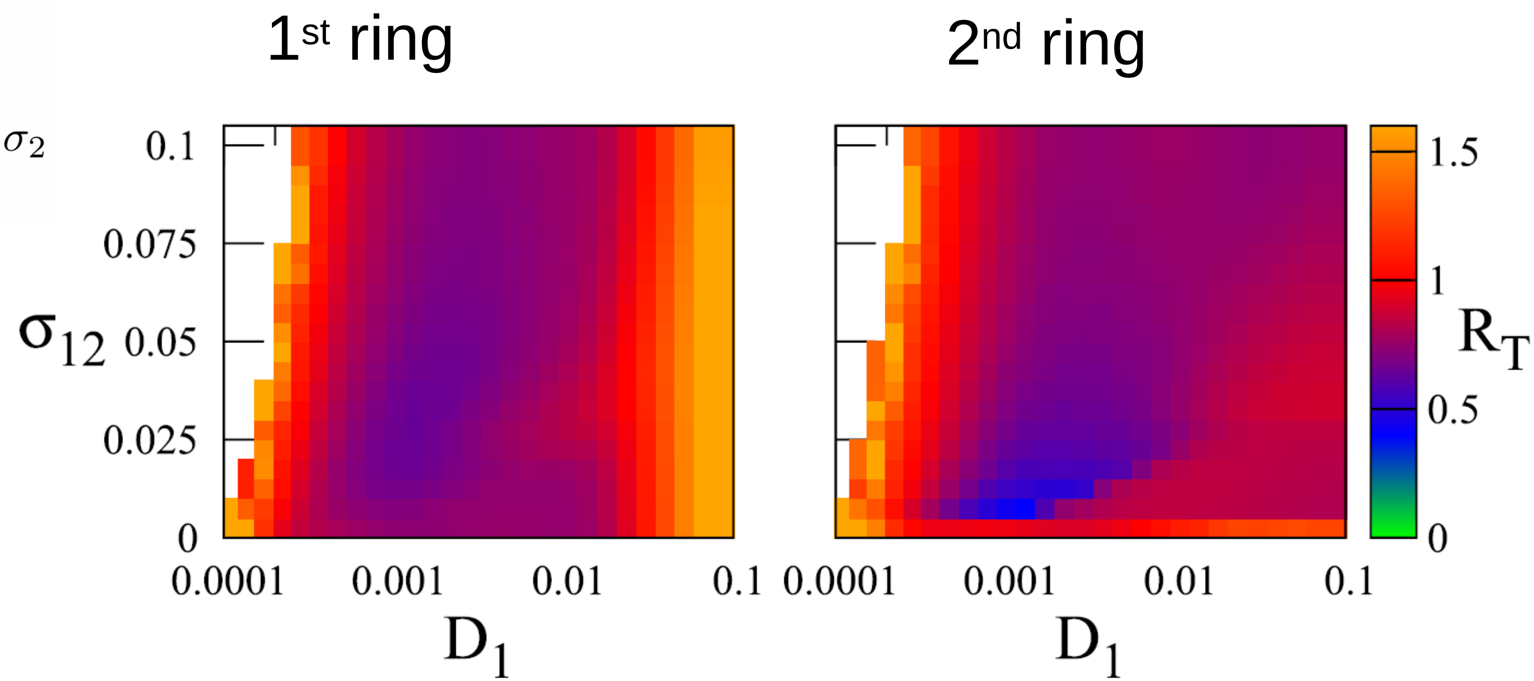
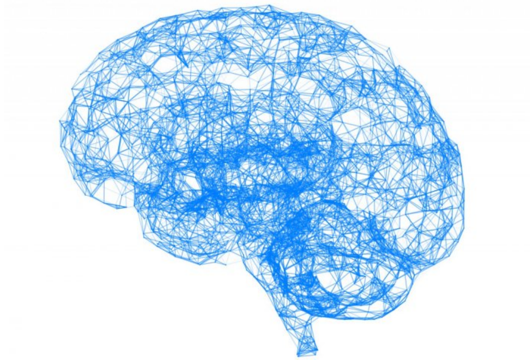
Deterministic layer multiplexed with a noisy layer

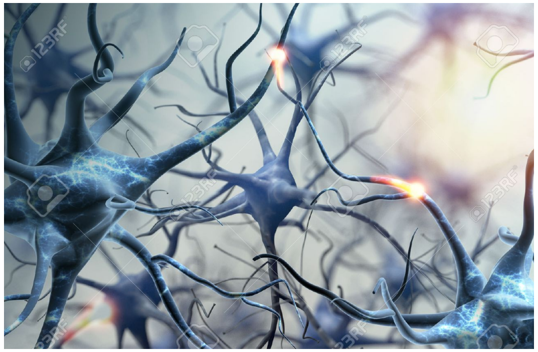




- Coherence resonance is more pronounced in the 2nd layer
- Stronger multiplexing shifts the minimum of R_T to larger values of noise
- Multiplexing induces coherence resonance for rather small values of σ_{12}

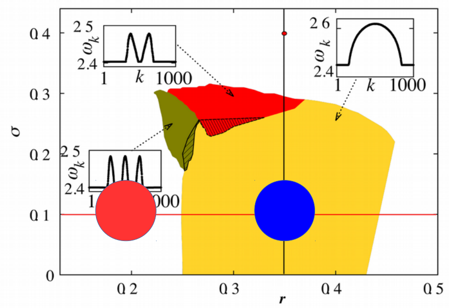


Conclusions

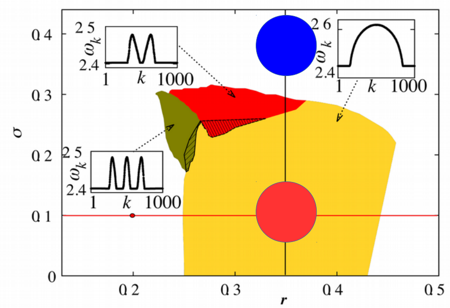


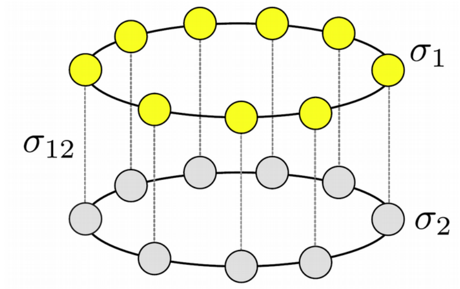
Weak multiplexing is a powerful method to control neural networks in both oscillatory and excitable regimes:

- induces chimeras with desired properties in the parameter regime where they do not occur in isolation

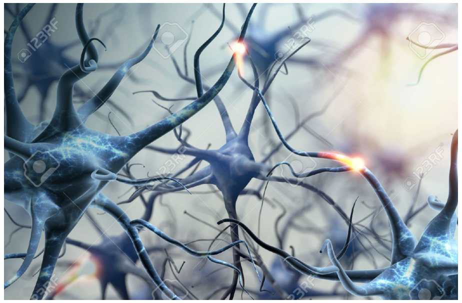


- suppresses chimeras in the parameter regimes where they occur in isolation and induces in-pase sync, two-headed chimeras, **solitary states**





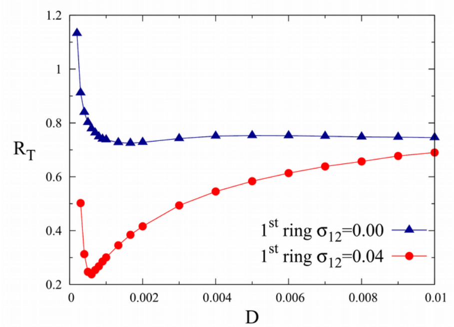
Conclusions

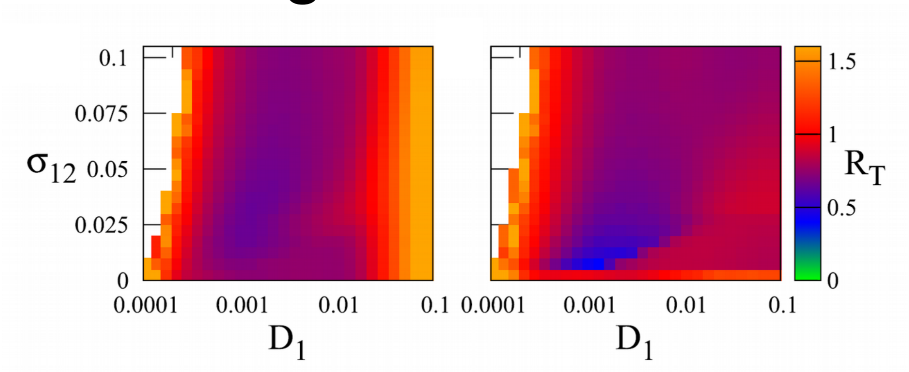


Weak multiplexing induces **coherence resonance** in the parameter regimes where it is absent for isolated networks

the coupling strength is not optimal

there is **no noise** noise exciting the elements



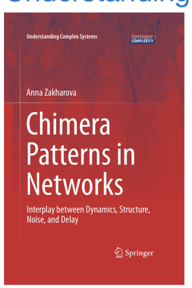


First book on chimera states

- To appear in 2019

» Physics » Complexity

Understanding Complex Systems



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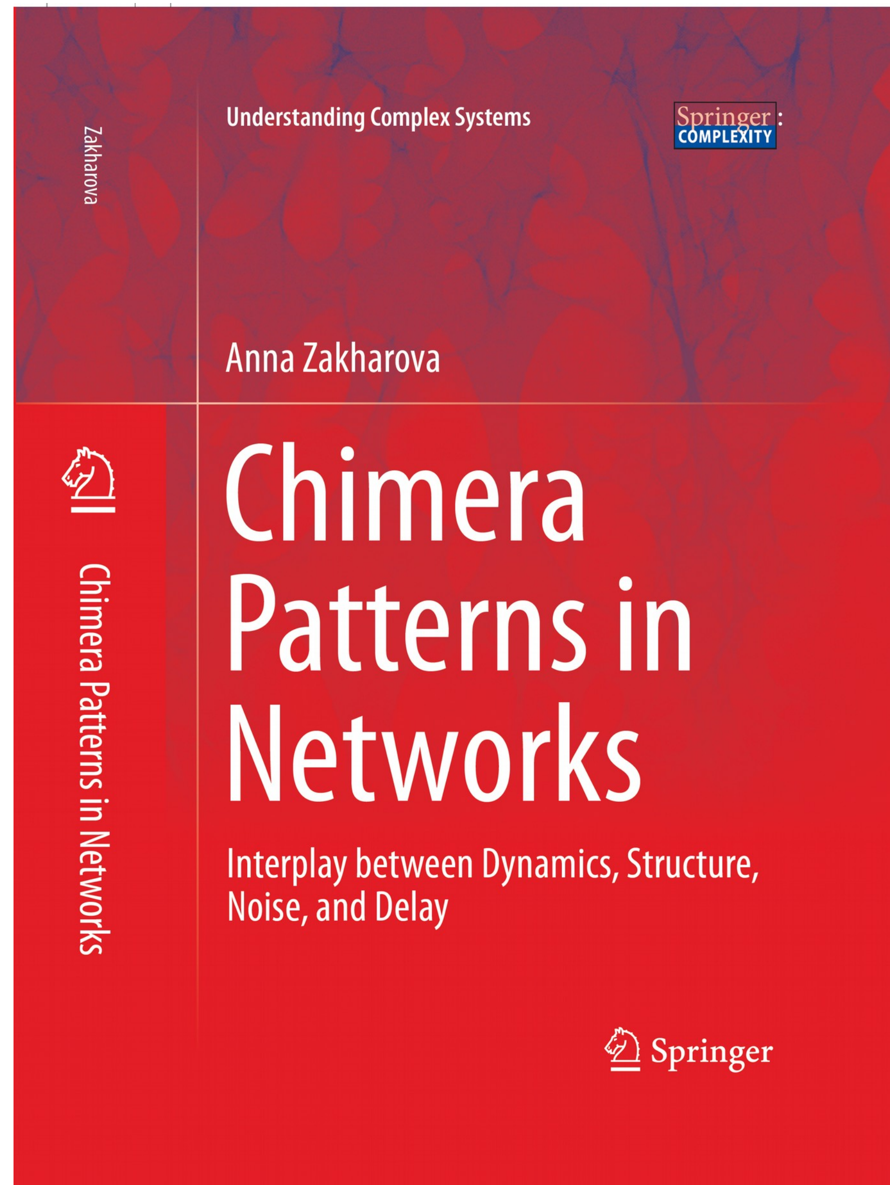

Thanks to my collaborators

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Sarika Jalan

Thank you!