# Spin glasses: the game

### Alexander K. Hartmann

#### Abstract

This document presents the rules of a tactical two-player boardgame which is inspired by spin glasses. The aim is, while placing bonds and spins, to achieve a majority of the spins facing the chosen direction of each player. The game has been already successfully used in university teaching but should be accessible to players from age 10 and up. (Note that there is a non-physics version of the rules available at www.compphys.uni-oldenburg.de which formulates the game in terms of a comptetion between party hosts.) Material is included such that a cheap version of the board game can be made using a color printer, scissors and glue. Alternatively, a professionally produced version of the game can be obtained online at a moderate price comparable to production costs.



## 1 Physical Background

Spin glasses are disordered magnetic alloys, for example iron-gold, which exhibit close to absolute zero temperature unusual properties. These materials, e.g., can remember their "magnetic history" they experienced at higher temperature, altough they seem not to be measurable at lower temperatures. The most important ingredient of spin glasses is that they exhibit ferromagnetic as well as antiferrmagnetic interactions. A ferromagnetic interaction connects spins such that equal orientations of the spins exhibit lower energies. On contrary, for antiferromagnetic interactions, opposite orientations of the spins are energetically favorable. Note that for all physical systems the basic principle holds that at low temperatures they converge to states of low energy.

Although thousands of scientists have worked on spin glasses during the past few decades and published their results in about 10000 scientific papers, still many fundamental properties of spin glasses are poorly understood.

Play the spin-glass game and learn to understand the principles of spin glasses, such that one day you can contribute to uncover the last secrets of spin glasses!

### 2 Game Material

The game contains the following items:

- These intructions
- Two different boards (which can be combined to make one big board)
- Two types of *pieces*:
  - 40 spins (two-coloured wooded discs with one white and one black side)
- 60 bonds (coloured wooden sticks) for ferromagnetic (blue, 40 pieces) and antiferromagnetic (red, 20 pieces) interactions
- one little sack
- two sets of each 6 action cards
- short instruction (in German) consisting of 4 cards
- eight additional cards with further information.

# 3 Aim of the game

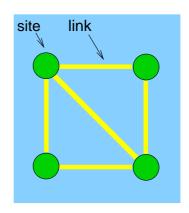
The game is for two players. One player ("player black") aims at having as many as possible of the spins being oriented in such a way that the black side is up. Correspondingly, the other player ("player white") tries to maximize the number of spins with an orientation such that the white side is up.

## 4 Game preparation

One of the boards is selected. The spins are placed next to the board, they form the *pool*. The bonds are put into the sack. Each player receives a complete set of 6 action cards, selects secretely 3 of them, and places them in front of him/her upside down, i.e., with the information side looking down such that it is not visible. The three other cards, respectively, are put concealed into the box.

### 5 The board

Both boards contain *sites* (circles) where during the games the spins are placed. The sites are connected by *links* where during the game the *bonds* are placed. The number of adjacent links per site is varying on the boards. At the boundary of the boards there are some "half" links, which are only used if both boards are joined to form a large board.

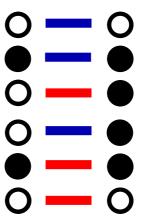


## 6 Basic principles/Denominations

A spin can be placed in two orientations, with the color white up or with the color black up.

Two spins which are connected by a ferromagnetic bond prefer to exhibit the same orientation (white/white or black/black). In the same way two spins joined by an antiferromagnetic bond prefer to take different orientations. In these two cases, one says the bond is *satisfied*. Then an *energy* of -1 is assigned to the bond.

In the opposite case a bond is called unsatisfied, i.e., if a ferromagnetic bond connects two spins of different orientations or if an antiferromagnetic bond connects two spins of equal orientation. To this bond an energy of +1 is assigned.



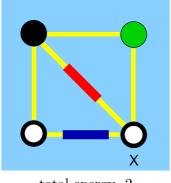
Remark: There are situations where it is impossible to satisfy all bonds, e.g., if three spins interact mutually (forming a triangle) via antiferromagnetic bonds.

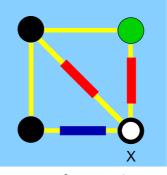
Bonds which are not adjacent to two spins are neutral (energy0).

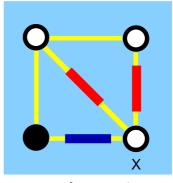
The most important quantity for each spin is the total energy of all bonds adjacent to the spin, i.e., the sum of the energy values. Links where (yet) no bonds are placed and also neutral bonds (where at least at one side no spin is placed) are not taken into account when calculating the total energy.

A spin is called *stable*, if for its adjacent bonds there are *more* satisfied than unsatisfied (total energy negative). A spin is called *unstable*, if for its adjacent bonds there are *less* satisfied than unsatisfied (total energy positive). A spin is called *free*, if for its adjacent bonds the number of satisfied and unsatisfied is *the same* (total energy zero). In particular a spin is free, if no bonds are adjacent.

Example: for the spin marked with 'X' we obtain:







total energy -2

total energy 0

total energy 3

## 7 The game

The players perform moves alternatingly. The junger one starts and gets assigned the color white. A move consits of *some* of the **steps** listed below. Each step can occur once, several times or zero times within one single move, while observing the basic rules mentioned here and on the next page:

- Blindly drawing a bond from the sack.
- Placing a bond on an unoccupied link, i.e., where so far no bond was placed, no matter what the orientation of the adjacent spins are, if there are any.
- Taking one spin from the pool.
- Placing a spin on an unoccupied site, i.e., where no spin was placed so far. The orientation (black/white) *must* be chosen such that the total energy of the spin is *NOT positive*, i.e., it may be negative or zero.
- Flipping of spins located on the board: a spin showing black on top before will show white on top afterwards, and vice versa. It is only allowed to flip spins, where the total energy (sum of energies) is positive or zero, i.e., where the spin is unstable or free.

Remark: spins exhibiting a positive total energy may be flipped but are not required to be flipped, in contrast to the strict rule when placing a spin. This freedom holds in particular if first a spin is placed on a site without adjacent bonds (all adjacent links are not occupied, leading to total energy zero) and next a bond is placed adjacent to the spin. (Also it may just happen and it is allowed that a player does not spot a spin which can be flipped, or forgets to flip a spin.)

**Basic rules**: For s standard move, a total of three pieces (bonds/spins) is taken from the sack and/or pool and placed on the board. This may be three bonds in one move, or one bond and two spins, etc. (Exceptions may occur when using action cards, see section 9.) All pieces taken within a move have to be placed on the board during the move.

The order of the steps within a move is arbitrary. *Example*: a player may first draw a bond and then take a spin from the pool. Next the player places the bond and then the spin, or the other way round. The player is also allowed to take a third piece first. During a move one or several spins may be flipped, if allowed, at any time. Also up to one action card may be used within one move any time.

A move is **finished** if a player has placed three pieces (spins/bonds) and announces that he or she is finished.

**Tactical advice**: Via placing bonds and via flipping spins it is possible to achieve that adjacent spins, which were stable before, become free or even unstable and thus can be flipped. This is the main mechanism to flip spins from the opponent's orientation to the own orientation.

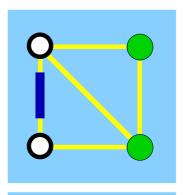
With this set of rules, together with the explenations of the action cards in section 9 and with the rules for the finishing the game (section 11), the game is completely described. For a better comprehendsion, next some examples are given, in particular for illustrating the tactical advice.

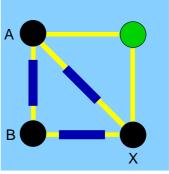
## 8 Example moves

Player white starts and draws one bond from the sack and happens to obtain a ferromagnetic (blue) bond.

The player places the bond on a link. Next, the player takes a spin from the pool and places it adjacent to the bond. Since the bond is still neutral, the spin is free and the player can chose the orientation of the spin. He/she choses the orientation white. The player takes another spin and places it on the other end of the bond. Due to the ferromagnetic bond and due to the white orientation of the first spin, now only the orientation white is possible. The situation shown to the right is obtained.

Now it is the turn of player black. He/she draws a bond from the sack and happens to obtain also a ferromagnetic (blue) bond. The player decides to draw another bond and obtaines by chance another ferromagnetic bond. Now the player first places a spin on the site marked by X. He/she is allowed to chose the orientation black since currently no bonds are adjancent to the site. Next, the player places the two ferromagnetic bonds between site X and the sites A and B which are already occupied by spins. Now the two white spins A and B have become free, since each of them is adjacent to one satisfied bond (connecting the spins



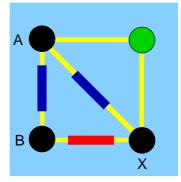


A and B) and one unsatisfied bond (adjacent to spin X). Player black now is allowed to flip any of the two free spins, e.g., A. Now spin B has become unstable and can be flipped as well. This results in the situation shown, where now all spins are oriented in favor of player black.

Remark: If player black had placed first a bond (or two) adjacent to the two white spins, he/she would have been forced to place the spin at site X with orientation white, because it is not allowed to place a spin with positive energy. Hence, the order of the steps must be chosen carefully.

Note that within the resulting situation the spins are more stable, since it needs at least two unsatisfied bonds to free and thus flip a spin. As we have seen in the previous example move, pairs (or chains) of spins can be flipped easily.

Remark: if player black had obtained not blue but insetad one red and one blue bond, he/she could have turned the two white spins A and B as well. In this case the situation shown to the right could have been obtained. Currently, spin A is stable. Nevertheless, spins X and B are free since both are adjacent to one satisfied and one unsatisfied bond. Thus, player white could in his/her move flip spin B. Now spin A has been freed, which can be flipped as well to white. Finally, spin X is free and can be flipped as well. All spins have been flipped from black to white. Nevertheless, the situation is still not stable, player black could



in the same way flip all three spins again. Therefore, all three spins are directly and indirectly (through neighboring free spins) free. Note that such spins will be ignored in the final evaluation of the game (see section 11).

### 9 Action cards

At the beginning of the game, each player receives a complete set of 6 action cards. From this set each player selects three cards, hidden from the view of the other player. There are the following action cards which can be used at *any time* during a move, but *at most one* during one move. After an action card has been used, it is put aside and cannot be used again during the same game.

For the "special move" card, the player has to take and place four pieces instead of three. Apart from this, the rules for a move do not change.



When using the "master move" card, the player must take and place five peices instead of three. Apart from this, the rules for a move do not change.



Using the "new link" action card, the player places a bond between two sites which are not connected by a link. It does not matter whether there are already spins placed on theses sites. It is like a new link is created between the two sites, where the bond is placed. Requirement: the imaginary straight line between the two sites must not touch existing links or sites. Remark: still the total number of pieces placed is exactly three wthin the move.



Using the "revision" card, the player removes an arbitrary bond and puts it back into the sack.

Remark 1: if the removed bond was previously placed via a "new link" card, this link is destroyed as well.

Remark 2: Via well planned usage of this card, some spins might become free and thus can be flipped.





When using the "double bond" card, the player puts one of the bonds played during the move next to an exisiting bond.

Remark: Both bonds contribute to the calculation of the energy. Thus, the corresponding link attains twice the normal importance, if both bonds placed on it are the same. If the two bonds are different, they cancel each other, i.e., they neutralized each other.



The "fixing" card allows a player to orient and fix an arbitrary spin on the borad in an arbitrary orientation, independent from the adjacent bonds and the neighboring spins. This spin is marked by putting another spin on the top of the spin. Still, the fixed spin is considered and counted as one spin. During the remaining game, the fixed spin is not allowed to flip, even not when using the other "fixing" card.

Remark: This is probably the strongest action card. Via a carfully chosen application, one might create a large cascade of flipped spins.

### 10 Finish

The game is finished when all sites are occupied by spins and all links are occupied by bonds. Thus, it might happen that during the last move only less than three pieces can be placed, if no unoccupied site or bond is available. In particular, during the last move a player might not have the choice of the pieces. It might happen, e.g., that there are only two sites left and no links, thus only two spins can be placed.

After the final move, no action card can be used, by any player.

### 11 Final evaluation

There might be spins which are free after the game has finsied, i.e., they may be flipped. In particular, there might be spins, which are *indiretly* free, i.e., they may be flipped after a free neighboring spin has been fliped. This may lead to cascades of free spins.

Remark 1: spins which are fixed are by definition not free.

Remark 2: spins which are adjacent to an odd number of bonds can never be free.

Remark 3: if during the evaluation unstable spins are detected (which have been overseen before), any player is allowed to flip them.

Thus, first the total set of directly and indirectly free spins is identified. They are removed in one strike. This means, one does not remove a free spin once it is identified, but only after all free spins have been detected.

After removal of the free spins, the spins of each orientation are counted. The player wins who has the majority of spins showing his/her orientation. If the number of spins showing the two different orientations is the same, the game ends in a draw.

### 12 Ideas for variants

- Both boards may be joined to form a large board. In this case each player gets assigned all six action cards (they need not to be kept concealed).
- The players are not forced to place the pieces immediately. Instead, they can be (partially) collected such that during a later move more than three pieces can be placed.
- One can remove chance completely from the game, if the bonds are not taken randomly from the sack. Instead, each player receives the same set of bonds from which he/she may select some during a move.
- Pro version: You can divide the game into two phases: First, only bonds are placed. After all bonds are placed, the spins are placed using the usual rules. Hence, when putting the bonds a player has to have already a good plan (which, on the other hand, may be exploited by the opponent).

# Enjoy the game!

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# Appendix

For trying the game, you might want to print it and cut the pieces out. Note that you can buy the professionally produced game for 14.50 Euros (which is slightly below the production costs!) plus pp at the University of Oldenburg via emailing to bisverlag@uni-oldenburg.de.

We suggest to print the spins below and glue them on coins, one side black, one side white.

