Asymptotically Fast Arithmetic in the Picard Group of Algebraic Curves

Matthias Junge

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Abstract

We present the asymptotically fastest algorithm for computing in the degree zero Picard group of algebraic curves which need not necessarily be smooth.
The degree zero Picard group contains geometric objects (isomorphism classes of degree zero vector bundles of rank one). In the case of elliptic curves it is isomorphic to the curve itself. However, in the case of higher arithmetic genus it is still of interest. We will show that we are able to represent its elements as ideals which themselves are free modules over $k[x]$. That is we can work with polynomial matrices and linear algebra to cope with the initially geometric problem.
Our algorithm unifies the fastest running times for smooth curves of constant gonality (Heß) and for smooth curves with gonality in the order of the genus (Khuri-Makdisi); moreover, it generalizes the former algorithms to the non-smooth case. The problem of computing in the degree zero Picard group can thus be solved using $O^{\sim}(n^{\omega-1}g)$ operations in the ground field where g denotes the arithmetic genus and n the gonality of the curve.