Extended tables and figures to the M-estimator paper

Peter Ruckdeschel
Mathematisches Institut
Universität Bayreuth
D-95440 Bayreuth
Germany
e-Mail: peter.ruckdeschel@uni-bayreuth.de

5th October 2005

1 Extended / additional Tables

In this document, you find extended tables to Ruckdeschel (2005).

1.1 Extended Table 1

Optimal clipping heights and corresponding (numerically) exact MSE

r		$n=5$	$n=10$	$n=30$	$n=50$	$n=100$	$n=\infty$
0.1	c_0	1.948	1.948	1.948	1.948	1.948	1.948
	$\mathrm{MSE}_n(c_0)$	1.508	1.290	1.166	1.138	1.112	1.054
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_0)$	8.679%	4.065%	1.340%	0.836%	0.448%	–
	c_1	1.394	1.484	1.611	1.663	1.724	1.948
	$\mathrm{MSE}_n(c_1)$	1.399	1.242	1.151	1.129	1.107	1.054
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_1)$	0.833%	0.207%	0.027%	0.014%	0.010%	–
	c_2	1.309	1.428	1.585	1.644	1.713	1.948
	$\mathrm{MSE}_n(c_2)$	1.392	1.240	1.151	1.129	1.107	1.054
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_2)$	0.332%	0.066%	0.008%	0.004%	0.006%	–
	c_{FZY}	1.368	1.370	1.610	1.668	1.756	1.939
	$\mathrm{MSE}_n(c_{\mathrm{FZY}})$	1.397	1.239	1.151	1.129	1.107	1.054
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_{\mathrm{FZY}})$	0.658%	0.002%	0.026%	0.021%	0.031%	–
	c_{ex}	1.167	1.358	1.560	1.630	1.704	–
	$\mathrm{MSE}_n(c_{\mathrm{ex}})$	1.388	1.239	1.151	1.129	1.107	–

r		$n=5$	$n=10$	$n=30$	$n=50$	$n=100$	$n=\infty$
0.25	c_0	1.339	1.339	1.339	1.339	1.339	1.339
	$\mathrm{MSE}_n(c_0)$	2.365	1.768	1.454	1.390	1.335	1.220
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_0)$	6.280%	3.681%	1.108%	0.656%	0.330%	–
	c_1	0.994	1.059	1.147	1.181	1.219	1.339
	$\mathrm{MSE}_n(c_1)$	2.246	1.713	1.438	1.381	1.330	1.220
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_1)$	0.933%	0.415%	0.055%	0.023%	0.009%	–
	c_2	0.890	0.990	1.114	1.159	1.207	1.339
	$\mathrm{MSE}_n(c_2)$	2.230	1.707	1.438	1.381	1.330	1.220
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_2)$	0.241%	0.104%	0.009%	0.002%	0.003%	–
	c_{FZY}	0.924	1.020	1.205	1.177	1.211	1.338
	$\mathrm{MSE}_n(c_{\mathrm{FZY}})$	2.234	1.709	1.441	1.381	1.330	1.220
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_{\mathrm{FZY}})$	0.417%	0.215%	0.233%	0.018%	0.002%	–
	c_{ex}	0.783	0.921	1.092	1.140	1.205	–
	$\mathrm{MSE}_n(c_{\mathrm{ex}})$	2.225	1.705	1.438	1.381	1.330	–
0.5	c_0	0.862	0.862	0.862	0.862	0.862	0.862
	$\mathrm{MSE}_n(c_0)$	4.768	3.120	2.180	2.017	1.883	1.636
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_0)$	2.930%	2.655%	0.792%	0.446%	0.218%	–
	c_1	0.650	0.690	0.746	0.767	0.790	0.862
	$\mathrm{MSE}_n(c_1)$	4.667	3.058	2.164	2.008	1.879	1.636
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_1)$	0.756%	0.615%	0.087%	0.036%	0.013%	–
	c_2	0.547	0.620	0.712	0.744	0.777	0.862
	$\mathrm{MSE}_n(c_2)$	4.643	3.045	2.163	2.008	1.879	1.636
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_2)$	0.230%	0.191%	0.015%	0.008%	0.003%	–
	c_{FZY}	0.539	0.632	0.716	0.749	0.782	0.866
	$\mathrm{MSE}_n(c_{\mathrm{FZY}})$	4.641	3.047	2.163	2.008	1.879	1.636
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_{\mathrm{FZY}})$	0.200%	0.248%	0.021%	0.011%	0.008%	–
	c_{ex}	0.413	0.531	0.686	0.728	0.770	–
	$\mathrm{MSE}_n(c_{\mathrm{ex}})$	4.632	3.039	2.162	2.008	1.879	–
1.0	c_0	0.436	0.436	0.436	0.436	0.436	0.436
	$\mathrm{MSE}_n(c_0)$	12.970	8.710	4.985	4.311	3.793	2.964
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_0)$	2.716%	3.132%	0.746%	0.348%	0.149%	–
	c_1	0.320	0.340	0.369	0.380	0.394	0.436
	$\mathrm{MSE}_n(c_1)$	12.805	8.581	4.961	4.299	3.788	2.964
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_1)$	1.411%	1.610%	0.251%	0.076%	0.021%	–
	c_2	0.255	0.291	0.342	0.361	0.382	0.436
	$\mathrm{MSE}_n(c_2)$	12.737	8.530	4.954	4.297	3.788	2.964
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_2)$	0.876%	0.999%	0.123%	0.027%	0.006%	–
	c_{FZY}	–	0.281	0.344	0.375	0.387	0.440
	$\mathrm{MSE}_n(c_{\mathrm{FZY}})$	–	8.521	4.955	4.298	3.788	2.964
	$\mathrm{relMSE}_n^{\mathrm{ex}}(c_{\mathrm{FZY}})$	–	0.892%	0.132%	0.063%	0.012%	–
	c_{ex}	0.001	0.125	0.286	0.334	0.366	–
	$\mathrm{MSE}_n(c_{\mathrm{ex}})$	12.627	8.445	4.948	4.296	3.787	–

c	order	determined by	optimal among
c_0	f-o-o	num. solution of (1.9)	all IC's
c_1	s-o-o	num. solution of (7.4)	all IC's acc. to (bmi),(D'),(Vb) and (C')
c_2	t-o-o	num. optimization of (3.22)	all Hampel-type IC's
c_{FZY}	—	num. optimization of ()	all (4.45)-type IC's
c_{ex}	—	num. optimization of the (num.) exact MSE	all Hampel-type IC's

where (7.4) is the s-o analogue to (1.9), which is derived in Corollary 7.2. A description to this table is located on page Ruckdeschel (2005).

1.2 Additional tables to subsection 5.3.1

emp., num., and as. MSE **at** $r = 0.1$**,** $c = c_0(r) = 1.9483$

n/ situation		simulation $\bar{S}_n$	[low; up]	numeric Algo C	Algo D	asymptotics n^0	$n^{-1/2}$	n^{-1}
5	id	0.981	[0.954 ;1.009]	1.008	1.007	1.012	1.012	1.007
	cont	1.471	[1.419 ;1.532]	1.501	1.612	1.054	1.292	1.331
10	id	1.001	[0.973 ;1.029]	1.010	1.009	1.012	1.012	1.010
	cont	1.288	[1.248 ;1.328]	1.290	1.296	1.054	1.222	1.242
30	id	1.028	[1.000 ;1.057]	1.011	1.011	1.012	1.012	1.011
	cont	1.192	[1.158 ;1.226]	1.165	1.167	1.054	1.151	1.158
50	id	1.027	[0.998 ;1.056]	–	1.011	1.012	1.012	1.012
	cont	1.142	[1.110 ;1.174]	–	1.138	1.054	1.129	1.133
100	id	0.984	[0.956 ;1.011]	–	1.010	1.012	1.012	1.012
	cont	1.081	[1.050 ;1.111]	–	1.111	1.054	1.107	1.109

emp., num., and as. MSE **at** $r = 0.5$**,** $c = c_0(r) = 0.862$

n/ situation		simulation $\bar{S}_n$	[low; up]	numeric Algo C	Algo D	asymptotics n^0	$n^{-1/2}$	n^{-1}
5	id	1.117	[1.086 ;1.148]	1.124	1.121	1.139	1.139	1.124
	cont	3.061	[2.962 ;3.161]	3.084	12.557	1.636	2.558	3.170
10	id	1.144	[1.112 ;1.177]	1.131	1.128	1.139	1.139	1.132
	cont	2.993	[2.893 ;3.093]	2.908	4.905	1.636	2.288	2.594
30	id	1.149	[1.117 ;1.180]	1.137	1.134	1.139	1.139	1.137
	cont	2.199	[2.135 ;2.263]	2.183	2.185	1.636	2.013	2.115
50	id	1.120	[1.089 ;1.151]	–	1.134	1.139	1.139	1.138
	cont	1.956	[1.903 ;2.009]	–	2.018	1.636	1.928	1.989
100	id	1.142	[1.111 ;1.174]	–	1.135	1.139	1.139	1.139
	cont	1.894	[1.845 ;1.944]	–	1.882	1.636	1.842	1.873

1.3 Additional table to subsection 5.3.2

emp., num., and as. MSE **at** $n = 50$**,** $c = 2.0$

r	simulation			numeric		asymptotics		
	$\bar{S}_n$	[low;	up]	Algo C	Algo D	n^0	$n^{-1/2}$	n^{-1}
0.00	1.001	[0.973 ;	1.029]	1.009	1.009	1.010	1.010	1.010
0.10	1.145	[1.112 ;	1.177]	1.139	1.141	1.054	1.132	1.136
0.25	1.518	[1.475 ;	1.560]	1.535	1.546	1.285	1.495	1.524
0.50	2.753	[2.682 ;	2.823]	2.824	2.866	2.108	2.645	2.775
1.00	8.620	[8.446 ;	8.794]	8.490	8.750	5.401	7.406	8.148

1.4 Extended / additional tables to subsection 5.3.3

emp., num., and as. MSE **at** $n = 100$**,** $r = 0.5$

estimator/		simulation			num	asymptotics		
situation		$\bar{S}_n$	[low;	up]	$*$	n^0	$n^{-1/2}$	n^{-1}
Med	id	1.562	[1.518 ;	1.605]	1.549	1.571	1.571	1.526
	cont	2.165	[2.106 ;	2.223]	2.171	1.963	2.241	2.251
$c = 0.5$	id	1.273	[1.237 ;	1.308]	1.258	1.263	1.263	1.262
	cont	1.926	[1.875 ;	1.978]	1.912	1.689	1.880	1.907
$c = 0.7$	id	1.192	[1.159 ;	1.226]	1.182	1.187	1.187	1.186
	cont	1.894	[1.844 ;	1.944]	1.879	1.647	1.844	1.873
$c = 1.0$	id	1.108	[1.078 ;	1.139]	1.103	1.107	1.107	1.107
	cont	1.913	[1.864 ;	1.963]	1.904	1.644	1.860	1.893
$c = 1.5$	id	1.035	[1.006 ;	1.063]	1.034	1.037	1.037	1.037
	cont	2.125	[2.072 ;	2.179]	2.133	1.786	2.063	2.108
$c = 2.0$	id	1.008	[0.980 ;	1.036]	1.009	1.010	1.010	1.010
	cont	2.569	[2.507 ;	2.632]	2.601	2.108	2.488	2.553
$c = c_0 = 0.8616$	id	1.142	[1.111 ;	1.174]	1.135	1.139	1.139	1.139
	cont	1.894	[1.845 ;	1.944]	1.882	1.636	1.842	1.873

emp., num., and as. relMSE **at** $n = 100$**,** $r = 0.5$ **relative to** $\mathrm{Var}[\bar X_n]$ **for** id **and** $\mathrm{MSE}(c_0(r))$ **for** cont**,** $c_0(r) = 0.8616$

estimator/ situation		simulation	numeric	asymptotics	
			$*$	n^0	$n^{-1/2}$
Med	id	1.367	1.365	1.379	1.379
	cont	1.142	1.154	1.200	1.153
$c = 0.5$	id	1.144	1.108	1.108	1.108
	cont	1.017	1.016	1.020	1.018
$c = 0.7$	id	1.044	1.041	1.041	1.041
	cont	0.999	0.999	1.001	1.000
$c = 1.0$	id	0.970	0.972	0.972	0.972
	cont	1.010	1.012	1.009	1.010
$c = 1.5$	id	0.906	0.911	0.910	0.910
	cont	1.122	1.134	1.120	1.125
$c = 2.0$	id	0.883	0.889	0.887	0.887
	cont	1.356	1.382	1.350	1.363

emp., num., and as. MSE **at** $n = 30$**,** $r = 0.25$

estimator/ situation		simulation		num	asymptotics		
		$\bar S_n$	[low; up]	ex	n^0	$n^{-1/2}$	n^{-1}
Med	id	1.492	[1.451 ;1.532]	1.501	1.571	1.571	1.496
	cont	1.786	[1.736 ;1.835]	1.779	1.669	1.821	1.767
$c = 0.5$	id	1.250	[1.216 ;1.284]	1.259	1.263	1.263	1.259
	cont	1.545	[1.502 ;1.588]	1.545	1.369	1.514	1.532
$c = 0.7$	id	1.175	[1.142 ;1.207]	1.183	1.187	1.187	1.184
	cont	1.482	[1.440 ;1.523]	1.483	1.302	1.450	1.469
$c = 1.0$	id	1.092	[1.062 ;1.122]	1.105	1.107	1.107	1.105
	cont	1.433	[1.393 ;1.473]	1.440	1.241	1.402	1.425
$c = 1.5$	id	1.018	[0.990 ;1.046]	1.036	1.037	1.037	1.036
	cont	1.462	[1.421 ;1.503]	1.478	1.224	1.426	1.458
$c = 2.0$	id	0.991	[0.963 ;1.018]	1.010	1.010	1.010	1.010
	cont	1.611	[1.566 ;1.656]	1.633	1.285	1.556	1.604
$c = c_0 = 1.3393$	id	1.035	[1.006 ;1.063]	1.051	1.139	1.053	1.052
	cont	1.438	[1.398 ;1.479]	1.452	1.220	1.405	1.434

emp., num., and as. relMSE **at** $n = 30$**,** $r = 0.25$ **relative to** $\mathrm{Var}[\bar X_n]$ **for** id **and** $\mathrm{MSE}(c_0(r))$ **for** cont**,** $c_0(r) = 1.3393$

estimator/ situation		simulation	numeric ex	asymptotics n^0	$n^{-1/2}$
Med	id	1.435	1.427	1.379	1.379
	cont	1.241	1.224	1.320	1.263
$c = 0.5$	id	1.202	1.197	1.199	1.198
	cont	1.073	1.064	1.077	1.068
$c = 0.7$	id	1.130	1.126	1.127	1.126
	cont	1.029	1.021	1.032	1.025
$c = 1.0$	id	1.051	1.051	1.051	1.051
	cont	0.995	0.991	0.998	0.994
$c = 1.5$	id	0.980	0.985	0.985	0.985
	cont	1.016	1.018	1.014	1.017
$c = 2.0$	id	0.953	0.960	0.959	0.960
	cont	1.119	1.125	1.107	1.119

1.5 Extended Table 8

Minimal n_0 such that for $n \geq n_0$ the relative error using first to third order asymptotics for approximating $\mathrm{MSE}_n(\psi_c)$ for $c = 0.7$ is smaller than 1% resp. 5%

rel.err	order	$r = 0.00$	$r = 0.10$	$r = 0.25$	$r = 0.50$	$r = 1.00$
1%	1st order asy.	9	$> 640^*$	$> 3927^*$	$> 14425^*$	$> 49220^*$
	$[\varepsilon]$	[0.00]	[3.95E−3]	[3.99E−3]	[4.16E−3]	[4.51E−3]
	2nd order asy.	9	15	60	196	$> 580^*$
	$[\varepsilon]$	[0.00]	[2.58E−2]	[3.23E−2]	[3.57E−2]	[4.15E−2]
	3rd order asy.	5	15	30	59	146
	$[\varepsilon]$	[0.00]	[2.58E−2]	[4.56E−2]	[6.51E−2]	[8.28E−2]
5%	1st order asy.	3	28	162	$> 590^*$	$> 1995^*$
	$[\varepsilon]$	[0.00]	[1.89E−2]	[1.96E−2]	[2.06E−2]	[2.24E−2]
	2nd order asy.	3	6	17	43	119
	$[\varepsilon]$	[0.00]	[4.08E−2]	[6.06E−2]	[7.62E−2]	[9.17E−2]
	3rd order asy.	3	6	12	23	49
	$[\varepsilon]$	[0.00]	[4.08E−2]	[7.21E−2]	[1.04E−1]	[1.43E−1]

The additional ε corresponds to the actual amount of contamination, i.e. $r/\sqrt{n_0}$.

1.6 Additional table to subsection 7.4

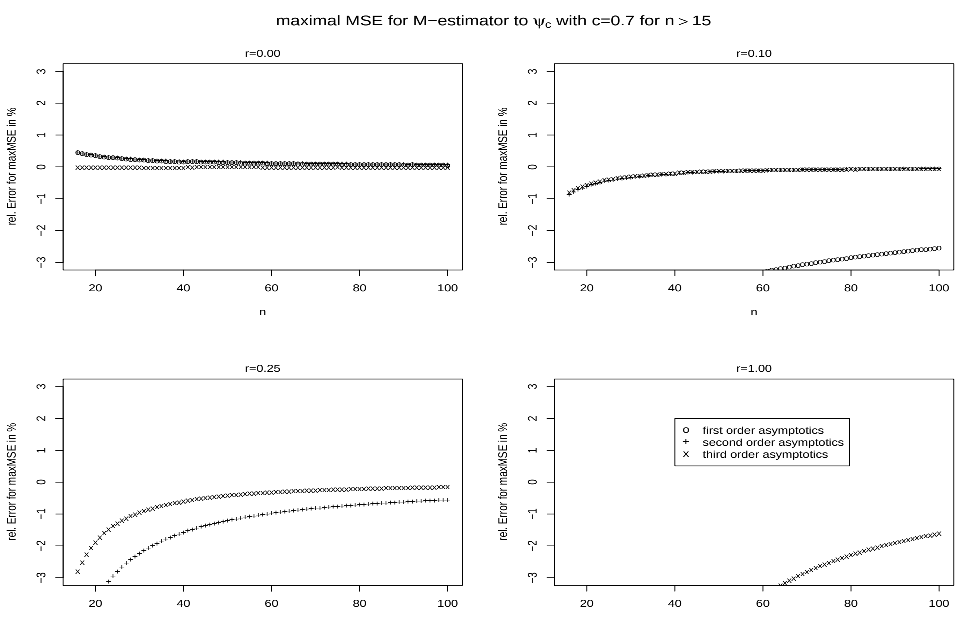
This table is Table 10 in Ruckdeschel (2005) for t-o risks. Again there is not much variation in both $c_2(r_\infty,\cdot)$, $\rho_{2;\gamma}(r_\gamma,\cdot)$ for varying n.

Minimax radii for second order asymptotics

		$n=5$	$n=10$	$n=30$	$n=50$	$n=100$	$n=\infty$
	r_γ	0.337	0.404	0.489	0.518	0.548	0.621
$\gamma=\infty$	$c_2(r_\gamma)$	0.742	0.733	0.725	0.722	0.721	0.718
	$\rho_{2;\gamma}(r_\gamma)$	15.58%	16.70%	17.56%	17.75%	17.89%	18.07%
	r_γ	0.384	0.436	0.492	0.507	0.525	0.548
$\gamma=3$	$c_2(r_\gamma)$	0.677	0.693	0.721	0.735	0.747	0.800
	$\rho_{2;\gamma}(r_\gamma)$	6.026%	6.708%	7.513%	7.784%	8.067%	8.836%
	r_γ	0.421	0.477	0.533	0.549	0.561	0.574
$\gamma=2$	$c_2(r_\gamma)$	0.632	0.644	0.675	0.688	0.707	0.770
	$\rho_{2;\gamma}(r_\gamma)$	2.869%	3.252%	3.703%	3.851%	4.005%	4.410%

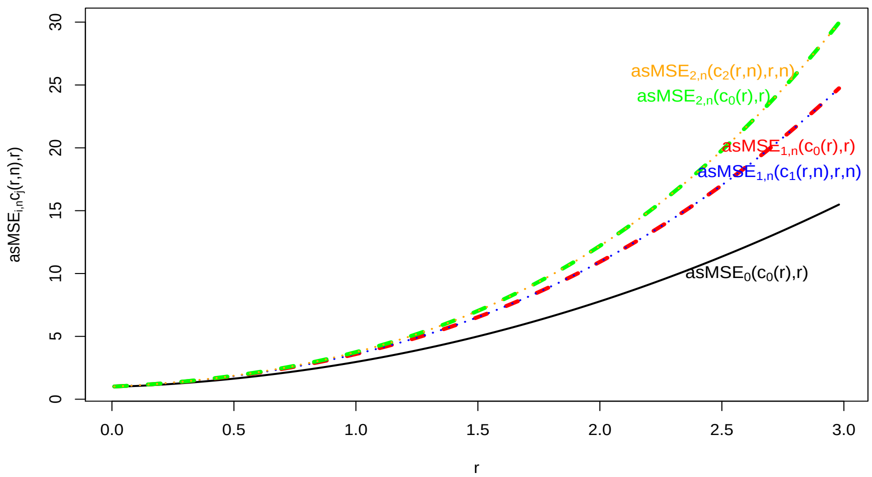
2 Additional Figures

2.1 Zoom into Figure 1

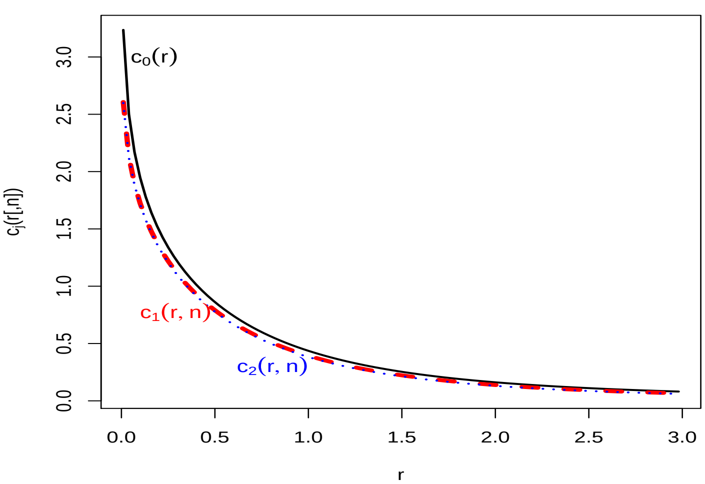


Zoom into the mapping $n \mapsto \mathrm{rel.error}(\mathrm{MSE}_n(\psi_c))$ **for** $c=0.7$ **and** $F=\mathcal{N}(0,1)$**.**

2.2 Additional figures to subsection 7.3.3



The mapping $r \mapsto \mathrm{asMSE}_{i[,n]}(\eta_{c_j(r[,n])}, r[,n])$ **for** $i = 0, 1, 2$**,** $j = 0, i$**,** $n = 100$ **and** $F = \mathcal{N}(0,1)$



The mapping $r \mapsto c_j(r[,n])$ **for** $j = 0, 1, 2$**,** $n = 100$ **and** $F = \mathcal{N}(0,1)$

References

Ruckdeschel P. (2005): Higher Order Asymptotics for the MSE of M-Estimators on Shrinking Neighborhoods. unpublished manuscript. Also available in
http://www.uni-bayreuth.de/departments/math/org/mathe7/RUCKDESCHEL/pubs/mest1.pdf
. 1, 1.1, 1.6