

Semiclassical quantization of collective excitations in a spatially inhomogeneous quantum plasma

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We study a quantum mechanical generalization of the Vlasov-Poisson system. In condensed matter physics, this system describes the collective excitations of electrons in metals, which are known as plasmons. When we consider a single material, our system of equations is translationally invariant and can be solved analytically using the Fourier transform. Experimentally, however, one typically uses a combination of different materials, which breaks the translational invariance. For these inhomogeneous media one often uses numerical approaches, which are however limited to smaller systems.

In this talk, I will show how we can use the semiclassical approximation to obtain an asymptotic solution when the medium is inhomogeneous. We regard the one-electron density operator as a pseudodifferential operator and determine its principal and subprincipal symbol from the equation of motion. With the principal symbol we derive a Hamilton-Jacobi equation, which describes the motion of plasmons in our material. We subsequently analyze the trajectories in phase space generated by the Hamiltonian flow.

We find that our system gives rise to two different types of classical turning points. The first of these corresponds to the conventional Lagrangian singularity A_2 , and consequentially the Maslov index can easily be determined. The second type of turning point is more peculiar, and does not correspond to one of the standard Lagrangian singularities. I will discuss the structure of the phase space curve in the vicinity of this point, and highlight the open questions regarding its nature. Finally, I will discuss the semiclassical quantization conditions and give an example of the plasmon spectrum.

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