Steering the Energy Transition in a World of Intermittent Electricity Supply: Optimal Subsidies and Taxes for Renewables and Storage*

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Abstract

Spurred by substantial subsidies, renewable energies have reduced their costs and captured a steadily growing market share. However, the variability of solar and wind power leads to new challenges for power systems. Policy instruments for steering the energy transition towards a zero-carbon future must account for this. We consider an economy in which competitive firms use pollutive fossils, intermittent renewables, and storage for electricity production. A Pigouvian tax is still efficient, because price fluctuations that result from intermittent renewables provide sufficient incentives to invest in storage capacities. However, governments have proved reluctant to impose carbon taxes. Therefore, we examine second-best subsidies when carbon pricing is imperfect. The optimal subsidy rate for renewables decreases as electricity production becomes less reliant on fossils. The optimal storage subsidy is usually negative as long as fossils are dispatched while filling the storage, but turns positive thereafter. This is because more storage capacity reduces the price when stored electricity is supplied to the market, but raises it when storage adds to demand. This has countervailing effects on firms' incentives to invest in fossil capacities. A numerical simulation illustrates that substantial subsidy payments are required even after fossils have been completely driven out of the market.

Keywords: intermittent renewable energies, electricity storage, carbon externality, subsidies, peak-load pricing, optimal control

JEL Classification: H23, Q42, Q58, O33.

1 Introduction

Dramatic cost reductions and substantial subsidies have created a worldwide boom of renewable energies, especially wind and solar power. In most parts of the world, they now have lower LCOE (levelised cost of electricity) than conventional fossil energies (IRENA, 2019). However, electricity from wind

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and solar power varies over time and depends on weather conditions. As its share rises, this intermittency becomes the dominating obstacle for transitioning to an energy system based primarily on renewables because supply must match demand at any time. For periods of low renewables availability, either a reduction of electricity demand or a back-up with fossil and/or electricity storage capacities is needed. By contrast, if their availability is high, they may produce more electricity than the market can absorb, leading to zero (or even negative) prices. For example, in Germany the number of intervals with more than six hours of negative prices more than doubled from 55 in 2016 to 123 in 2019, whereas installed wind and solar capacity increased only from 90 to 110 GW.[1] This makes renewables more costly than it appears on the basis of their LCOE, and it hampers their further market penetration. While this problem is now widely acknowledged, there is little research on how renewable support policies, which were initially targeted at inducing innovation and costs reductions, have to be adjusted as renewables make up an ever increasing share of our electricity supply. This paper aims contributing to close this gap.

An obvious response to the intermittency problem are measures to reduce its extent. This includes technological improvements such as taller wind turbines and sun tracking solar panels, enhanced power transmission grids to exploit spatial differences in the availability of intermittent renewables, as well as a mix of renewable energies whose temporal variations complement each other. However, such measure will not be sufficient as renewables continue to grow. Therefore, it is widely perceived that electricity storage is an essential part of the solution. For example, IRENA (2017) calculates that electricity storage capacity need to grow from an estimated 4.67 TWh in 2017 to at least 11.89 TWh if the share of renewable energy in the energy system doubles by 2030. Such storage will probably be a mix of traditional pumped hydro storage, small- (as in electric vehicles) and large-scale batteries, power-to-gas (mainly hydrogen), and compressed air storage. Since the deployment of pumped hydro storage is limited (Gimeno-Gutiérrez and Lacal-Arántegui, 2015; Sinn, 2017), its share is projected to fall from 97% to 51% in 2030 (IRENA, 2017), and much of the build-up must come from technologies like batteries that are not competitive yet.[2]

In this paper, we analyse the optimal policy mix for supporting renewables as well as storage to counterbalance their intermittency. In particular, we examine how these policies vary as the share of renewables grows and, thus, the extent of intermittent supply that they bring with them. For illustration, consider an initially low share of renewables. At times of high availability, they would make part of the fossil capacities redundant. In this case, adding storage capacities would absorb electricity from these idle fossil capacities and allow them to continue operating. This improves the profitability of fossils and the optimal policy intervention is a tax on storage. By contrast, once having attained a high enough share in the power system, renewables provide sufficient electricity to fill the storage. In addition, storage replaces fossil capacities as a back-up for times with low availability of renewables. The optimal policy intervention is now a subsidy for storage.

For our analysis, we build on the standard peak-load pricing model and extent it to an economy with three types of firms: those that produce with a polluting fossil energy, those that use carbon-neutral but intermittent renewable energies, and those that engage in electricity storage. We assume that intermittency follows a repetitive pattern—like a day-night cycle of solar power—and is perfectly forecastable. The latter is consistent with the empirical findings of Gowrisankaran, Reynolds, and Samano (2016) that social costs of unforecastable intermittency are small in comparison to those of intermittency overall. Moreover, short-term forecasts over a day-night cycle are quite accurate (e.g., Iversen, Morales, Møller, and Madsen, 2016), and seasonal wind availability is, at least in the historic average, well known.

First, firms make long-term investments in their respective capacities. Thereafter, they produce

[1] See https://www.smard.de/page/en/topic-article/5892/15618.

[2] Costs of batteries fell by 22% from 2016 to 2017 (https://www.iea.org/tcep/energy-integration/energystorage/). Schmidt, Hawkes, Gambhir, and Staffell (2017) predict (using experience curves) that battery storage will be competitive in the next 10 (electric vehicle transportation) to 20 years (residential energy storage) (see Kittner, Lill, and Kammen (2017) for similar predictions), although other studies are less optimistic (e.g., Brouwer, van den Broek, Zappa, Turkenburg, and Faaij, 2016).

electricity and interact with consumers in a perfectly competitive market. Storage firms have a dual role. They buy electricity—that is, act like consumers—at the low prices that prevail during times of high availability of renewables, but supply electricity at the high prices that obtain during times of low availability. This exploitation of price differences, which we model as an optimal control problem, as well as the increasing role of flexible pricing schemes motivate our assumption of dynamic pricing.[3]

The first policy that we consider is a Pigouvian tax per unit of carbon emissions from fossils. Given our assumptions of competitive markets, dynamic pricing, and lump-sum taxation, this instrument would implement the first-best solution; at least as long as there is no other externality such as R&D spillovers that requires a separate intervention. Since we neglect dynamic aspects of resource extraction and of the climate system, the tax would even be constant for constant marginal damage costs (see Lemoine and Rudik (2017) for dynamic taxing schemes).

However, a Pigouvian tax may not be implementable, especially due to political economy reasons.[4] For example, after popular protests by the "Yellow Vests" movement the French government suspended its plans to raise the carbon tax from 44.6 €/ton in 2018 to 86.2 €/ton by 2022 (Douenne and Fabre, 2021). Indeed, most countries have relied on a mix of putting a price on carbon and extensive subsidies for renewable energies. Abrell, Kosch, and Rausch (2019) estimate that for the period 2011-2015 the implicit abatement costs for reducing CO_2 through subsidies for wind have been in the range of 105 to 276 €/ton for Germany and 82 to 258 €/ton for Spain, with even substantially larger values for solar. By contrast, the carbon price in the EU ETS was only around 5 to 10 €/ton in this period.

Therefore, our analysis focuses on subsidies for renewables and storage technologies, which reduce pollution only indirectly. First, more renewables capacities lower the expected electricity price and, thus, incentives to invest in fossils. Second, fossil capacities may remain unused when the availability of intermittent renewables, which have lower variable costs, is high. As the share of fossils in the energy system falls, these effects have less impact and the subsidy rate for renewables should be reduced.

Storage capacities even out the intermittent supply of renewables and, thereby, alleviate their main disadvantage. But this intuition for subsidising storage is flawed. If the level of renewables is efficient—as under a Pigouvian tax—the market provides sufficient incentives to exploit price variations by investing in storage. If it is inefficiently low, then directly subsidising renewables, rather than storage, is a better policy.

Nevertheless, there is a rationale for incentivising storage due to its effects on fossils. Storage reduces the electricity price when stored energy is supplied to the market, and raises it when the storage is filled. This has countervailing effects on average electricity prices and, therefore, on the incentives to invest in fossil capacities. Due to round-trip efficiency losses during a storage cycle, more electricity has to be taken from the market than can be supplied to it during times of destorage.[5] Therefore, as long as fossils contribute to electricity production during times of storage, the price increasing effect dominates and storage capacities raise the profitability of fossils. Hence storage should be taxed. Once renewable capacities are sufficient to fill the storage, fossils no longer benefit from the price increasing effect and it becomes optimal to subsidise storage. This subsidy is constant until fossils are no longer used; under the same conditions that lead to a decreasing renewable subsidy. Roughly speaking, as the market share of fossil energies falls, it is optimal to gradually shift from the subsidisation of renewables to subsidising storage.

It has been argued that optimal subsidies for renewables (and storage) even implement the first-best solution, provided that they are complemented by a consumption tax (Abrell, Rausch, and Streitberger 2019, Ambec and Crampes 2019). We show that this finding does no longer hold once renewables

[3] Dynamic pricing of electricity is still often restricted to larger commercial customers (e.g., Borenstein and Holland, 2005; Joskow and Wolfram, 2012), but according to Helm and Mier (2019) this may be sufficient to create appropriate price signals. Moreover, recent technological advances have dramatically lowered the costs of smart metering technologies, and many regions have set ambitious targets for their deployment (e.g., in the EU Third Energy Package). In addition, several studies have found evidence that households actually do respond to higher electricity prices by reducing usage (e.g., Faruqui and Sergici, 2010; Jessoe and Rapson, 2014).

[4] The literature discusses equity issues (e.g., Polinsky, 1979), lobbying and rent seeking (e.g., Fredriksson, 1997), and distributional implications (see Goulder and Parry (2008) for a discussion and Reguant (2019) for empirical evidence).

[5] Round-trip efficiency is usually in the range of 65 to 90 per cent, depending on storage technology (IRENA, 2017).

have reached a capacity where supply at times of high availability exceeds electricity demand at a price of zero.[6]

The analytical model is restricted to the most interesting case where all three technologies are used. However, in a numerical simulation we also consider cost parameters for which fossils are no longer employed in the efficient solution. We find that implementing this as a decentralised solution still requires substantial subsidies in order to keep fossils out of the market.

The remainder of the paper is structured as follows. After discussing the related literature in Section 2, we introduce the model and the timing of decisions (Section 3). We then examine electricity production and storage decisions (Section 4), capacity choices (Section 5), and policy instruments (Section 6). A numerical simulation in Section 7 illustrates and extends the results. Section 8 concludes, and an appendix contains the proofs.

2 Related Literature

The first strand of literature to which our paper connects is the economics of intermittent sources of electricity production, of which Ambec and Crampes (2012, p. 321) wrote some years ago that they are "still in their infancy". Since then, the literature has grown substantially, but most contributions rely heavily on numerical simulations (e.g., Després, Mima, Kitous, Criqui, Hadjsaid, and Noirot, 2017) or are empirical (e.g., Abrell et al., 2019; Liski and Vehviläinen, 2020). Ambec and Crampes (2019) share our focus on optimal support policies, and they also consider a storage technology. However, the storage pattern is trivial because the availability of renewables is restricted to be binary, i.e., either 0 or 1. This binary pattern and their assumption of non-reactive consumer demand imply that fossils never contribute to filling the storage.

Abrell et al. (2019), using a simulation and a simpler analytical model, analyse a larger set of renewables support policies but abstract from storage. More importantly, they deviate from the standard peak-load pricing paradigm by not distinguishing between production and capacity choices of fossil energies. This neglects that fossils need prices above their marginal costs to recoup capacity costs, which becomes increasingly difficult as capacities are underutilised more often when supply from renewable energies rises. Like Fell and Linn (2013), the authors include two renewable technologies (wind and solar) with different times of binary availability. Our model could be extended relatively straightforwardly to several renewable technologies too, but this would raise the notational complexity. Moreover, the main effects are very intuitive; hence we only provide an informal discussion in the concluding section. Andor and Voss (2016) also consider subsidies for renewables, but their model includes neither fossils nor a storage technology. Finally, Helm and Mier (2019) use a peak-load pricing model with a very general intermittency pattern similar to this paper. However, they do not account for storage and do not examine policy instruments.

Another strand of literature to which this paper relates is the economics of storage. Traditional applications include balancing stochastic production disturbances in agriculture (e.g., Newbery and Stiglitz, 1979; Wright and Williams, 1984) and the combination of thermal capacity with mainly pumped hydro storage (e.g., Crampes and Moreaux, 2001). In a seminal contribution, Gravelle (1976) studies the implications of storage for peak-load pricing with variable demand. He finds that peak consumption increases less than off-peak production increases, due to round-trip losses of storage. This is similar to the effect of storage during times with high and low availability of intermittent renewables in our model. More recently, the focus has shifted toward the role of pumped storage as a natural complement to the intermittency of renewables (e.g., Crampes and Moreaux, 2010; Heal, 2016; Schmalensee, 2019). Similar to us, Steffen and Weber (2013) determine optimal capacity investments, but only for the fossil and storage technologies. They then use a load duration curve to determine the effect of intermittent renewable energies and demonstrate their results numerically by using a case study for Germany. In a related contribution, Steffen and Weber (2016) use optimal control

[6] Such situations of excess generation, that countries with a high market penetration of renewables already experience occasionally, cannot obtain in the different model set-ups of the just mentioned contributions.

theory to provide a more precise representation of storage dynamics. However, like Horsley and Wrobel (2002), they only consider the problem of an individual storage firm, and they focus on differences between large (unconstrained) and small (constrained) reservoirs. Durmaz (2014) uses discrete time and dynamic programming to determine the optimal storage pattern. However, he does not consider policy instruments and his problem is analytically not fully tractable. Finally, Pommeret and Schubert (2019) also integrate storage into a model with electricity production from renewable and fossil technologies. Their focus is on the optimal allocation of a fixed carbon budget over time, whereas the availability of sufficient storage capacities is taken as exogenously given.

Our paper also contributes to the more general literature on second-best policies and the ranking of policy instruments to incentivize pollution abatement. For a given abatement cost function, pollution taxes and abatement subsidies are usually seen as equivalent in the short run, whereas in the long run subsidies lead to excessive firm entry (e.g., Kohn, 1992). In an extension of this literature that is more similar to our approach, firms can decide whether to incur the fixed cost of a new technology that reduces costs of emission abatement. In this framework, taxes on emissions and subsidies for emission abatement are usually equivalent (e.g., Milliman and Prince, 1989; Requate and Unold, 2003). Although this literature is often motivated by the problem of mitigating CO_2 emissions, specific aspects of energy markets such as the intermittency of renewables and their interaction with storage are usually neglected (see also Fischer, Preonas, and Newell, 2017). We show that accounting for them fundamentally affects the comparison of instruments, as it compromises the efficiency of subsidies, but not that of a Pigouvian tax. Recent empirical findings point in the same direction (Gugler, Haxhimusa, and Liebensteiner, 2021).

In accordance with our results, there is a broad consensus that no additional subsidies are necessary to tackle an environmental externality if perfect carbon taxation is possible (Golosov, Hassler, Krusell, and Tsyvinski, 2014; Van Der Ploeg and Withagen, 2014). Positive externalities from R&D may require renewables subsidisation (Acemoglu, Aghion, Bursztyn, and Hemous, 2012), but Parry, Pizer, and Fischer (2003) argue that the welfare effect from tackling climate change externalities is greater than the positive effect of R&D subsidisation (see also Goulder and Parry, 2008). Other reasons that have been put forward to motivate renewables subsidies are international tax competition with mobile capital (Eichner and Runkel, 2014), learning externalities and imperfect competition (Reichenbach and Requate, 2012), lumpy entry cost (Antoniou and Strausz, 2017), and imperfections in demand for energy efficiency (Fischer et al., 2017). We account for learning externalities, but focus on the role of intermittency of renewable energies and of storage when addressing the carbon externality.

3 The Model

Consider an electricity market with three technologies, indexed $j = f, r, s$. Technology f represents a dispatchable fossil technology—like conventional power plants that burn coal or gas. Dispatchability means that electricity production can be freely varied at every point in time up to the limit of its installed capacity (see Joskow, 2011). Technology r is a renewable technology with intermittent supply—like wind turbines, solar PV, or solar thermal plants. The third technology s does not generate electricity, but is able to store it for later usage.

For each of the three technologies there are a large number, n_j, of identical firms that interact on competitive markets. We use lower-case letters to denote choices of firms and upper-case letters for aggregate values. Accordingly, q_j are capacities of an individual firm that produce with technology j and the overall capacity level is $Q_j = n_j q_j$. To avoid tedious case distinctions, the formal analysis is restricted to the most interesting situation where strictly positive capacities are installed for all three technologies. The numerical simulation in Section 7 extends this to situations where only a subset of technologies is used. Obviously, which of the cases occurs depends on the relative costs of the technologies.

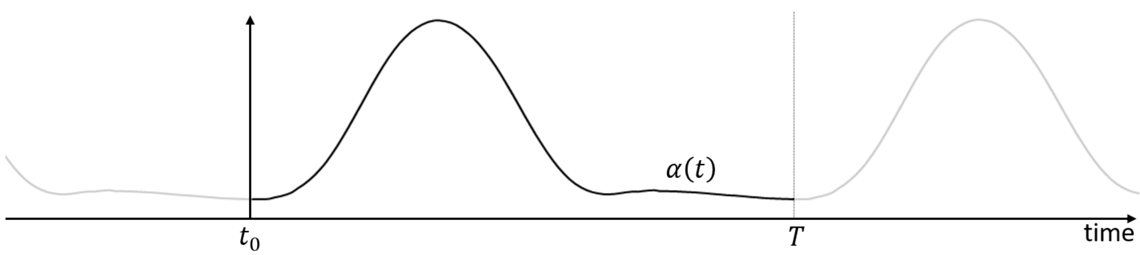
The costs of one unit of capacity is $c_j(Q_j) > 0$. It is constant from the perspective of an individual firm. If one thinks of $c_j\left(Q_j\right)$ as the unit costs of, e.g., solar panels or batteries for electricity storage, this coincides with the standard assumption that firms on competitive markets are too small to affect input prices. However, unit costs may depend on the overall capacity level, which allows us to account for different assumptions in the literature regarding renewables. In particular, $c_j'(Q_j) < 0$ would capture the idea that economy-wide economies of scale or learning reduce unit costs (as in Green and Léautier, 2017).[7] By contrast, if one wants to emphasize that the most efficient sites for wind and solar energies are used first, then $c_j'(Q_j) > 0$ seems more appropriate (as in Abrell et al. (2019) and Ambec and Crampes (2019)).[8] For storage, increasing unit costs could result from less suitable pump storage locations and the scarcity of rare earths that are needed for batteries. Note that due to our assumption that individual firms take unit costs as given, these effects constitute an externality. For the renewable and storage technology, we impose no restriction on the sign of $c_j'(Q_j)$. For the established fossil technology, we assume $c_f'(Q_f) = 0$ and often denote the unit costs simply by c_f.

For renewables, the actual output of installed capacities depends on the intensity of solar radiation and wind conditions that vary over time. We represent this intermittency by an availability factor $\alpha(t)$ that is a continuous function of time and satisfies $\alpha(t) \in [\alpha_{\min}, \alpha_{\max}]$, where $\alpha_{\min} \geq 0$ and $\alpha_{\max} \leq 1$. Therefore, renewable capacities available at time t are $\alpha(t) Q_r$. We now specify the assumption addressed in the introduction that $\alpha(t)$ can be forecasted perfectly and follows a repetitive pattern.

Assumption 1. *The availability of renewable energies, $\alpha(t)$, follows a pattern of repetitive cycles whose initial and terminal time are denoted t_0 and T, respectively. For each such cycle, $\alpha(t)$ is the same single-peaked function with $\alpha(t_0) = \alpha(T) = \alpha_{\min}$.*

The bold $\alpha(t)$-curve in Figure 1 depicts the availability from a mix of solar PV, wind onshore, and wind offshore in Germany, for a (representative) cycle of 24 hours.[9] The cycle has been normalized such that the availability of the mix of renewables is the lowest at its initial and terminal point t_0 and T, that is $\alpha(t_0) = \alpha(T) = \alpha_{\min}$. The transparent segments to the left and to the right illustrate our assumption that the availability follows a repetitive pattern. This distribution satisfies Assumption 1, except that there is a light wind peak during the night. Moreover, the reason for Assumption 1 is that it leads to a regular pattern of completely filling and depleting the storage (see Subsection 4.2). Minor violations of it—such as the light peak during night—do not affect this outcome and, thus, the subsequent analysis would still hold.

Fig. 1: Availability of renewables



Annual seasonal fluctuations in the availability of wind power also roughly correspond to Assumption 1 (see Sinn, 2017), but they are less regular. Moreover, pump storage capacities are limited and new seasonal storage technologies (e.g., power-to-gas) are still very costly—mainly due to high conversion losses. Hence we have written this paper with the storage of daily fluctuations in mind.

Electricity produced by the fossil and renewable technology is $y_j \geq 0$, $j = f, r$. Fossils have constant costs, $k_f > 0$, of producing one unit of output, which are mainly variable costs for coal,

[7] In a seminal paper, Ghemawat and Spence (1985) argue that unit costs are decreasing in accumulated output of the industry. The simple specification $c_j'(Q_j) < 0$ reflects this idea, but omits the time dimension of accumulating capacity.

[8] For a simple two-period model that accounts for learning and site scarcity see Lancker and Quaas (2019).

[9] We calculate $\alpha(t)$ by aggregating production of solar PV, wind onshore, and wind offshore for every quarterly hour of a day from 2016 to the end of 2018, and divide by the installed capacity. The data is downloaded from https://data.open-power-system-data.org/time_series/ on 13 January 2020.

oil, or natural gas. Moreover, fossil production leads to an environmental unit cost, $\delta > 0$, that can be (fully or partly) internalised by a carbon tax, τ. Hence a fossil firm's total unit production costs are $b_f = k_f + \tau$, which equals social costs if $\tau = \delta$. Variable costs of renewables are negligible and, therefore, ignored.

For the storage technology, $y_s(t) > 0$ is supply of stored electricity, $y_s(t) < 0$ is demand for electricity to fill the storage, and $s(t)$ is the level of stored electricity. The change in this level, $\dot{s} := \frac{ds}{dt}$, differs from the related demand and supply due to conversion losses over a storage cycle. In particular, $\dot{s} = -\eta(y_s) y_s(t)$ where

$$\eta(y_s) = \begin{cases} \eta_s \in (0,1] & \text{if } y_s < 0 \\ 1 & \text{if } y_s = 0 \\ \eta_d \geq 1 & \text{if } y_s > 0 \end{cases} \tag{1}$$

is a piecewise constant function that represents the conversion losses. This specification reflects that during times of storage, for which $y_s < 0$, less than one unit of electricity taken from the market arrives in the storage, whereas supplying one unit of electricity to the market ($y_s > 0$) requires more than one unit from the storage. For intermediate periods during which the storage capacity is not used ($y_s = 0$), we assume that no electricity is lost ($\dot{s} = 0$) and $\eta(y_s) = 1$. We do not model limits or costs of the charging speed, but assume that firms prefer a smoother storage pattern if this does not lead to additional costs.

The timing is as follows. In Stage 1, the government chooses one or several of the following policy instruments: a tax on fossil production τ, a tax on electricity consumption χ, and subsidies σ_r, σ_s for renewable and storage capacities. We focus on per unit subsidies for capacity investments rather than feed-in tariffs, market premiums, and, more recently, tenders that are widely used for renewables.[10] These instruments also provide incentives for capacity investments so that their effects are quite similar if subsidies are financed by lump-sum taxation.[11] In Stage 2, competitive firms build their respective fossil, renewable, or storage capacities. In line with the literature on peak-load pricing, we assume a greenfield setting that disregards any capacity that is currently in place. Finally, in Stage 3, firms choose production levels and interact with consumers on a competitive electricity market. We now solve the game backwards, starting with stage 3.

4 Production and Consumption Decisions

4.1 Derivation of Optimality Conditions

We now determine the optimality conditions for production and consumption decisions during a representative cycle $t \in [t_0, T]$. They follow from firms' profit maximisation and consumers' utility maximisation, subject to electricity prices, $p(t)$, that balance supply and demand.

First, consider production decisions of fossil and renewable firms. Capacity costs are sunk so that firms' objective is to maximise revenues, $p(t) y_j(t)$, minus variable production and carbon tax costs, $(k_f + \tau) y_f(t)$, for fossil firms. Production is restricted by the (available) capacity, $y_f(t) \leq q_f$, $y_r(t) \leq \alpha(t) q_r$, and must be non-negative, $y_f(t), y_r(t) \geq 0$. The latter constraint can be ignored because profit maximising renewable and fossil firms will never choose negative quantities in the unconstrained equilibrium. Thus, a fossil firm's profit maximisation problem for a representative cycle is

[10] See, e.g., Eichner and Runkel (2014) for a similar approach. In 2016, 83 countries used feed-in tariffs or premiums to promote renewable energy, 58 countries used investment subsidies (capital subsidies, grants, or rebates), and 73 countries used auctions that do not exclude the use of an investment subsidy (IRENA and CPI, 2018). Moreover, most of storage subsidisation is constructed as an investment subsidy (ESC, 2015).

[11] Using data from a Belgian program, De Groote and Verboven (2019) find that investment subsidies are more effective than production subsidies like feed-in tariffs because households significantly discount their future benefits.

$$\pi_f\left(y_f^*\left(q_f\right)\right) := \max_{y_f(t)} \int_{t_0}^{T} \left(p\left(t\right)-\tau-k_f\right) y_f\left(t\right) dt \text{ such that} \tag{2}$$

$$y_f\left(t\right) \leq q_f. \tag{3}$$

Using asterisks to characterise values in the competitive market solution, $\pi_f(y_f^*(q_f))$ denotes the value function of this problem, that is, the maximum profits a firm can achieve by optimising production y_f for all $t \in [t_0, T]$, given the fixed capacity parameter q_f. Differentiation of the corresponding Lagrangian yields the first-order and complementary slackness conditions ($\mu_f\left(t\right)$ is the Lagrangian multiplier) for each $t \in [t_0, T]$:

$$p\left(t\right)-\tau-k_f-\mu_f\left(t\right) \leq 0 \qquad \left[=0, \text{ if } y_f^*\left(t\right)>0\right], \tag{4}$$

$$q_f-y_f\left(t\right) \geq 0, \qquad \mu_f\left(t\right) \geq 0,\ \mu_f\left(t\right)\left[q_f-y_f\left(t\right)\right]=0. \tag{5}$$

Due to the linearity of the objective function, the first-order condition is sufficient and leads to corner solutions. Specifically, if the price exceeds variable production and carbon costs, the firm produces at full capacity; i.e., $y_f\left(t\right)=q_f$ if $p\left(t\right)>b_f=k_f+\tau$. By contrast, fossil firms do not produce during times t for which $p\left(t\right)<b_f$, while any $y_f\left(t\right) \in [0, q_f]$ is optimal if $p\left(t\right)=b_f$.

Renewable firms face no variable costs, but their output is constrained by the availability of installed capacities. Thus, the profit maximisation problem and the resulting value function are

$$\pi_r\left(y_r^*\left(q_r\right)\right) := \max_{y_r(t)} \int_{t_0}^{T} p\left(t\right) y_r\left(t\right) dt \text{ such that} \tag{6}$$

$$y_r\left(t\right) \leq \alpha\left(t\right) q_r. \tag{7}$$

For each $t \in [t_0, T]$, the first-order and complementary slackness conditions are

$$p\left(t\right)-\mu_r\left(t\right)=0, \tag{8}$$

$$\alpha\left(t\right) q_r-y_r(t) \geq 0, \qquad \mu_r\left(t\right) \geq 0,\ \mu_r\left(t\right)\left[\alpha\left(t\right) q_r-y_r\left(t\right)\right]=0. \tag{9}$$

Here, the binding condition (8) reflects that $y_r^*\left(t\right)>0$ for any $\alpha\left(t\right), p\left(t\right)>0$ because, in contrast to fossils, renewables have no variable costs. Moreover, the complementary slackness condition in (9) then implies $y_r^*\left(t\right)=\alpha\left(t\right) q_r$ for all $p\left(t\right)>0$, i.e., renewables are used at full capacity. However, if the level of available renewable capacities is very large, supply may exceed demand from consumers and storage firms, leading to an equilibrium price of zero.

Storage firms control the level of stored electricity $s(t)$ (the state variable) so as to exploit price differences. They buy and store electricity ($y_s\left(t\right)<0$) during times of low prices, and they destore ($y_s\left(t\right)>0$) when prices are high. To avoid double taxation, the consumption tax χ is only paid by final consumers and not by storage firms when buying electricity. This yields the profit maximisation problem

$$\pi_s\left(y_s^*\left(q_s\right)\right) := \max_{y_s(t)} \int_{t_0}^{T} p\left(t\right) y_s\left(t\right) dt \text{ such that} \tag{10}$$

$$\dot{s}\left(t\right) = -\eta\left(y_s\right) y_s\left(t\right), \tag{11}$$

$$s\left(t_0\right) = s\left(T\right), \tag{12}$$

$$s\left(t\right) \leq q_s, \tag{13}$$

$$s\left(t\right) \geq 0. \tag{14}$$

The first constraint (11) is the equation of motion for the level of stored energy, $s(t)$. Condition (12) requires that the initial and terminal storage level must be the same, which results from Assumption

1 that $\alpha(t)$ follows an identical repetitive pattern. Finally, (13) is the capacity constraint of storage firms, and (14) is the constraint that the level of stored energy must be non-negative. The Hamiltonian is

$$\mathcal{H}_s\left(y_s\left(t\right)\right) = p\left(t\right) y_s\left(t\right) - \lambda\left(t\right) \eta\left(y_s\right) y_s\left(t\right), \tag{15}$$

where $\lambda\left(t\right)$ is the adjoint variable of $s\left(t\right)$. Conditions (13) and (14) are pure state space constraints that can be accounted for by forming the Lagrangian

$$\mathcal{L}_s\left(t\right) = \mathcal{H}_s\left(y_s\left(t\right)\right) + \varphi_s\left(t\right)\left(q_s - s\left(t\right)\right) + \varphi_d\left(t\right) s\left(t\right), \tag{16}$$

where $\varphi_s\left(t\right)$ and $\varphi_d\left(t\right)$ are the Lagrangian multipliers for the respective constraints. As we show in Appendix A, the following conditions are sufficient for optimality:

$$\begin{aligned}
\max_{y_s(t)} \mathcal{H}_s\left(y_s\left(t\right)\right) &= \left[p\left(t\right) - \lambda\left(t\right)\eta\left(y_s\right)\right] y_s\left(t\right), && (17)\\
\dot{s}\left(t\right) &= \frac{\partial \mathcal{L}_s(t)}{\partial \lambda\left(t\right)} = -\eta\left(y_s\right) y_s\left(t\right), && (18)\\
\dot{\lambda}\left(t\right) &= -\frac{\partial \mathcal{L}_s(t)}{\partial s\left(t\right)} = \varphi_s\left(t\right) - \varphi_d\left(t\right), && (19)\\
\frac{\partial \mathcal{L}_s(t)}{\partial \varphi_s\left(t\right)} &= q_s - s\left(t\right) \geq 0,\ \ \varphi_s\left(t\right) \geq 0,\ \varphi_s\left(t\right)\left[q_s - s\left(t\right)\right] = 0, && (20)\\
\frac{\partial \mathcal{L}_s(t)}{\partial \varphi_d\left(t\right)} &= s\left(t\right) \geq 0,\ \ \varphi_d\left(t\right) \geq 0,\ \varphi_d\left(t\right) s\left(t\right) = 0, && (21)\\
s\left(t_0\right) &= s\left(T\right). && (22)
\end{aligned}$$

Here, (17) is the optimality condition for the control variable $y_s\left(t\right)$, conditions (18) and (19) are the differential equations for the state and adjoint variable, and conditions (20) and (21) account for the pure state space constraints.

Turning to consumers, utility maximisation leads to a demand function $x\left(t\right) = x\left(p\left(t\right) + \chi\right)$, for which we impose no restrictions other than $\frac{\partial x}{\partial p} < 0$. Consumption choices on the competitive electricity market maximise consumer surplus and are restricted by aggregate production. It is straightforward to show that in equilibrium $x\left(t\right) = \sum_j Y_j\left(t\right)$, that is, demand equals supply. In conclusion, this market clearing condition, the inverse demand function, $p\left(x\left(t\right)\right)$, and the optimality conditions of fossil firms, (4) and (5), renewable firms, (8) and (9), and storage firms, (17) to (22) determine electricity production, demand and the electricity price as functions of the environmental and consumption tax τ, χ, and of installed capacities Q_j, which in turn depend on related subsidies and taxes $\sigma_r, \sigma_s, \tau, \chi$ (see Section 5).

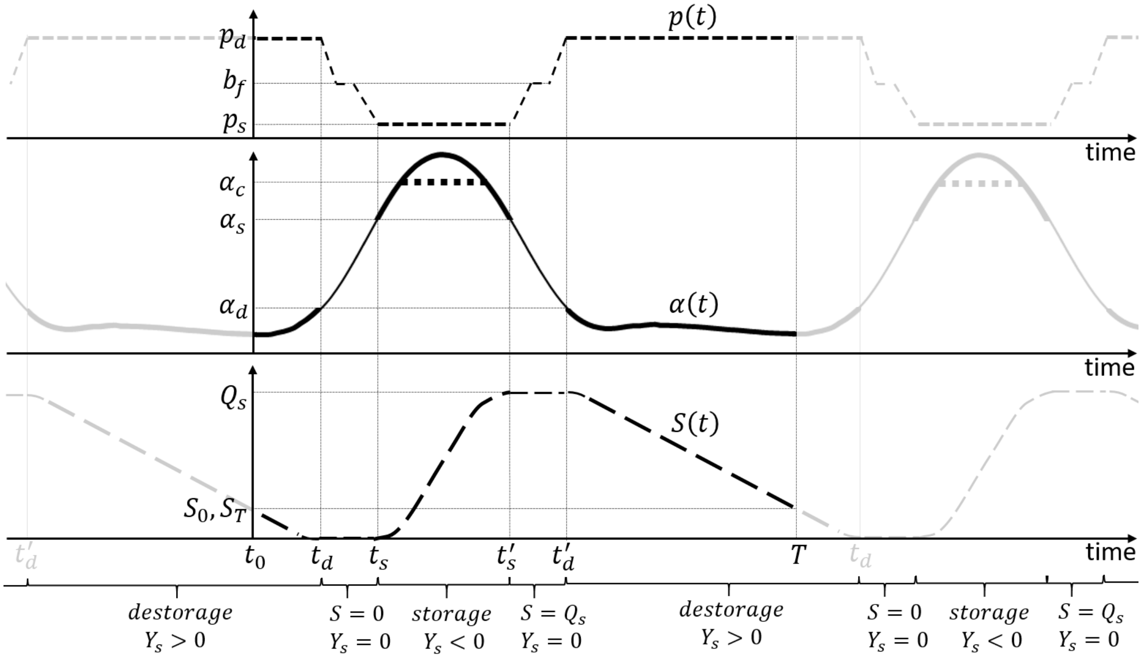
In the next subsection, we derive an intuitive solution for the competitive equilibrium and show in Appendix A that it satisfies all optimality conditions. For later reference, we state the solution in terms of aggregate values, $Y_j = n_j y_j$, $Q_j = n_j q_j$, and $S = n_s s$.

4.2 Determination of Competitive Equilibrium

The middle panel in Figure 2 depicts again the availability function $\alpha(t)$ from Figure 1. Obviously, firms should destore electricity when the availability of renewables is low (lower bold parts of $\alpha(t)$-curve), and store electricity when the availability is high (upper bold part of $\alpha(t)$-curve). Denoting the associated threshold levels by α_d and α_s, this leads to the depicted sequence of periods for a representative cycle $t \in [t_0, T]$: *destorage* ($Y_s(t) > 0$) for $\alpha \leq \alpha_d$, *intermediate* ($Y_s(t) = 0$) for $\alpha \in (\alpha_d, \alpha_s]$, *storage* ($Y_s(t) < 0$) for $\alpha > \alpha_s$; and as α decreases again *intermediate* and *destorage*.

Dispatchable electricity from storage and fossils is most valuable when the availability of renewables is minimal, i.e., at t_0. Hence the electricity price (top panel in Figure 2) is maximal, electricity is

Fig. 2: Availability of renewables and competitive equilibrium



Top panel shows electricity price $p(t)$. It has the constant values p_s during storage, b_f when fossils are price setting, and p_d during destorage. Middle panel shows availability of renewables $\alpha(t)$, with destorage for $\alpha \leq \alpha_d$, storage for $\alpha > \alpha_s$ and capped production of renewables for $\alpha > \alpha_c$. Bottom panel shows stored electricity $S(t)$. S_0, S_T are the starting and final storage volume, at Q_s the storage is fully filled.

destored (bottom panel), and fossils are fully used as, otherwise, some capacities would always lie idle. Moreover, the price during destorage must be constant (denoted p_d in Figure 2). If not, firms would have arbitrage opportunities and shift their sales of stored electricity to times of higher prices. Given the constant price, fossils—and obviously also renewables due to their lower variable costs—are fully used during a whole destorage period.

From Figure 2, the two destorage intervals are the segments $[t_0, t_d]$ and $[t'_d, T]$ that comprise availabilities below the critical level $\alpha_d := \alpha(t_d) = \alpha(t'_d)$. The quantity of destored electricity balances the variability of renewables so as to keep electricity supply from these two sources—and, thus, the price—constant at the level at which the intermediate period starts, i.e., $Y_s(t) + \alpha(t) Q_r = \alpha_d Q_r$. Hence,

$$Y_s(t) \quad = \quad (\alpha_d - \alpha(t)) Q_r \quad \text{for all } t \in [t_0, t_d] \quad \text{and } t \in [t'_d, T]. \tag{23}$$

The first line in Table 1 summarises production in the destorage period. Noting that we consider a representative cycle, the destorage interval $[t'_d, T]$ is identical to the one that precedes t_0. Accordingly, the two destorage intervals can be viewed as being connected. This implies that the storage must be full at $t = t'_d$ where destorage begins (i.e., $S(t'_d) = Q_S$), and run empty at $t = t_d$ where destorage ends. Otherwise, some stored electricity and/or some storage capacity would never be used, which cannot be optimal. This is represented by the $S(t)$-curve in the bottom panel in Figure 2.

Tab. 1: Solution of production stage for fossils, renewables, and storage

period	availability of renewables		$Y_r\left(t\right)$	$Y_f\left(t\right)$	$Y_s\left(t\right)$
d	$0 \leq \alpha\left(t\right) \leq \alpha_d$		$\alpha\left(t\right)Q_r$	Q_f	$\left(\alpha_d - \alpha\left(t\right)\right)Q_r$
case 1	$\alpha_d < \alpha\left(t\right) \leq \min\left\{\alpha_1, \alpha_s\right\}$		$\alpha\left(t\right)Q_r$	Q_f	0
case 2	$\alpha_1 < \alpha\left(t\right) \leq \min\left\{\alpha_2, \alpha_s\right\}$		$\alpha\left(t\right)Q_r$	$x\left(b_f + \chi\right) - \alpha\left(t\right)Q_r$	0
case 3	$\alpha_2 < \alpha\left(t\right) \leq \alpha_s$		$\alpha\left(t\right)Q_r$	0	0
s	$\alpha_s < \frac{x(0+\chi)}{Q_r}$	$\alpha_s < \alpha\left(t\right) \leq \alpha_{\max}$,	$\alpha\left(t\right)Q_r$	$Y_f\left(\alpha_s\right)$	$\left(\alpha_s - \alpha\left(t\right)\right)Q_r$
	$\alpha_s = \frac{x(0+\chi)}{Q_r}$	$\alpha_s < \alpha\left(t\right) \leq \alpha_c$	$\alpha\left(t\right)Q_r$	0	$x\left(0+\chi\right) - \alpha\left(t\right)Q_r$
		$\alpha_c < \alpha\left(t\right) \leq \alpha_{\max}$	a_cQ_r		$x\left(0+\chi\right) - a_cQ_r$

$\alpha_1 = \frac{x\left(b_f+\chi\right)-Q_f}{Q_r}$, $\alpha_2 = \frac{x\left(b_f+\chi\right)}{Q_r}$, α_d implicitly solves (40),
$\alpha_s = \min\left\{\alpha_s \text{ that solves (41)}, \frac{x(0+\chi)}{Q_r}\right\}$, α_c implicitly solves (42)

When $\alpha\left(t\right)$ starts to exceed α_d, we enter the first intermediate period where neither storage nor destorage occurs so that $Y_s\left(t\right) = 0$ (similar to Helm and Mier (2019)). By continuity of $\alpha(t)$, fossils and renewables continue to be fully used initially. This is case 1 in Table 1. As renewable supply rises together with $\alpha\left(t\right)$, the equilibrium price $p\left(t\right)$ falls until it equals the total unit costs of fossils, $b_f = k_f + \tau$. Therefore, fossils would make losses for any further price reduction and respond to a rising availability of renewables by taking some of their supply from the market. This keeps the price constant at the after tax level $p\left(t\right) + \chi = b_f + \chi$. Renewable capacities continue to be fully used (due to lower variable costs), whereas fossils serve the remaining demand, $x\left(b_f + \chi\right) - \alpha\left(t\right)Q_r$. This is case 2 in Table 1. Once $\alpha\left(t\right)$ is sufficiently large so that renewables can serve the whole demand at the after tax price $b_f + \chi$, a further rising $\alpha(t)$ pushes $p\left(t\right)$ below b_f so that fossils drop out of the market. Hence only renewables are used, but still at full capacity (case 3).

Let α_1 and α_2 denote the critical availabilities where case 1 (renewables and fossils fully used) and case 2 (renewables fully and fossils partially used) start, with associated times $t_1 < t_2$. Hence $\alpha_i = \alpha\left(t_i\right)$ for $i = 1, 2$. Depending on the size of storage and renewable capacities, not all cases need obtain. Ceteris paribus, a larger storage capacity takes longer to fill so that storage starts at a lower $\alpha(t)$, that is, already during case 1 or 2. Conversely, larger renewable capacities imply that a given storage can be filled faster; hence storage starts only at higher $\alpha(t)$ and more cases obtain. In Table 1, this is represented by the minimum operator in the column for availability, where $\alpha_s := \alpha\left(t_s\right) = \alpha\left(t'_s\right)$ is the availability when the intermediate period ends and storage starts (t'_s denotes the end of the storage period; see Figure 2).

Accordingly, any one of cases 1 to 3 can prevail at the start of the storage period, during which the price remains constant. Otherwise, firms would shift their storage to times of lower prices. By continuity of the available production capacities, this price, p_s, must be the same as that at the end of the intermediate period, i.e., $p_s = p\left(t_s\right)$. This results in constant demand, $x_s = x\left(t_s\right)$, and constant supply from fossils, $Y_f(t) = Y_f\left(t_s\right)$, during storage. Moreover, supply of renewables, $Y_r\left(t\right)$, above the level required to satisfy constant demand, $Y_r\left(t_s\right) = \alpha_s Q_r$, is used to fill the storage, i.e.,

$$-Y_s\left(t\right) \quad = \quad Y_r\left(t\right) - \alpha_s Q_r \;\text{ for all } t \in \left[t_s, t'_s\right]. \tag{24}$$

To determine $Y_r(t)$, we need to account for the possibility that the level of available renewable capacities exceeds demand at an equilibrium price $p(t) = 0$ plus the quantity required to fill the storage. If this is not the case, renewables always produce at full capacity and substitution of $Y_r\left(t\right) = \alpha\left(t\right)Q_r$ into (24) yields $-Y_s(t) = \left(\alpha\left(t\right) - \alpha_s\right)Q_r$ (first line of case s in Table 1).[12] By contrast, excess

[12] Note that if storage starts in case 2, the storage period is characterised by excess capacities of fossils and a price that equals variable production costs. These idle fossil capacities could be used to reschedule some storage without affecting profits. However, if fossil generators prefer a smooth pattern of production (due to ramping cost and constraints), the pattern in Table 1 is the only optimal one.

capacities of renewables lead to an equilibrium price $p_s = 0$, demand $x_s = x(0+\chi)$, and no production of fossils throughout the whole storage period, including its boundaries so that $\alpha_s Q_r = x(0+\chi)$.[13] Given our assumption that firms prefer smoother storage patterns, only the production peaks of renewables where storage would be maximal will be capped. Let α_c denote the critical availability that separates the uncapped from the capped part of the storage period. This is illustrated by the horizontal dotted bold segment of the $\alpha(t)$-curve in Figure 2. Then renewable production is $Y_r(t) = \alpha(t) Q_r$ for $\alpha(t) \in [\alpha_s, \alpha_c]$, and $Y_r(t) = \alpha_c Q_r$ for $\alpha(t) > \alpha_c$. Substitution of this into (24) yields storage volumes, $-Y_s(t)$, for the uncapped and the capped region, which are stated in the last two lines in Table 1. Finally, the threshold levels at which the various cases start are listed below the table (see Appendix A for their derivation).

For the second intermediate period from t'_s to t'_d (see Figure 2) the solution follows from the same equilibrium conditions as for the first one. Thus, for each $\alpha(t)$, the solution is the same as already summarised by cases 1 to 3 in Table 1, but the cases obtain in reverse order because $\alpha(t)$ is now (weakly) decreasing in t. Lemma 1 summarises these results.

Lemma 1. *Equilibrium levels for production and storage are as given in Table 1. Demand and prices follow straightforwardly from the market clearing condition,* $x(t) = \sum_j Y_j(t)$*, and the inverse demand function* $p(x(t))$*. They are constant during each storage and destorage period.*

4.3 Comparative Statics of Production and Demand

The later analysis of optimal subsidies depends on how the triggered changes in capacities affect production and demand. For the intermediate period, this follows straightforwardly from the expressions in Figure 1, but for the storage and destorage periods it depends in a non-trivial way on effects via the boundaries α_d, α_s that determine the lengths of these periods. Lemma 2 summarises the relevant comparative statics for later reference ($\int_s dt := \int_{t_s}^{t'_s} dt$ and $\int_d dt := \int_{t_0}^{t_d} dt + \int_{t'_d}^{T} dt$ are shorthand notation for the storage and destorage periods).

Lemma 2. *Marginal changes in capacities* Q_f, Q_r, Q_s *have the following comparative static effects for the storage and destorage periods.*

(a) Fossil capacities: $\frac{\partial x_d}{\partial Q_f} = 1$ *and* $\frac{\partial \alpha_s}{\partial Q_f} = \frac{\partial \alpha_d}{\partial Q_f} = 0$*. Moreover,* $\frac{\partial x_s}{\partial Q_f} = 1$ *if storage starts during case 1, whereas for all other cases* $\frac{\partial x_s}{\partial Q_f} = 0$.

(b) Renewable and storage capacities for $\alpha_s Q_r < x(0+\chi)$ *(no excess capacities of renewables): If case 2 prevails at the beginning of the storage period, then* $\frac{\partial x_s}{\partial Q_r} = \frac{\partial x_s}{\partial Q_s} = 0$*. Otherwise,*

	$\partial\alpha_d/\partial$	$\partial x_d/\partial$	$\partial\alpha_s/\partial$	$\partial x_s/\partial$
Q_r	$-\frac{\int_d (\alpha_d - \alpha(t))dt}{Q_r \int_d dt} < 0$	$\frac{\int_d \alpha(t)dt}{\int_d dt} > 0$	$-\frac{\int_s (\alpha_s - \alpha(t))dt}{Q_r \int_s dt} > 0$	$\frac{\int_s \alpha(t)dt}{\int_s dt} > 0$
Q_s	$\frac{1}{\eta_d Q_r \int_d dt} > 0$	$\frac{1}{\eta_d \int_d dt} > 0$	$-\frac{1}{\eta_s Q_r \int_s dt} < 0$	$-\frac{1}{\eta_s \int_s dt} < 0$

(c) Renewable and storage capacities for $\alpha_s Q_r = x(0+\chi)$ *(excess capacities of renewables): For the destorage period, derivatives are as in (b). For the storage period, demand is constant at* $x(0+\chi)$ *so that* $\frac{\partial x_s}{\partial Q_r} = \frac{\partial x_s}{\partial Q_s} = 0$*,* $\frac{\partial \alpha_s}{\partial Q_s} = 0$*, and* $\frac{\partial \alpha_s}{\partial Q_r} = -\frac{x(0+\chi)}{Q_r^2} < 0$.

The non-trivial effects that require some intuition are those in the table and concern the most relevant case of no excess capacities. With higher *capacities of renewables*, storage starts later ($\frac{\partial \alpha_s}{\partial Q_r} > 0$) because the storage can be filled faster. The magnitude of this effect is given by the additional

[13] Remember that $Y_f(t) = 0$ if $p(t) = 0$ and $Y_s(t_s) = 0$ so that at the boundary of the storage cycle available renewable capacities are equal to demand.

production of a marginal renewable capacity unit over the storage cycle, $\int_s (\alpha_s - \alpha(t))\,dt$, weighted by the overall available capacity during this cycle, $Q_r \int_s dt$. Similarly, the destorage period lasts shorter ($\frac{\partial \alpha_d}{\partial Q_r} < 0$) because a given level of stored electricity, Q_s, has to substitute for a larger amount of renewables over the destorage period. The corresponding marginal changes in demand during destorage and storage, $\frac{\partial x_d}{\partial Q_r}, \frac{\partial x_s}{\partial Q_r}$, are simply average additional renewable production of a marginal capacity unit over the destorage and storage period.

An increase in *storage capacities* leads to longer storage and destorage periods ($\frac{\partial \alpha_s}{\partial Q_s} < 0$ and $\frac{\partial \alpha_d}{\partial Q_s} > 0$). The size of this effect is smaller if more intermittent renewables, Q_r, have to be balanced during storage and destorage. In addition, with larger conversion losses of storage (small η_s) it takes longer to fill the storage, and with larger conversion losses of destorage (high η_d) the storage is depleted more quickly. Turning to demand, a larger storage requires more electricity to be filled. This raises the price and reduces demand ($\frac{\partial x_s}{\partial Q_s} < 0$), and conversion losses (small $\eta_s \int_s dt$) accentuate this effect. Conversely, during the destorage period the additional electricity feed-in from a larger storage reduces the price and raises demand ($\frac{\partial x_d}{\partial Q_s} > 0$), especially when conversion losses, η_d, are low.

5 Capacity Choices of Competitive Firms

We now turn to Stage 2, in which fossil, renewable, and storage firms choose their respective capacities, thereby anticipating the outcome of production decisions in Stage 3. Remember that the value functions, $\pi_j\left(y_j^*(q_j)\right)$, $j = f, r, s$ as given by (2), (6), and (10), represent the maximum profits that the respective firms can achieve for given capacities, q_j, during one representative cycle, $t \in [t_0, T]$. We assume that the lifetime of installed capacities is the same for all technologies and consists of m such cycles. Therefore, the net present value of profits over this lifetime is $\sum_{\kappa=1}^{m} \frac{1}{(1+r)^\kappa} \pi_j\left(y_j^*(q_j)\right) = \rho \pi_j\left(y_j^*(q_j)\right)$, where $\rho := \frac{1}{r} - \frac{1}{r(1+r)^m}$ and r is the discount factor. Substitution of the value functions from stage 3 and accounting for capacity costs, $c_j(Q_j)\,q_j$, as well as subsidies, σ_r, σ_s, yields firms' profit maximization problem and the resulting value function at Stage 2 ($\boldsymbol{\theta} = (\sigma_r, \sigma_s, \tau, \chi)$ denotes the vector of policy instruments):

$$\pi_f\left(q_f^*(\boldsymbol{\theta}), \boldsymbol{\theta}\right) := \max_{q_f} \rho \int_{t_0}^{T} (p(t) - \tau - k_f)\, y_f^*(t, q_f) dt - c_f q_f, \tag{25}$$

$$\pi_r\left(q_r^*(\boldsymbol{\theta}), \boldsymbol{\theta}\right) := \max_{q_r} \rho \int_{t_0}^{T} p(t)\, y_r^*(t, q_r) dt - (c_r(Q_r) - \sigma_r)\, q_r, \tag{26}$$

$$\pi_s\left(q_s^*(\boldsymbol{\theta}), \boldsymbol{\theta}\right) := \max_{q_s} \rho \int_{t_0}^{T} p(t)\, y_s^*(t, q_s) dt - (c_s(Q_s) - \sigma_s)\, q_s. \tag{27}$$

When choosing capacity levels, competitive firms take as given the capacity choices of other firms, unit capacity costs, $c_j(Q_j)$, the equilibrium electricity demand and price, $x(t), p(t)$, as well as the occurrence of cases and the t where they start (columns 1 and 2 of Table 1). Using this, differentiation of the objective functions in (25) to (27) with respect to the respective capacities yields the following first-order conditions for fossil, renewable, and storage firms:

$$\rho \int_{t_0}^{T} (p(t) - \tau - k_f) \frac{\partial y_f^*(t, q_f)}{\partial q_f} dt - c_f = 0, \tag{28}$$

$$\rho \int_{t_0}^{T} p(t) \frac{\partial y_r^*(t, q_r)}{\partial q_r} dt - c_r(Q_r) + \sigma_r = 0, \tag{29}$$

$$\rho \int_{t_0}^{T} p(t) \frac{\partial y_s^*(t, q_s)}{\partial q_s} dt - c_s(Q_s) + \sigma_s = 0, \tag{30}$$

where the derivatives $\partial y_j^* (t, q_j) / \partial q_j$ follow straightforwardly from Table 1 for the respective cases. Intuitively, firms equalise the net present value of additional production from a marginal capacity unit—the integral terms—and its costs, $c_j (Q_j)$, thereby accounting for subsidies, and a tax τ on fossils if implemented. A priori, corner solutions might obtain. However, our focus on situations where optimal capacity levels are positive for all three technologies excludes the case that marginal profits are negative. Conversely, positive marginal (and total) profits would lead to entry until the conditions bind. Indeed, by solving (28) to (30) for $c_f, c_r (Q_r)$ and $c_s (Q_s)$ and substituting this into the profit functions (25) to (27), it is straightforward to see that all firms make zero profits in equilibrium.

6 First- and Second-best Policies

Now consider the regulator's choice of the optimal tax-subsidy scheme for fossil production, electricity consumption as well as renewable and storage capacities in Stage 1. Denote by $\mathbf{Q} = (Q_f, Q_r, Q_s)$ the vector of overall capacities and remember that $\boldsymbol{\theta} = (\sigma_r, \sigma_s, \tau, \chi)$ is the vector of policy instruments. We use (non-bold) $\theta = \sigma_r, \sigma_s, \tau, \chi$ to refer to an element of this vector. Assuming lump-sum taxation, welfare is given by (omitting asterisks for optimised values from stages 2 and 3)

$$W := \rho \int_{t_0}^{T} \left(\int_{0}^{x(t,\chi,\tau,\mathbf{Q})} p(\tilde{x})\, d\tilde{x} \right) dt - \rho \int_{t_0}^{T} (k_f + \delta)\, Y_f (t, \chi, \tau, \mathbf{Q})\, dt - \sum_{j=f,r,s} c_j (Q_j)\, Q_j, \tag{31}$$

where the notation clarifies that equilibrium demand, $x(t, \chi, \tau, \mathbf{Q})$, and production of fossils, $Y_f (t, \chi, \tau, \mathbf{Q})$, at time t depend directly on taxes χ, τ, but only indirectly via capacity levels $\mathbf{Q}$ on subsidies σ_r, σ_s (see Table 1). The first term is the net present value of gross consumer surplus, i.e., the area under the inverse demand function $p(x)$. The second term is the net present value of variable production and environmental damage costs. The third term are capacity costs. This takes into account that revenues and costs of taxes and subsidies cancel in the aggregate.

The optimal policy vector maximises welfare W. Differentiation of (31) yields for $\theta = \sigma_r, \sigma_s, \tau, \chi$ (skipping arguments for parsimony):[14]

$$\frac{dW}{d\theta} = \rho \int_{t_0}^{T} \left(\frac{dx(t)}{d\theta} (p(t) + \chi) - (k_f + \delta) \frac{dY_f(t)}{d\theta} \right) dt - \sum_{j=f,r,s} c_j (Q_j) \frac{dQ_j}{d\theta} - \sum_{i=r,s} Q_i \frac{\partial c_i}{\partial Q_i} \frac{dQ_i}{d\theta}. \tag{32}$$

The first term under the integral reflects that the value of the inverse demand function at the equilibrium consumption level equals the equilibrium price plus consumption tax, i.e., $p(x(t, \chi, \tau, \mathbf{Q})) = p(t) + \chi$. Accordingly, the integral terms give the difference between the marginal value of electricity consumption and the marginal environmental and production cost that result from a marginal increase in θ. The remaining two terms extend this to effects via capacity costs, where the summation in the last term takes into account that $\frac{\partial c_f}{\partial Q_f} = 0$ for the established fossil technology. Using $\sum_j Y_j(t) = x(t)$, rearranging terms and substituting from conditions (28) to (30) for firms' capacity choices, the four first-order conditions $\frac{dW}{d\theta} = 0$ for the optimal policy instruments $\theta = \sigma_r, \sigma_s, \tau, \chi$ are (see Appendix C for the calculations):

$$\sum_{i=r,s} \left(\sigma_i + Q_i \frac{\partial c_i}{\partial Q_i} - \rho \chi \int_{t_0}^{T} \frac{\partial Y_i(t)}{\partial Q_i} dt \right) \frac{dQ_i}{d\theta} + \rho (\delta - \tau - \chi) \int_{t_0}^{T} \frac{dY_f(t)}{d\theta} dt - z = 0, \tag{33}$$

where

$$z = \begin{cases} 0 & \text{if } \alpha_s Q_r < x(0+\chi) \\ \rho \chi \frac{1}{\eta_s} \frac{dQ_s}{d\theta} & \text{if } \alpha_s Q_r = x(0+\chi) \text{ for } \theta = \sigma_r, \sigma_s, \tau \\ \rho \chi \left(\int_s \frac{\partial x(0+\chi)}{\partial \chi} dt + \frac{1}{\eta_s} \frac{dQ_s}{d\chi} \right) & \text{if } \alpha_s Q_r = x(0+\chi) \text{ for } \theta = \chi \end{cases} \tag{34}$$

[14] All terms under the integral as well as their derivatives are continuous so that one can apply the Leibniz rule and differentiate under the integral sign (see Sydsaeter, Hammond, Seierstad, and Strom 2005, p. 156).

Accordingly, z is only non-zero when there are excess capacities of renewables—i.e., if $\alpha_s Q_r = x(0+\chi)$. Regarding the other terms in (33), $\rho(\delta-\tau-\chi)\int_{t_0}^{T}\frac{dY_f(t)}{d\theta}dt$ represents the marginal effect of the instrument θ on the net present value of damages from fossil production that are not internalised by the taxes $\tau+\chi$. Similarly, the summation term adds up the marginal effect of θ on renewable and storage capacities, $dQ_i/d\theta$, weighted by extent to which the capacity subsidy, σ_i, fails to internalise the capacity cost externality, $Q_i\frac{\partial c_i}{\partial Q_i}$, and the distortionary costs, $\rho\chi\int_{t_0}^{T}\frac{\partial Y_i(t)}{\partial Q_i}dt$, that result because the consumption tax χ must also be paid for electricity from renewable production and from the storage. We now examine the different options for policy interventions.

6.1 Pigouvian Taxation

If the consumption tax is set at $\chi = 0$, all related terms in (33) cancel and the first-order optimality conditions for the remaining policy instruments $\theta = \sigma_r, \sigma_s, \tau$ simplify to

$$\sum_{i=r,s}\left(\sigma_i + Q_i\frac{\partial c_i}{\partial Q_i}\right)\frac{dQ_i}{d\theta} = -\rho(\delta-\tau)\int_{t_0}^{T}\frac{dY_f(t)}{d\theta}dt. \tag{35}$$

It is straightforward to see that these are satisfied by a Pigouvian tax on fossils, $\tau = \delta$, and subsidies for renewable and storage capacities that equal the cost externality, $\sigma_i = -Q_i\frac{\partial c_i}{\partial Q_i}$. The latter may be positive or negative, depending on whether $\frac{\partial c_i}{\partial Q_i} < 0$ (e.g., due to economy-wide economies of scale or learning) or $\frac{\partial c_i}{\partial Q_i} > 0$ (e.g., due to the scarcity of suitable sites for wind and solar power).

To see that this instrument mix even implements the first-best solution, note that production and consumption choices on competitive markets as analysed in Section 4.2 are only distorted by the pollution externality. Hence the resulting equilibrium levels in Lemma 1 are obviously first-best if no consumption tax is levied and the environmental externality is internalised by a Pigouvian tax $\tau = \delta$, provided that the underlying capacity levels are first-best. These follow from maximizing welfare W in (31) with respect to capacities for $\chi = 0$. Using $\frac{\partial c_f}{\partial Q_f} = 0$, we have for $j = f, r, s$:

$$\frac{\partial W}{\partial Q_j} = \rho\int_{t_0}^{T}\left(\frac{\partial x(t)}{\partial Q_j}p(t) - (k_f+\delta)\frac{\partial Y_f(t)}{\partial Q_j}\right)dt - c_j(Q_j) - \frac{\partial c_j}{\partial Q_j}Q_j = 0. \tag{36}$$

In the proof of Proposition 1, we show that with a Pigouvian tax and a subsidy that corrects the capacity cost externality, profit maximizing firms have exactly the same first-order conditions and, hence, choose the same capacity levels. This leads to the following result.

Proposition 1. *The social optimum can be implemented by a Pigouvian tax on fossils, $\tau^* = \delta$, subsidies for renewable and storage capacities that equal the cost externality, $\sigma_i^* = -Q_i^*\frac{\partial c_i}{\partial Q_i}$, $i = r, s$, and no consumption tax, $\chi^* = 0$.*

Proposition 1 confirms the expectation that Pigouvian taxation also works in a model that accounts for intermittency of renewables and storage. However, this provides no guidance for second-best policies when Pigouvian taxes are not feasible—for example, due to political economy reasons. In the next subsection, we consider the alternative policy option of subsidising renewable and storage capacities in combination with a consumption tax on electricity.

6.2 Consumption Taxation

Now suppose that—in contrast to the previous subsection—an incomplete fossil tax, $\tau < \delta$, is taken as given. Remember that the first-order conditions for policy instruments are as stated in (33). First, consider the situation without excess capacities of renewables ($\alpha_s Q_r < x(0+\chi)$) so that $z = 0$ (from 34). Moreover, from Table 1 and Lemma 2 the effect of renewable and storage capacities on associated production values are $\frac{\partial Y_r(t)}{\partial Q_r} = \alpha(t)$ and $\int_{t_0}^{T}\frac{\partial Y_s(t)}{\partial Q_s}dt = \int_d \frac{\partial \alpha_d}{\partial Q_s}Q_r dt + \int_s \frac{\partial \alpha_s}{\partial Q_s}Q_r dt = \frac{1}{\eta_d} - \frac{1}{\eta_s} \leq 0.$

Intuitively, additional storage and destorage volumes that result from a larger storage capacity cancel each other over a cycle, except for conversion losses $\frac{1}{\eta_s} - \frac{1}{\eta_d}$.[15] Substituting this into (33), the optimal policy instruments $\theta = \sigma_r, \sigma_s, \chi$ must satisfy

$$\left(\sigma_r + Q_r \frac{\partial c_r}{\partial Q_r} - \rho\chi \int_{t_0}^{T} \alpha(t)\, dt\right) \frac{dQ_r}{d\theta} + \left(\sigma_s + Q_s \frac{\partial c_s}{\partial Q_s} - \rho\chi \left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right)\right) \frac{dQ_s}{d\theta} = -\rho(\delta - \tau - \chi) \int_{t_0}^{T} \frac{dY_f(t)}{d\theta} dt. \tag{37}$$

Denoting the optimal tax/subsidy scheme with a consumption tax by superscript $^{\#}$, it is straightforward to see that $\chi^{\#} = \delta - \tau$, $\sigma_r^{\#} = -Q_r \frac{\partial c_r}{\partial Q_r} + \rho\chi^{\#} \int_{t_0}^{T} \alpha(t)\, dt$, and $\sigma_s^{\#} = -Q_s \frac{\partial c_s}{\partial Q_s} + \rho\chi^{\#} \left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right)$ satisfies all three first-order conditions. Moreover, as shown in the proof of Proposition 2 it implements the first-best solution.

For the fossil technology this is very intuitive. The total tax on electricity from fossils is the Pigouvian level, $\tau + \chi^{\#} = \delta$, and the tax incidence does not depend on who pays the tax; producers in the case of τ or consumers in the case of χ. Moreover, from Table 1 supply $Y_j(t)$ and, thus, also demand $x(t) = \sum_{j=f,r,s} Y_j(t)$ are the same under the Pigouvian and under the consumption tax (remember that $b_f = k_f + \tau$). Therefore, $x\left(p^{\#}(t) + \chi^{\#}\right) = x(p^*(t))$, where $p^*(t)$ and $p^{\#}(t)$ denote the (before tax) equilibrium prices with the Pigouvian and the consumption tax, respectively. It follows that $p^{\#}(t) = p^*(t) - \chi^{\#}$, i.e., the equilibrium price under the consumption tax is lower. Ceteris paribus, this distorts investment decisions. For renewables, the net present value of losses from the lower equilibrium price is compensated by the subsidy component $\rho\chi^{\#} \int_{t_0}^{T} \alpha(t)\, dt$. By contrast, for storage firms $p^{\#}(t) < p^*(t)$ implies that they pay and receive a lower price during storage and destorage, respectively. As the level of stored electricity exceeds that of destored electricity by conversion losses $\frac{1}{\eta_s} - \frac{1}{\eta_d}$, this is beneficial for storage firms. To balance this requires a tax, which explains the "subsidy" component $\rho\chi^{\#}\left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right) < 0$.

Next, consider the situation with excess capacities of renewables ($\alpha_s Q_r = x(0 + \chi)$). The consumption tax χ must also be paid during times when renewables are in excess supply. This reduces demand, creating a standard tax distortion. In particular, from the last two lines in Table 1, overall supply during storage is now $\sum_j Y_j(t) = x(0 + \chi)$ and, thus, lower with a consumption tax $\chi > 0$ than with a Pigouvian tax. Therefore, even at first-best capacity levels, consumption and production decisions are distorted and the social optimum cannot be attained. The proposition summarizes these results.

Proposition 2. *Suppose that we have an incomplete carbon tax $\tau < \delta$.*

(a) If there are no excess capacities of renewables ($\alpha_s Q_r < x(0 + \chi)$), then the social optimum can still be implemented by a combination of the following policies: (i) A tax on electricity consumption, $\chi^{\#} = \delta - \tau$, that closes the gap between the incomplete carbon tax and its Pigouvian level. (ii) Subsidies for renewable and storage capacities, $\sigma_r^{\#} = -Q_r^* \frac{\partial c_r}{\partial Q_r} + \rho(\delta - \tau) \int_{t_0}^{T} \alpha(t)\, dt$ and $\sigma_s^{\#} = -Q_s^* \frac{\partial c_s}{\partial Q_s} + \rho\chi^{\#}\left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right)$, that internalize the capacity cost externality and compensate the distortion that the consumption tax must also be paid for electricity from renewables and for electricity that goes through the storage. As $\frac{1}{\eta_d} - \frac{1}{\eta_s} < 0$, storage capacities should be taxed unless the capacity cost externality dominates.

(b) With excess capacities of renewables ($\alpha_s Q_r = x(0 + \chi)$), no policy mix of consumption taxes and capacity subsidies can attain the social optimum.

[15] Remember that $\eta_d \geq 1 \geq \eta_s$. $\frac{1}{\eta_s}$ is the quantity of electricity needed to fill the storage by one unit so that $\frac{1}{\eta_s} - 1$ is electricity lost during storage. Similarly, $\frac{1}{\eta_d}$ is the quantity of electricity that arrives at the market for each unit taken from the storage so that $1 - \frac{1}{\eta_d}$ is electricity lost during destorage. Summing up yields $\frac{1}{\eta_s} - \frac{1}{\eta_d} \geq 0$.

This restricts the finding of Abrell et al. (2019) as well as Ambec and Crampes (2019) that subsidies for the renewable (and storage) technology and a consumption tax can also decentralise the first-best solution to situations in which maximal generation from renewables does not exceed demand at a price equal to the consumption tax. Moreover, the usual framing of this policy is to emphasize the need to subsidise renewables and to consider the consumption tax as a complementary policy. The above analysis suggests a reverse perspective. The consumption tax brings the carbon tax to the Pigouvian level. The subsidy for renewables and the tax on storage correct the distortions for capacity investments that result because the consumption tax must also be paid for electricity from renewables and from the storage.

Even though this policy shares some similarities with the financing of renewables subsidies by levies on electricity consumption that some countries have implemented, from an economics perspective this is an awkward policy. It results in Pigouvian taxation of fossils, but as a by-product leads to unintended taxes for renewables and storage, which in turn must be compensated by another round of subsidies and taxes. Moreover, in the real world these tax/subsidy streams lead to costs that we have ignored in our simple model. In addition, the policy fails for excess capacities of renewables that we occasionally observe already now in some countries. Therefore, it is hard to see any advantage over a simple Pigouvian tax, even if one accounts for political economy issues that impede their implementation.

6.3 Renewable and Storage Subsidies

In this section, we assume that it is not possible to implement a tax that fully internalises the carbon externality, neither directly by a Pigouvian tax nor through the backdoor by a consumption tax. Instead, noting that subsidies for renewable energies and, more recently, also for storage capacities have been the dominating policy instrument in many countries, we examine their optimal levels and how these change as the share of renewables in the energy mix rises.

For a given imperfect carbon tax and no tax on electricity consumption ($\chi = 0, \tau < \delta$), the first-order conditions (33) for optimal subsidy levels become

$$\left(\sigma_r + Q_r \frac{\partial c_r}{\partial Q_r}\right) \frac{dQ_r}{d\sigma_i} + \left(\sigma_s + Q_s \frac{\partial c_s}{\partial Q_s}\right) \frac{dQ_s}{d\sigma_i} = -\rho\left(\delta - \tau\right) \int_{t_0}^{T} \frac{dY_f\left(t, \mathbf{Q}\right)}{d\sigma_i} dt,\ i = r, s. \tag{38}$$

This is the same as condition (35) in the section on Pigouvian taxation, except that τ is no longer available as a policy instrument. Therefore, subsidies for renewables and storage are not only targeted at the capacity cost externality, but also at the non-internalised carbon externality from fossils. The term $(\delta - \tau) \int_{t_0}^{T} \frac{dY_f(t,\mathbf{Q})}{d\sigma_i} dt$ that captures this can be written as $(\delta - \tau) \int_{t_0}^{T} \sum_{j=f,r,s} \frac{\partial Y_f}{\partial Q_j} \frac{dQ_j}{d\sigma_i} dt$.[16] From Table 1, $\frac{\partial Y_f}{\partial Q_j}$ depends on which of the intermediate cases 1 to 3 obtain in equilibrium, which also affects the usage of fossils during storage. Therefore, also optimal subsidies hinge on this, as the following proposition shows (subscripts to the integral sign denote the periods over which the integration applies).

Proposition 3. *For an incomplete carbon and no consumption tax ($\tau < \delta, \chi = 0$), optimal subsidies of renewable and storage capacities for the respective cases of full ($\alpha_s \leq \alpha_1$), partial ($\alpha_s \in (\alpha_1, \alpha_2]$) and no ($\alpha_s > \alpha_2$) usage of the fossil technology during the storage period are*

$$\sigma_r^* = -Q_r \frac{\partial c_r}{\partial Q_r} + \rho\left(\delta - \tau\right) \begin{cases} \left(\int_{d,1,s} dt \frac{\int_{d,1,s} \frac{\partial p(t)}{\partial x(t)} \alpha(t) dt}{\int_{d,1,s} \frac{\partial p(t)}{\partial x(t)} dt}\right) & \text{if} \quad \alpha_s \leq \alpha_1, \\ \left(\int_{d,1} dt \frac{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} \alpha(t) dt}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt} + \int_{2,s} \alpha\left(t\right) dt\right) & \text{if} \quad \alpha_s \in \left(\alpha_1, \alpha_2\right], \\ \left(\int_{d,1} dt \frac{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} \alpha(t) dt}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt} + \int_{2} \alpha\left(t\right) dt\right) & \text{if} \quad \alpha_s > \alpha_2, \end{cases}$$

[16] Note that renewable and storage subsidies have no direct effect on fossil production, i.e., $\frac{\partial Y_f}{\partial \sigma_i} = 0$.

and

$$\sigma_s^* = -Q_s \frac{\partial c_s}{\partial Q_s} + \rho\left(\delta - \tau\right) \begin{cases} \left(\int_{d,1,s} dt \frac{\frac{\partial p_d}{\partial x_d}\frac{1}{\eta_d} - \frac{\partial p_s}{\partial x_s}\frac{1}{\eta_s}}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt + \frac{\partial p_s}{\partial x_s}\int_s dt}\right) & \text{if} \quad \alpha_s \leq \alpha_1, \\ \left(\int_{d,1} dt \frac{\frac{\partial p_d}{\partial x_d}\frac{1}{\eta_d}}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt} - \frac{1}{\eta_s}\right) & \text{if} \quad \alpha_s \in (\alpha_1, \alpha_2], \\ \left(\int_{d,1} dt \frac{\frac{\partial p_d}{\partial x_d}\frac{1}{\eta_d}}{\int_{d,1} \frac{\partial p(t)}{\partial x(t)} dt}\right) & \text{if} \quad \alpha_s > \alpha_2. \end{cases}$$

All expressions for the subsidies have the same intuitive structure: The first term, $Q_i \frac{\partial c_i}{\partial Q_i}$, accounts for the capacity cost externality. According to the other terms, optimal subsidies are higher if less damages are internalised by a carbon tax (large $\delta - \tau$), and if the marginal effect of renewable respectively storage capacities on reducing fossil production is higher (the terms after the curly brackets). The parameter ρ converts this to the net-present value. We now examine the latter effect in more detail.

Start with the *renewable subsidy* σ_r^*. If $\alpha_s \leq \alpha_1$ (first line), then fossils produce at full capacity during storage, destorage, and in the intermediate period that consists of case 1 only (Table 1). This explains the integration range. More renewable capacities crowd fossil capacities out of the market ($\frac{\partial Q_f}{\partial Q_r} < 0$, see appendix) and, thus, reduce fossil production. This effect is larger when the availability of renewables, $\alpha(t)$, is large. Finally, the weighting by $\frac{\partial p(t)}{\partial x(t)}$ captures that this availability is more relevant at times where it reduces the electricity price more strongly because this makes investments in fossils less attractive.

If $\alpha_s \in (\alpha_1, \alpha_2]$ (second line), there are two changes. First, also case 2 where fossils produce only at partial capacity obtains in the intermediate period, which is added to the integration range. Second, during this case and during storage the equilibrium electricity price is constant at $p(t) = k_f + \tau$ so that there are no price effects. Hence a marginal increase in renewable capacities simply raises renewable production by $\alpha(t)$ and replaces fossil production by this amount. Finally, for $\alpha_s > \alpha_2$ (third line), there exist sufficient renewable capacities such that fossils no longer contribute to filling the storage. Therefore, additional renewable capacities have no effect on fossil production during storage and this period is dropped from the integration range.

Now consider the *storage subsidy,* σ_s^*, for which the fractions after the curly bracket represent the marginal effect of storage capacities on fossils, $-\frac{\partial Q_f}{\partial Q_s}$. A higher storage capacity reduces the price of the destorage period as more electricity from the storage is fed into the market ($\int_d \frac{\partial p_d}{\partial Q_s} dt = \frac{\partial p_d}{\partial x_d}\frac{1}{\eta_d} < 0$). This makes investment in fossils less attractive and provides an argument for subsidising storage. Moreover, the effect is smaller if efficiency losses are large (high η_d) so that only a small share of electricity from the storage arrives in the market.

However, as long as fossils contribute to filling the storage—i.e., for $\alpha_s \leq \alpha_2$—there are countervailing effects. If fossils are fully used during storage ($\alpha_s \leq \alpha_1$), a higher storage capacity raises the price of the storage period ($\int_s \frac{\partial p_s}{\partial Q_s} dt = -\frac{\partial p_s}{\partial x_s}\frac{1}{\eta_s} > 0$) and, thus, the profitability of investment in fossils. This provides an argument for taxing storage, especially when efficiency losses are large (low η_s) so that more electricity has to be taken from the market to fill the storage. Moreover, one would expect that demand is more price responsive when prices are high—that is, during destorage—than at the low prices during storage (see Faruqui and Sergici, 2010). This implies that $|\frac{\partial x_d}{\partial p_d}| > |\frac{\partial x_s}{\partial p_s}| \iff |\frac{\partial p_d}{\partial x_d}| < |\frac{\partial p_s}{\partial x_s}|$, further supporting the rationale for taxing storage capacities.

No such price effects obtain if fossils are only partly used during storage ($\alpha_s \in (\alpha_1, \alpha_2]$), because the storage price is constant at $p_s = k_f + \tau$ in this case. Hence $\frac{\partial p_s}{\partial x_s} = 0$ and the corresponding terms for the storage period are dropped. However, now an additional marginal unit of storage capacities allows fossils to employ $\int_s \frac{\partial Y_f}{\partial Q_s} dt = \frac{1}{\eta_s}$ of their idle capacity because more fossils are needed to supplement renewables in filling a larger storage.[17] This provides another reason for taxing storage, which again

[17] From Table 1, $Y_f = x(b_f) - \alpha_s Q_r$ during storage so that $\int_s \frac{\partial Y_f}{\partial Q_s} dt = \int_s \left(-\frac{\partial \alpha_s}{\partial Q_s} Q_r\right) dt = \frac{1}{\eta_s}$, where the last step

increases for higher efficiency losses during storage (low η_s). Finally, in the third line ($\alpha_s > \alpha_2$) renewable capacities are large enough so that fossils no longer contribute to filling the storage. Hence the above effects and, thus, also the puzzling arguments for taxing storage vanish.

Price effects that captured differences in consumers' price-responsiveness at times of low and high prices featured prominently in the above elaborations. If we simplify by assuming linear demand, and ignore the cost externalities, then optimal subsidies have a very simple structure.

Corollary 1. *For linear demand, $\frac{\partial^2 p}{\partial x^2} = 0$, and constant unit costs, $\frac{\partial c_r}{\partial Q_r}, \frac{\partial c_s}{\partial Q_s} = 0$, optimal subsidies are*

$$\sigma_r^* = \rho\left(\delta - \tau\right) \begin{cases} \int_{t_0}^{T} \alpha\left(t\right) dt > 0 & \text{if} \quad \alpha_s \leq \alpha_2, \\ \int_{d,1,2} \alpha\left(t\right) dt > 0 & \text{if} \quad \alpha_s > \alpha_2, \end{cases}$$

and

$$\sigma_s^* = \rho\left(\delta - \tau\right) \begin{cases} \left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right) \leq 0 & \text{if} \quad \alpha_s \leq \alpha_2, \\ \frac{1}{\eta_d} > 0 & \text{if} \quad \alpha_s > \alpha_2. \end{cases}$$

The respective inequalities are strict if there are conversion losses of storage so that storage capacities should be taxed as long as fossils contribute to filling the storage ($\alpha_s \leq \alpha_2$).

Intuitively, the renewable subsidy σ_r^* reflects that an additional unit of renewable capacities displaces fossil production by $\alpha\left(t\right)$ as long as the latter are used. Accordingly, it is constant in the first line where fossils produce for all t. By contrast, the period $\int_{d,1,2} dt$ in the second line is shorter and decreasing in Q_r because the availability of renewables for which case 2 ends, $\alpha_2 = x\left(b_f + \chi\right)/Q_r$, is decreasing in Q_r. Hence the optimal renewable subsidy σ_r^* now falls in the level of installed renewable capacities. This reflects that fossil capacities are used less often so that there is less reason to subsidise their replacement by renewable.

Turning to σ_s^*, storage capacities should be taxed as long as their price increasing effect during storage dominates their price reducing effect during destorage due to conversion losses. This reflects that fossil capacities are more profitable if electricity prices are higher. For $\alpha_s > \alpha_2$, storage capacities only contribute to destorage and, therefore, σ_s^* turns strictly positive.

7 Numerical Illustration of Optimal Subsidies and Discussion

Figure 3 presents results of a numerical simulation of the optimal subsidy scheme with linear demand, constant unit costs and no consumption tax, as in Corollary 1. The parameters are loosely calibrated to German data (see Appendix G for details). Values for optimal unit subsidies are depicted on the right axis (in €/MW and €/MWh, respectively); capacities (in MW and MWh) and total subsidy payments, $\Sigma := \sigma_r Q_r + \sigma_s Q_s$, on the left axis.[18] The (small) diamonds show efficient capacities (Q_r^*, Q_s^*, Q_f^*) that would occur with a Pigouvian tax $\tau^* = \delta$.

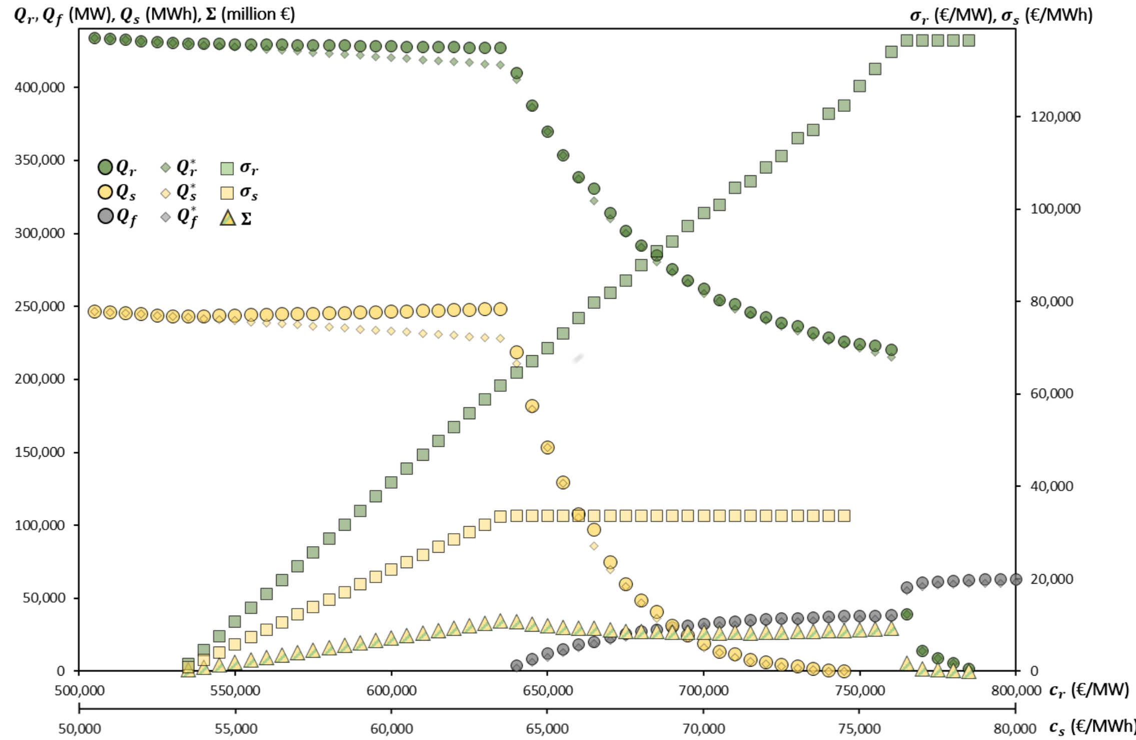
The figure should be read from the right to the left. Then all values are depicted as a function of unit capacity costs of renewables (c_r) and storage (c_s) that are falling at the same rate. Whereas the preceding analysis was restricted to the case where positive quantities of all three technologies are installed in equilibrium, we now consider a broader cost range. It also includes the situations where renewables and storage enter the market, and where they have fully captured it.

Intuitively, renewable capacities enter the market first, supported by a subsidy that is constant at the level 136,600 €/MW as long as fossils produce for all t (in line with Corollary 1). These are roughly 17% of initial capacity costs. Storage capacities follow once the resulting volume of intermittent supply is large enough to make buffering electricity economically viable. Due to relatively high capacity costs and conversion losses (19 per cent in our calibration), this only happens when renewables have reached

follows by substitution for $\frac{\partial \alpha_s}{\partial Q_s}$ from Lemma 2.

[18] Note that storage capacity is measured in MWh. This distinction between power and energy was irrelevant in the theoretical model, but is important now.

Fig. 3: Market diffusion and optimal subsidies



Q_r, Q_s, Q_f are capacities of renewables, storage and fossils that result from optimal subsidies (σ_r, σ_s), depicted as functions of unit capacity costs c_r, c_s. Q_r^*, Q_s^*, Q_f^* are efficient capacity levels. Σ are total subsidy payments.

a capacity of 224 GW.[19] This is large enough to completely satisfy electricity demand at times of high availability during which storage takes place. Therefore, the situations where fossils profited from the higher electricity prices due to storage, which provided the rationale for taxing storage capacities, are leapfrogged.[20] Once storage capacities enter the market, they receive a constant subsidy that initially makes up roughly 50% of their capacity costs. By contrast, the renewable subsidy is gradually decreasing (both values are exactly those of Corollary 1 for $\alpha_s > \alpha_2$). However, the lower rate is paid for larger capacities so that total subsidy payments, Σ, are even slightly increasing until fossils are completely driven out of the market.

Even thereafter, falling but still substantial subsidies are required to prevent fossils from re-entering the market. In the analytical model, this boundary case where subsidies are chosen such that they are just sufficient to keep fossil firms out requires that their first-order condition (28) is satisfied at $Q_f = 0$. In particular, noting that a re-entering fossil firm would produce at full capacity during destorage and case 1, the first-order condition becomes (during case 2 fossils make zero profits)

$$\rho \int_{d,1} \left(p\left(t, \mathbf{Q}\right) - \tau - k_f \right) dt - c_f = 0, \text{ where } \mathbf{Q} = \left(0, Q_r, Q_s\right). \tag{39}$$

[19] This value is similar to current wind and solar capacities in Germany, where dividing maximum production of 59 GW (attained on April 23, 2019, see https://energy-charts.info/charts/power/chart.htm?l=de&c=DE&year=2019&interval=year) by $\alpha_{\max} = 0.29$ (see Appendix G) yields 203 GW. Corresponding optimal subsidies for renewables of 136,600 €/MW translate into a market premia of 68.24 €/MWh when paid for 20 years (applying 3% discounting). The observed market premia in Germany in 2019 was 37.76 €/MWh (https://www.netztransparenz.de/EEG/Marktpraemie/Marktwerte).

[20] Results of a model calibration that leads to an earlier build-up of storage capacities and, thus, taxes in the initial stages are available upon request. In a nutshell, they require lower costs and lower conversion losses of storages.

Obviously, this condition would be met if renewable and storage capacities, Q_r, Q_s, were kept constant at the level that solves this equation, which is roughly the case in our numerical simulations. From the first-order conditions (29) and (30) for capacity choices of renewable and storage firms, this requires that falling capacity costs are balanced by lower subsidies; as in Figure 3. By contrast, when renewable and storage capacities have become cheap enough to defend the market without subsidies, both rise in response to further falling costs.

Next, consider the evolution of capacities in the initial stages. The market diffusion of renewables is slow in the beginning, then accelerates rapidly, and thereafter slows down again. The evolution of fossil capacities matches this pattern in opposite direction. This is in line with the result in Helm and Mier (2019), but the additional storage technology accelerates the build-up of renewables and the phase out of fossils. Storage capacities are increasing exponentially despite constant subsidies. The reason is their rising market value as there are more variable renewables and less reliable fossils.

Finally, remember that for the situations analysed in Section 6.2, efficient capacities would also obtain from a consumption tax $\chi^{\#} = \delta - \tau$, if it is complemented by subsidies that correct the resulting distortions. For storage firms, the benefits of a lower before tax price during storage dominate, requiring a constant tax of $\sigma_s^{\#} = \rho\chi^{\#}\left(\frac{1}{\eta_d} - \frac{1}{\eta_s}\right) = -7,490$ €/MWh. By contrast, renewables must receive a *constant* subsidy $\sigma_r^{\#} = \rho\left(\delta - \tau\right)\int_{t_0}^{T} \alpha\left(t\right) dt = 136,600$ €/MW (not depicted), which leads to substantially higher subsidy costs than the second-best subsidies in Figure 3. This reflects that as fossils are increasingly driven out of the market, the consumption tax must be paid for ever higher shares of electricity from renewables, making the necessary compensation increasingly costly.

8 Concluding Remarks

Using a peak-load pricing model, we started by verifying that intermittent supply of renewable technologies and electricity storage do not compromise the efficiency of a Pigouvian tax on fossil electricity production. Nonetheless, countries usually fail to impose such a first-best carbon tax. Instead, they have often implemented subsidies, especially for renewables, that are financed by taxes on electricity consumption. If the consumption tax is not determined by the aim to finance subsidies but set at the Pigouvian level, then it provides the same incentives as a Pigouvian tax on fossil production; simply because the tax incidence is independent of who pays it. However, the consumption tax also penalizes electricity that comes from renewables and from the storage. This distorts investment decisions that must be corrected by subsidising renewables and *taxing* storage capacities. Moreover, the efficiency of this tax/subsidy scheme breaks down for high shares of renewables in the energy mix. Finally, a model with several fossil technologies that have different carbon intensities would require differentiated Pigouvian and, therefore, differentiated consumption taxes for the *homogeneous* good electricity (see also Abrell et al. (2019)).

We then analysed second-best subsidies for installing capacities of intermittent renewable energies and storage that are financed by lump-sum taxes. Renewables reduce the profitability of fossil investments by lowering expected prices and by displacing fossil production. This provides a rationale for the subsidisation of renewable energies. However, these effects lose relevance as the share of fossils in the energy mix falls; hence also the subsidy rate should fall. Storage capacities raise the electricity price when the storage is filled and lower it during destorage. This has countervailing effects on the profitability of fossils. Due to round-trip efficiency losses the volume of electricity taken from the market during storage exceeds the volume provided during destorage. Hence the price increasing effect during storage dominates, and as long as fossils benefit from this it is usually optimal to tax storage capacities. As this result is driven by conversion losses over a storage cycle, technologies for which these are low—like batteries—should be charged with a lower tax. Moreover, once renewables have risen sufficiently so that fossils no longer contribute to electricity storage, it is optimal to switch to a subsidy.

Thus, our results provide an argument for gradually reducing the subsidy for renewables as their market penetration rises, and raising the subsidy for storage instead. Importantly, the latter is not targeted at supporting the rising share of renewables, because the market provides sufficient incentives to build storage capacities if there is more fluctuating electricity from renewables. Rather, storage of electricity is subsidised because it substitutes fossil production when the availability of renewables is low.

Our analysis also accounts for a cost externality, e.g., from learning or economy-wide economies of scale. Given the current technological progress in storage technologies such as batteries and power-to-gas, these externalities may well provide an overriding argument for always subsidising storage. An equivalent argument strengthens the initial case for subsiding renewables. However, as they become more prevalent, problems such as the scarcity of suitable sites for wind gain relevance. These constitute a negative cost externality, providing an additional argument to reduce the subsidy rate for renewables.

In a calibrated numerical optimization of our analytical model that abstracts from cost externalities, storage capacities only enter the market once their electricity demand can be fully met by renewables. Hence it is always optimal to subsidise storage. However, in the real world fossil capacities nearly always produce at times of electricity storage and, thus, benefit from the price increase of the resulting higher electricity demand.[21] Similarly, electric vehicles are usually charged to a substantial extent with electricity from burning fossil fuels. Even worse, batteries in electric vehicles are almost never used for destorage. Hence they never exert a price dampening effect that would erode the competitiveness of fossils. Both effects currently weaken the case for subsidising electric vehicles to reduce CO_2 emissions; although other reasons such as reducing local air pollution may still justify subsidies (see Holland, Mansur, Muller, and Yates, 2016).

In summary, even if capacity levels under the optimal subsidy scheme are relatively close to their efficient values—as is the case in our numerical simulation—the required subsidies for renewables and storage are substantially more complex than a first-best carbon tax. They vary, sometimes even in their sign, depending on the relative shares of the three technologies in the power system. They also require substantial knowledge about the electricity market—such as demand sensitivity. This complexity is driven by the intermittency of renewables, an aspect that is still often neglected in the analytical literature. Given the large public funds that currently subsidise renewables, and, increasingly, storage, a better understanding of this is highly policy relevant. The Pigouvian tax, by contrast, simply equals the environmental unit costs of fossil production. This reflects its central advantage that it directly addresses the externality.

We now discuss some further limitations and potential extensions of our analysis that need be taken into account when drawing policy recommendations. First, we have ignored effects that may result from the interaction with existing overlapping instruments. Most importantly, an increasing number of countries are implementing cap-and-trade systems that substantially impact the effectiveness of subsidies for renewables (Jarke and Perino, 2017). Second, we only considered subsidies for capacities, whereas the dominating instruments for renewables have been feed-in-tariffs and market premia—that is, a subsidisation of electricity output. However, these are quite similar in that both are paid independently of the price that obtains on the market for electricity. Moreover, in our numerical simulation renewable capacities are always fully used due to a sufficient storage capacity. In this case, a subsidy per unit of output is equivalent to a subsidy per unit of capacity that is available on average. Note that this argument does not apply to the storage technology. The reason is that a subsidy per unit of electricity that is stored or/and destored would not only affect decisions to build up capacity, but also distort storage decisions—in contrast to renewables for which production is driven by the exogenous availability parameter $\alpha(t)$.

Third, we have restricted the analysis to one renewable and one fossil technology. This could be extended relatively straightforwardly to several renewable technologies—e.g., PV, offshore and onshore wind—with technology specific availability factors, $\alpha_l(t)$.[22] There would then be a separate optimal subsidy rate for each renewable technology such that technologies which (on average) reduce

[21] Note that the largest share of storage is done by commercial customers that face variable prices, as in our model.

[22] There would then be a profit maximisation problem for each type l of renewable firms (Eqs. (6) and (7)), and supply from renewables would be given by the sum over all technologies, $\sum_l \alpha_l(t) Q_{lr}$.

electricity prices more strongly receive a higher subsidy rate.[23] With peak prices around midday, this would suggest a subsidy mark-up for solar power. By contrast, with high PV shares there may be consistently higher prices during the winter season, which would suggest a subsidy mark-up for more stable offshore wind power. If several fossil technologies were considered, then a different emission intensity of the marginal technology during storage and destorage periods would affect the analysis (see Carson and Novan, 2013). As destorage takes places during high price periods, one might expect that it displaces primarily gas with comparatively low emission intensity.

A related extension would include several storage technologies so as to better address the different storage needs that result for different renewable technologies (see Sinn (2017) and Zerrahn, Schill, and Kemfert (2018) for a discussion). For example, battery storage appears most suitable to buffer the daily intermittency of solar power, whereas power-to-gas better fits the long-term storage requirements of seasonal fluctuations. Just as for the case of several renewables, optimal subsidy rates would differ across technologies. Moreover, due to cost considerations it is unlikely that fossils will contribute to power-to-gas storage to any significant extent. This makes the optimality of a storage tax seem much less likely for the seasonal storage technology.

Fourth, one could relax Assumption 1 that the availability of renewables follows an identical repetitive pattern. As long as it remains optimal to completely fill and empty the storage during one period, the optimisation problems at stage 3 (production, storage and consumption) would still be as analysed in Section 4.1. Similarly, profits of capacity investments would still be based on the net present value of the resulting profit streams, the only difference being that per period profits may differ. This suggests that also the basic results regarding policy instruments should continue to hold. Similar arguments apply to the integration of some unforcastability into the model.

Finally, the model could be extended by including other market failures—e.g., non-reactive consumer demand, distortionary taxation, and imperfect competition—or further aspects of electricity markets, such as variable demand, trade, and the transmission grid. However, the first extension would make it more difficult to isolate the effects of the pollution externality that was the focus of this contribution, and the second would probably come at the cost of greater reliance on numerical simulations.

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[23] For example, in Proposition 3 this would be captured by terms $\int_{t_0}^{T} \frac{\partial p(t)}{\partial x(t)} \alpha_l(t)\, dt$, where $\alpha_l(t)$ is technology specific.

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Appendix

A Proof of Lemma 1

As shown in the main text, for fossil and renewable firms conditions (4) and (5) as well as (8) and (9) are sufficient for optimality. For storage firms, note that $\eta\left(y_s\right)$ has the constant values η_d, η_s and 1 during the destorage, storage and intermediate period. Hence the Hamiltonian $\mathcal{H}_s$ as given in (15) is piecewise linear in $y_s\left(t\right)$. Moreover, the constraints (13) and (14) are linear in $s\left(t\right)$. Therefore, conditions (17) to (22) are sufficient for optimality, if $\lambda(t)$ is continuous (see Seierstad and Sydsaeter (1987, p. 317-318)), which we show below.

We now prove that the equilibrium values in Lemma 1 satisfy these conditions, starting with the constant electricity price during storage and destorage, that is, for $y_s\left(t\right) \neq 0$. If $p\left(t\right) \neq \lambda\left(t\right)\eta\left(y_s\right)$, there can be no $y_s^*\left(t\right) \neq 0$ that maximizes (17); hence $p\left(t\right) = \lambda\left(t\right)\eta\left(y_s\right)$ during storage and destorage periods. Intuitively, the adjoint variable $\lambda\left(t\right)$ is usually interpreted as the change in the value function due to a unit increase in the state variable, $s\left(t\right)$. Thus, $\lambda\left(t\right)$ is the value of stored electricity which, after being weighted by conversion losses, must equal the price of electricity. Moreover, during storage and destorage periods the storage can neither be full nor empty (except at the boundaries), i.e., $s\left(t\right) < q_s$ and $s\left(t\right) > 0$. Thus, $\varphi_s\left(t\right) = \varphi_d\left(t\right) = 0$ from the complementary slackness conditions in (20) and (21) so that $\dot{\lambda}\left(t\right) = \varphi_s\left(t\right) - \varphi_d\left(t\right) = 0$ from (19). Finally, by assumption the round-trip efficiency loss parameter is constant at $\eta\left(y_s\right) = \eta_s$ during storage and at $\eta\left(y_s\right) = \eta_d$ during destorage. Using $p\left(t\right) = \lambda\left(t\right)\eta\left(y_s\right)$ it follows that not only $\lambda\left(t\right)$, but also prices and, therefore, demand, are constant.

Regarding the other values during the *destorage period*, it is straightforward to see that the solution $y_r\left(t\right) = \alpha\left(t\right)q_r$ and $y_f\left(t\right) = q_f$ satisfies (4), (5), (8), and (9) for $\mu_r, \mu_f > 0$, and results in a price $p\left(t\right)$ above the total unit costs of fossils, b_f.[24] Turning to storage firms, conditions (17) and (19) to (21) have already been used to derive the result of a constant price during destorage. The remaining condition (18) describes how an initially full storage is completely depleted during the destorage period

[24] Note that firms are identical so that $y_r\left(t\right) = \alpha\left(t\right)q_r$ implies $Y_r\left(t\right) = n_r y_r\left(t\right) = \alpha(t)Q_r$, where n_r is the number of renewable firms. An equivalent argument applies to the other quantities in the proof.

(see main text). Therefore, integrating this equation of motion (18) over the two destorage intervals must satisfy $Q_s = \eta_d \int_{t_0}^{t_d} Y_s(t)\, dt + \eta_d \int_{t'_d}^{T} Y_s(t)\, dt$. Substitution from (23) yields the condition that implicitly determines the critical availability α_d below which destorage obtains ($\int_d dt := \int_{t_0}^{t_d} dt + \int_{t'_d}^{T} dt$ is a shorthand notation for the combined duration of the two destorage intervals):

$$Q_s \quad = \quad \eta_d \int_d (\alpha_d - \alpha(t))\, Q_r dt. \tag{40}$$

Once the storage has run empty, we have $s(t) = 0$ so that $\varphi_d(t)$ turns (weakly) positive (see (21)). This initiates the *first intermediate period*, $t \in (t_d, t_s)$, during which $\dot{s}(t) = -y_s(t) = 0$ (eq. (18)). For renewables, (8) and (9) imply that $y_r(t) = \alpha(t)\, q_r$ for any $p(t) > 0$, whereas for $p(t) = 0$ any output $y_r(t)$ is profit maximizing due to marginal costs of zero. Moreover, the fact that $\alpha(t)$ is increasing during the first intermediate period implies that, ceteris paribus, $p(t)$ is decreasing. As long as $p(t) > b_f$, we have $\mu_f(t) > 0$ from (4) so that $y_f(t) = q_f$ from (5). This *case* 1 continues until full usage of fossil and available renewable capacities have lowered the price to the total unit costs of fossils, $p(t) = b_f$.

Noting that consumers may face a consumption tax χ, *case* 2 starts when $x(b_f + \chi) = \alpha(t)\, Q_r + Q_f$ which yields $\alpha_1 = \frac{x(b_f+\chi)-Q_f}{Q_r}$. During case 2, fossils continue to be used so that (4) still binds. Hence, $p(t) = b_f$ implies $\mu_f(t) = 0$ from (4) so that $0 < y_f(t) < q_f$ by the complementary slackness conditions in (5). Once available renewable capacities, $\alpha(t)\, Q_r$, are large enough to satisfy demand at consumers' after tax price $b_f + \chi$, we enter *case 3*. Accordingly, case 3 starts when $x(b_f + \chi) = \alpha(t)\, Q_r$ so that $\alpha_2 = \frac{x(b_f+\chi)}{Q_r}$. For $\alpha(t) > \alpha_2$, we have $p(t) - b_f < 0$ so that $y_f(t) = 0$ from (4).

It remains to consider *storage firms* during the first intermediate period, for which $q_s > s(t)$ so that $\varphi_s(t) = 0$ by condition (20). Two paragraphs above, we have already addressed conditions (21) and (18). It remains to show that $y_s(t) = 0$ maximizes $[p(t) - \lambda(t)\, \eta(y_s)]\, y_s(t)$ (eq. (17)). Using $\varphi_s(t) = 0$, condition (19) implies $\dot{\lambda}(t) = -\varphi_d(t) \leq 0$. Moreover, $\eta_d \geq 1$ and $\eta_s \leq 1$ so that $\lambda(t)\, \eta_d \geq \lambda(t) \geq \lambda(t)\, \eta_s$, with strict inequality if there are conversion losses. Figure 4 depicts this situation during the first intermediate period, $t \in [t_d, t_s]$, such that $\lambda(t)$ is continuous, which was a precondition for (17) to (22) being sufficient (λ_d and λ_s are the constant values of $\lambda(t)$ during destorage and storage). Moreover, during $t \in [t_d, t_s]$ the price $p(t)$ is monotonically decreasing from its level during destorage, $p_d = \lambda_d \eta_d$, to its level during storage, $p_s = \lambda_s \eta_s$. This is represented by the dashed line. Using $\dot{\lambda}(t) = -\varphi_d(t)$ it is straightforward to see that the values of the multiplier $\varphi_d(t)$ can be chosen such that $\lambda(t)\, \eta_d > p(t) > \lambda(t)\, \eta_s$. Using this, $y_s(t) > 0$ would lead to $[p(t) - \lambda(t)\, \eta_d]\, y_s(t) < 0$, and $y_s(t) < 0$ to $[p(t) - \lambda(t)\, \eta_s]\, y_s(t) < 0$. Therefore, $y_s(t) = 0$ must be optimal.

As shown in the main text, the *storage period* ($y_s(t) < 0$) can start during either of the cases 1 to 3 and it has a price that equals the one at the end of the intermediate period, i.e., $p_s = p(t_s)$. Hence, $Y_f = Y_f(t_s)$ satisfies (4) and (5). Moreover, for $p_s > 0$ conditions (8) and (9) imply $Y_r = \alpha(t) Q_r$, whereas for $p_s = 0$ any Y_r is profit maximizing due to the assumption of no variable costs. Turning to storage firms, the argument parallels that during the destorage period: Conditions (17) and (19) to (21) have already been addressed. The empty storage is completely filled during the storage period so that integration of the equation of motion (18) yields $Q_s = -\eta_s \int_{t_s}^{t'_s} Y_s(t)\, dt$. In the case of no excess capacities, $-Y_s(t) = (\alpha(t) - \alpha_s)\, Q_r$ and substitution yields ($\int_s dt := \int_{t_s}^{t'_s} dt$ denotes the duration of the storage period)

$$Q_s \quad = \quad \eta_s \int_s (\alpha(t) - \alpha_s)\, Q_r dt, \tag{41}$$

Similarly, substitution of the storage values for the alternative case of excess capacities from Table

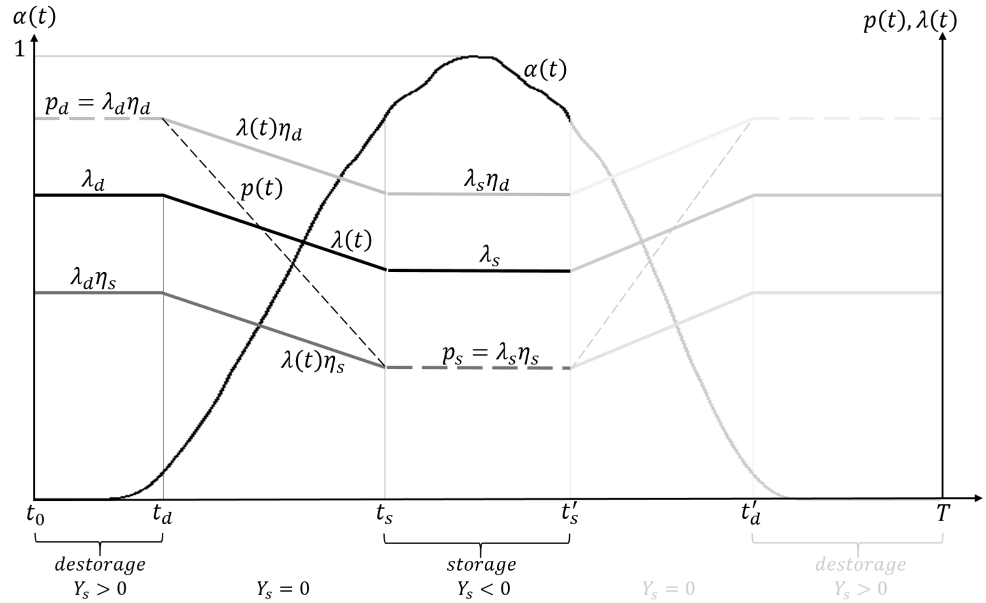

Fig. 4: Availability of renewables and resulting values of the adjoint variable

1 gives

$$Q_s = -\eta_s \left[\int_{\alpha_s}^{\alpha_c} \left(x\left(0+\chi\right) - \alpha Q_r \right) d\alpha + \int_{\alpha_c}^{\alpha_{\max}} \left(x\left(0+\chi\right) - \alpha_c Q_r \right) d\alpha \right], \tag{42}$$

where we integrate over α to simplify notation.

It remains to determine α_s, α_c and which of the two cases obtains. Without excess capacities, the critical availability, α_s, when the storage period starts is implicitly determined by (41). With excess capacities, it follows immediately from $\alpha_s Q_r = x\left(0+\chi\right)$. Moreover, $\alpha_s Q_r \leq x\left(0+\chi\right)$ as otherwise there would be excess capacities during case 3. Therefore, $\alpha_s = \min\left\{ \alpha_s \text{ that solves (41)}, \frac{x(0+\chi)}{Q_r} \right\}$ and for the case of no excess capacities $\alpha_s < \frac{x(0+\chi)}{Q_r}$. Finally, α_c only exists for the situation with excess capacities and follows implicitly from (42) after substitution of $\alpha_s = \frac{x(0+\chi)}{Q_r}$.

B Proof of Lemma 2

The statements in (c) that relate to the situation $\alpha_s Q_r = x\left(0+\chi\right)$ (excess capacities of renewables) follow immediately from implicit differentiation of this expression and the fact that demand is constant at $x\left(0+\chi\right)$. Hence we only need to prove the statements in (a) and (b) that concern the situation without excess capacities ($\alpha_s Q_r < x\left(0+\chi\right)$).

Conditions (40) and (41) that implicitly determine α_d and α_s can be written as

$$f_d := \eta_d \int_{t_0}^{t_d} \left(\alpha_d - \alpha\left(t\right)\right) Q_r dt + \eta_d \int_{t_d'}^{T} \left(\alpha_d - \alpha\left(t\right)\right) Q_r dt - Q_s = 0, \tag{43}$$

$$f_s := -\eta_s \int_{t_s}^{t_s'} \left(\alpha_s - \alpha\left(t\right)\right) Q_r dt - Q_s = 0. \tag{44}$$

The comparative static effects of a change in Q_f, Q_r, or Q_s, thereby taking the other capacities as given, follow from applying the implicit function theorem, i.e., $\frac{\partial \alpha_u}{\partial Q_j} = -\frac{\partial f_u}{\partial Q_j} / \frac{\partial f_u}{\partial \alpha_u}$ for $u = d, s$ and

$j=f,r,s$. It follows that $\frac{\partial \alpha_s}{\partial Q_f}=\frac{\partial \alpha_d}{\partial Q_f}=0$. Next, note that $\alpha_d=\alpha(t_d)=\alpha(t_d')$ and $\alpha_s=\alpha(t_s)=\alpha(t_s')$. This implies that the integral terms in (43) and (44) are zero if evaluated at the boundaries of the integral, t_d, t_d' and t_s, t_s', respectively. Using this when applying the implicit function theorem yields the comparative statics $\frac{\partial \alpha_d}{\partial Q_r}, \frac{\partial \alpha_d}{\partial Q_s}$, and $\frac{\partial \alpha_s}{\partial Q_s}, \frac{\partial \alpha_s}{\partial Q_r}$ in Lemma 2.

Demand during destorage, $x_d=\sum_j Y_j(t)=Q_f+\alpha_d Q_r$, follows straightforwardly from Lemma 1 and Table 1. Differentiation yields $\frac{\partial x_d}{\partial Q_r}=\alpha_d+\frac{\partial \alpha_d}{\partial Q_r}Q_r$ and $\frac{\partial x_d}{\partial Q_s}=\frac{\partial \alpha_d}{\partial Q_s}Q_r$. Substitution of $\frac{\partial \alpha_d}{\partial Q_r}, \frac{\partial \alpha_d}{\partial Q_s}$ yields the values in Lemma 2, where we have used

$$\frac{\partial x_d}{\partial Q_r} = \alpha_d-\frac{\int_d(\alpha_d-\alpha(t))\,dt}{\int_d dt}=\alpha_d-\alpha_d\frac{\int_d dt}{\int_d dt}+\frac{\int_d \alpha(t)\,dt}{\int_d dt}. \tag{45}$$

For storage, demand depends on the case that obtains at the beginning of the storage period. From Table 1, $x_s=\sum_j Y_j(t)=Q_f+\alpha_s Q_r$ if it starts during case 1, and $x_s=\alpha_s Q_r$ if it starts during case 3. In both situations, $\frac{\partial x_s}{\partial Q_r}=\alpha_s+\frac{\partial \alpha_s}{\partial Q_r}Q_r$ and $\frac{\partial x_s}{\partial Q_s}=\frac{\partial \alpha_s}{\partial Q_s}$. The values in the Table in Lemma 2 follow again after substituting for $\frac{\partial \alpha_s}{\partial Q_r}, \frac{\partial \alpha_s}{\partial Q_s}$, thereby applying the same steps as in (45) to x_s. Finally, if storage starts during case 2, then $x_s=\sum_j Y_j(t)=x(b_f+\chi)$ so that $\frac{\partial x_s}{\partial Q_j}=0$ for $j=f,r,s$.

C Derivation of Equation (33)

In this proof we use the compact notations $\sum_j$ and $\sum_i$ for summation over all three technologies $j,i=f,r,s$. Using $x(t,\chi,\tau,\mathbf{Q})=\sum_j Y_j(t,\chi,\tau,\mathbf{Q})$, the first integrand term in (32) can be written as

$$\frac{dx(t,\chi,\tau,\mathbf{Q})}{d\theta}=\sum_j\left(\frac{\partial Y_j(t,\chi,\tau,\mathbf{Q})}{\partial\theta}+\sum_i\frac{\partial Y_j(t,\chi,\tau,\mathbf{Q})}{\partial Q_i}\frac{dQ_i}{d\theta}\right). \tag{46}$$

The first term represents the direct effects of policy instruments θ on production (it is zero for subsidies), and the second the indirect effects via capacity choices. Moreover, this latter effect can be written out as (dropping the arguments)

$$\sum_j\left(\sum_i\frac{\partial Y_j}{\partial Q_i}\frac{dQ_i}{d\theta}\right)=\sum_j\frac{\partial Y_j}{\partial Q_j}\frac{dQ_j}{d\theta}+\frac{\partial(Y_r+Y_s)}{\partial Q_f}\frac{dQ_f}{d\theta}+\frac{\partial(Y_f+Y_s)}{\partial Q_r}\frac{dQ_r}{d\theta}+\frac{\partial(Y_f+Y_r)}{\partial Q_s}\frac{dQ_s}{d\theta}, \tag{47}$$

where the first term sums up the effects of capacity on production within type-j firms, whereas the other terms summarise the cross effects. From Table 1, $\frac{\partial(Y_r(t)+Y_s(t))}{\partial Q_f}=0$ for all t. Moreover, also $\int_{t_0}^T(p(t)+\chi)\frac{\partial Y_s(t)}{\partial Q_r}\frac{dQ_r}{d\theta}dt=0$. To see this, note that $Y_s(t)=0$ during the intermediate period and, by construction, the available storage capacity is completely filled during storage and completely emptied during destorage, independent of Q_r. Hence $\int_d(p(t)+\chi)\frac{\partial Y_s(t)}{\partial Q_r}\frac{dQ_r}{d\theta}dt=(p_d+\chi)\frac{dQ_r}{d\theta}\frac{\partial}{\partial Q_r}\left(\int_d Y_s(t)\,dt\right)=0$, and equivalently for the storage period. These terms can be eliminated when substituting (46) together with (47) into (32). After rearranging terms, this yields

$$\begin{aligned}\frac{dW}{d\theta} &= \sum_j\left(\rho\int_{t_0}^T(p(t)+\chi)\frac{\partial Y_j(t)}{\partial Q_j}dt-c_j(Q_j)\right)\frac{dQ_j}{d\theta}-\rho\int_{t_0}^T(k_f+\delta)\frac{dY_f(t)}{d\theta}dt-\sum_{i=r,s}Q_i\frac{\partial c_i}{\partial Q_i}\frac{dQ_i}{d\theta}\\ &\quad+\rho\int_{t_0}^T(p(t)+\chi)\left(\sum_{i=r,s}\frac{\partial Y_f(t)}{\partial Q_i}\frac{dQ_i}{d\theta}+\frac{\partial Y_r(t)}{\partial Q_s}\frac{dQ_s}{d\theta}\right)dt+\rho\int_{t_0}^T(p(t)+\chi)\sum_j\frac{\partial Y_j(t)}{\partial\theta}dt,\end{aligned}$$

where the last integral sums up the direct effects. Noting that $\frac{\partial Y_j(t)}{\partial Q_j}=\frac{\partial y_j(t)}{\partial q_j}$, substitution from the first-order conditions (28) to (30) for firms' capacity choices and collecting terms gives

$$
\begin{aligned}
\frac{dW}{d\theta} \;=\; & \sum_{i=r,s}\left(\rho\int_{t_0}^{T}\chi\frac{\partial Y_i\left(t\right)}{\partial Q_i}dt - Q_i\frac{\partial c_i}{\partial Q_i}-\sigma_i\right)\frac{dQ_i}{d\theta}+\rho\int_{t_0}^{T}\left(\tau+k_f+\chi\right)\frac{\partial Y_f\left(t\right)}{\partial Q_f}\frac{dQ_f}{d\theta}dt \\
& -\rho\int_{t_0}^{T}\left(k_f+\delta\right)\frac{dY_f\left(t\right)}{d\theta}dt+\rho\int_{t_0}^{T}\left(p\left(t\right)+\chi\right)\left(\sum_{i=r,s}\frac{\partial Y_f\left(t\right)}{\partial Q_i}\frac{dQ_i}{d\theta}+\frac{\partial Y_r\left(t\right)}{\partial Q_s}\frac{dQ_s}{d\theta}\right)dt \\
& +\rho\int_{t_0}^{T}\left(p\left(t\right)+\chi\right)\sum_{j}\frac{\partial Y_j\left(t\right)}{\partial\theta}dt. \qquad (48)
\end{aligned}
$$

Using $\frac{dY_f(t,\chi,\tau,\mathbf{Q})}{d\theta}=\frac{\partial Y_f(t)}{\partial\theta}+\sum_j\frac{\partial Y_f(t)}{\partial Q_j}\frac{dQ_j}{d\theta}$, we have

$$
-\left(k_f+\delta\right)\frac{dY_f\left(t\right)}{d\theta}=-\left(\delta-\tau-\chi\right)\frac{dY_f\left(t\right)}{d\theta}-\left(\tau+\chi+k_f\right)\left(\frac{\partial Y_f\left(t\right)}{\partial\theta}+\frac{\partial Y_f\left(t\right)}{\partial Q_f}\frac{dQ_f}{d\theta}+\sum_{i=r,s}\frac{\partial Y_f\left(t\right)}{\partial Q_i}\frac{dQ_i}{d\theta}\right).
$$

Substitution and using this to cancel the second term in (48) yields

$$
\begin{aligned}
\frac{dW}{d\theta} \;=\; & \sum_{i=r,s}\left(\rho\int_{t_0}^{T}\chi\frac{\partial Y_i\left(t\right)}{\partial Q_i}dt-Q_i\frac{\partial c_i}{\partial Q_i}-\sigma_i\right)\frac{dQ_i}{d\theta}-\rho\int_{t_0}^{T}\left(\delta-\tau-\chi\right)\frac{dY_f\left(t\right)}{d\theta}dt \\
& +\rho\int_{t_0}^{T}\left(p\left(t\right)-k_f-\tau\right)\sum_{i=r,s}\frac{\partial Y_f\left(t\right)}{\partial Q_i}\frac{dQ_i}{d\theta}dt+\rho\int_{t_0}^{T}\left(p\left(t\right)-\tau-k_f\right)\frac{\partial Y_f\left(t\right)}{\partial\theta}dt \\
& +\rho\int_{t_0}^{T}\left(p\left(t\right)+\chi\right)\sum_{i=r,s}\frac{\partial Y_i\left(t\right)}{\partial\theta}dt+\rho\int_{t_0}^{T}\left(p\left(t\right)+\chi\right)\frac{\partial Y_r\left(t\right)}{\partial Q_s}\frac{dQ_s}{d\theta}dt.
\end{aligned}
$$

Note that $\left(p\left(t\right)-\tau-k_f\right)\frac{\partial Y_f}{\partial Q_i}=0$, $i=r,s$, because $\frac{\partial Y_f}{\partial Q_i}=0$ except during stage 2 (and during storage if the storage period starts in case 2) for which, however, $p\left(t\right)=\tau+k_f$. An equivalent argument yields $\left(p\left(t\right)-\tau-k_f\right)\frac{\partial Y_f}{\partial\theta}=0$, so that the second line vanishes. Defining $z:=\rho\int_{t_0}^{T}\left(p\left(t\right)+\chi\right)\left(\sum_{i=r,s}\frac{\partial Y_i(t)}{\partial\theta}+\frac{\partial Y_r(t)}{\partial Q_s}\frac{dQ_s}{d\theta}\right)dt$, we obtain the wanted expression in (33).

It remains to specify z for the different periods and policy instruments. From Table 1, we have $\frac{\partial Y_s(t)}{\partial\theta},\frac{\partial Y_r(t)}{\partial\theta}=0$ for $\theta=\sigma_r,\sigma_s,\tau$. If $\alpha_s Q_r<x\left(0+\chi\right)$, then also $\frac{\partial Y_r(t)}{\partial Q_s},\frac{\partial Y_s(t)}{\partial\chi},\frac{\partial Y_r(t)}{\partial\chi}=0$. Therefore, z can only be non-zero if there are excess capacities of renewables ($\alpha_s Q_r=x\left(0+\chi\right)$). Remember that for this case $p(t)=0$ and $Y_r\left(t\right)=\alpha_c Q_r$ for all $\alpha\left(t\right)>\alpha_c$. Implicit differentiation of (42) at $\alpha_s=\frac{x(0+\chi)}{Q_r}$ gives $\frac{\partial a_c}{\partial Q_s}=\frac{1}{\eta_s\int_{\alpha_c}^{\alpha_{\max}}Q_r d\alpha}>0$. This reflects that with a larger storage capacity renewable production is capped later. Using this, $\int_{t_0}^{T}\frac{\partial Y_r(t)}{\partial Q_s}dt=\int_{\alpha_c}^{\alpha_{\max}}\frac{\partial\alpha_c Q_r}{\partial Q_s}d\alpha=\int_{\alpha_c}^{\alpha_{\max}}\frac{Q_r}{\eta_s\int_{\alpha_c}^{\alpha_{\max}}Q_r d\alpha}d\alpha=\frac{1}{\eta_s}$, which takes into account that $\frac{\partial Y_r(t)}{\partial Q_s}=0$ except during storage. Moreover, from Table 1, for $\alpha\left(t\right)\leq\alpha_s$ the term $\sum_{i=r,s}Y_i\left(t\right)$ does not *directly* depend on any instrument θ. This is also the case for $\alpha\left(t\right)>\alpha_s$, except for the instrument $\theta=\chi$. In particular, $\int_{t_0}^{T}\sum_{i=r,s}\frac{\partial Y_i(t)}{\partial\chi}dt=\int_{\alpha_s}^{\alpha_{\max}}\frac{\partial x(0+\chi)}{\partial\chi}d\alpha$, which explains the difference in the second and third line of (34).

D Proof of Proposition 1

It remains to show that the policy mix in the Proposition leads to first-best capacity levels, which we now determine. In equilibrium, demand equals supply so that $\frac{\partial x(t,\mathbf{Q})}{\partial Q_j}=\sum_{i=f,r,s}\frac{\partial Y_i(t)}{\partial Q_j}$. We have

$\int_{t_0}^{T} \frac{\partial Y_s(t)}{\partial Q_r} dt = 0$ because the available storage capacity is completely filled (emptied) during storage (destorage), independent of Q_r. Moreover, from Table 1, $\frac{\partial Y_f(t)}{\partial Q_s} = 0$ and $\frac{\partial (Y_r(t)+Y_s(t))}{\partial Q_f} = 0$ for all t. Finally, $p(t) \frac{\partial Y_r(t,\mathbf{Q})}{\partial Q_s} = 0$ because $\frac{\partial Y_r(t,\mathbf{Q})}{\partial Q_s} = 0$ except for excess capacities of renewables during storing ($\alpha_s Q_r = x(0+\chi)$), for which however $p(t) = 0$. Substitution of this into the first-order conditions for welfare maximizing capacity choices (36) yields

$$\frac{\partial W}{\partial Q_f} = \rho \int_{t_0}^{T} (p^*(t) - \delta - k_f) \frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_f} dt - c_f = 0, \tag{49}$$

$$\frac{\partial W}{\partial Q_r} = \rho \int_{t_0}^{T} p^*(t) \frac{\partial Y_r^*(t,\mathbf{Q})}{\partial Q_r} dt + \int_{t_0}^{T} (p^*(t) - \delta - k_f) \frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_r} dt - c_r(Q_r) - \frac{\partial c_r}{\partial Q_r} Q_r = 0, \tag{50}$$

$$\frac{\partial W}{\partial Q_s} = \rho \int_{t_0}^{T} p^*(t) \frac{\partial Y_s^*(t,\mathbf{Q})}{\partial Q_s} dt + \int_{t_0}^{T} (p^*(t) - \delta - k_f) \frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_s} dt - c_s(Q_s) - \frac{\partial c_s}{\partial Q_s} Q_s = 0, \tag{51}$$

where superscript * indicates that the outcome at the production stage is first-best.

Note that $\frac{\partial Y_j(t)}{\partial Q_j} = \frac{\partial y_j(t)}{\partial q_j}$. Moreover, $\frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_r} = \frac{\partial Y_f^*(t,\mathbf{Q})}{\partial Q_s} = 0$ except during case 2 and during storage if it starts during case 2, for which, however, $p^*(t) - \delta - k_f = 0$ if $\tau = \delta$. Using this, the above conditions are exactly the same as the first-order conditions (28) to (30) for firms' capacity choices if all conditions are evaluated at the first-best policy instruments, $\tau^* = \delta, \chi^* = 0$, $\sigma_i^* = -Q_i(\boldsymbol{\theta}) \frac{\partial c_i}{\partial Q_i}, i = r, s$.

E Proof of Proposition 2

It remains to prove that the optimal values $\chi^\#, \sigma_r^\#, \sigma_s^\#$ implement the social optimum for the situation of no excess capacities. Remember that production and consumption choices on competitive markets as analysed in Section 4.2 were only distorted by the pollution externality. Therefore, with a Pigouvian tax $\tau = \delta$ production levels as summarised in Table 1 and, thus, demand $x(t) = \sum_{j=f,r,s} Y_j(t)$ are obviously first-best, provided that the underlying capacity levels are first-best. In the main text we have shown that the combination of an incomplete carbon tax, $\tau < \delta$, and a correcting consumption tax $\chi^\# = \delta - \tau$ leads to exactly the same output and demand levels. Now we verify that for the policy vector in Proposition 2 also firms' capacity choices are first-best, i.e., satisfy conditions (49) to (51). In particular, upon substitution of $\sigma_r^\# = -Q_r^* \frac{\partial c_r}{\partial Q_r} + \rho(\delta - \tau) \int_{t_0}^{T} \alpha(t) dt$ and $\sigma_s^\# = -Q_s^* \frac{\partial c_s}{\partial Q_s} + \rho \chi^\# \left(\frac{1}{\eta_d} - \frac{1}{\eta_s} \right)$, the first-order conditions for firms' capacity choices are

$$\rho \int_{t_0}^{T} \left(p^\#(t) - \tau - k_f\right) \frac{\partial y_f^*(t,q_f)}{\partial q_f} dt - c_f = 0, \tag{52}$$

$$\rho \int_{t_0}^{T} p^\#(t) \frac{\partial y_r^*(t,q_r)}{\partial q_r} dt - c_r(Q_r) - Q_r^* \frac{\partial c_r}{\partial Q_r} + \rho(\delta - \tau) \int_{t_0}^{T} \alpha(t) dt = 0, \tag{53}$$

$$\rho \int_{t_0}^{T} p^\#(t) \frac{\partial y_s^*(t,q_s)}{\partial q_s} dt - c_s(Q_s) - Q_s^* \frac{\partial c_s}{\partial Q_s} + \rho \chi^\# \left(\frac{1}{\eta_d} - \frac{1}{\eta_s} \right) = 0. \tag{54}$$

After substitution of $p^\#(t) = p^*(t) - (\delta - \tau)$ (see main text), and using $\frac{\partial y_j(t)}{\partial q_j} = \frac{\partial Y_j(t)}{\partial Q_j}$ as well as $\int_{t_0}^{T} \frac{\partial Y_r^*(t,q_r)}{\partial Q_r} dt = \int_{t_0}^{T} \alpha(t) dt$ and $\int_{t_0}^{T} (\delta - \tau) \frac{\partial Y_s^*(t,q_s)}{\partial Q_s} dt = \chi^\# \left(\frac{1}{\eta_d} - \frac{1}{\eta_s} \right)$, these conditions are the same as those for first-best capacity choices in (49) to (51).

F Proof of Proposition 3

We start by determining the effect of renewable and storage subsidies on fossil capacities, $\frac{dQ_f}{d\sigma_j}$. For each t, the equilibrium electricity price that obtains in Stage 3 is a function of capacities that are given at this stage, i.e., $p(t) = p(t, \mathbf{Q})$, where $\mathbf{Q} = (Q_f, Q_r, Q_s)$ is the capacity vector. Moreover, $\frac{\partial y_f^*(t)}{\partial q_f} = 1$ during destorage and case 1, but $\frac{\partial y_f^*(t)}{\partial q_f} = 0$ for cases 2 and 3 (see Table 1). Therefore, during storage $\frac{\partial y_f^*(t)}{\partial q_f} = 1$ if storage starts during case 1 ($\alpha_s \leq \alpha_1$), and $\frac{\partial y_f^*(t)}{\partial q_f} = 0$ otherwise ($\alpha_s > \alpha_1$). Using this, total differentiation of fossil firms' first-order condition (28) for capacities yields ($\pi_{ff} := d\pi_f\left(q_f^*(\boldsymbol{\theta}), \boldsymbol{\theta}\right)/dq_f$ denotes the first-order derivative)

$$d\pi_{ff} = \begin{cases} \sum_{j=f,r,s} \rho \left(\int_1 \frac{\partial p(t)}{\partial Q_j} dt + \frac{\partial p_d}{\partial Q_j} \int_d dt + \frac{\partial p_s}{\partial Q_j} \int_s dt \right) dQ_j & \text{if } \alpha_s \leq \alpha_1, \\ \sum_{j=f,r,s} \rho \left(\int_1 \frac{\partial p(t)}{\partial Q_j} dt + \frac{\partial p_d}{\partial Q_j} \int_d dt \right) dQ_j & \text{if } \alpha_s > \alpha_1, \end{cases} \tag{55}$$

where p_d, p_s are the constant prices during destorage and storage. Storage capacities are not used during case 1 so that $\int_1 \frac{\partial p(t)}{\partial Q_s} dt = 0$. For the other derivatives in (55), applying the chain rule when partially differentiating equilibrium prices with respect to Q_j yields $\frac{\partial p(t)}{\partial Q_j} = \frac{\partial p(t)}{\partial x(t)} \frac{\partial x(t)}{\partial Q_j}$, where $\frac{\partial x(t)}{\partial Q_j}$ follows from Lemma 1 and 2. In particular, $\frac{\partial x_d}{\partial Q_s} \int_d dt = \frac{1}{\eta_d}$ and $\frac{\partial x_s}{\partial Q_s} \int_d dt = -\frac{1}{\eta_s}$. Using this, setting $d\pi_{ff} = 0$, dividing by $d\sigma_j$, and rearranging yields

$$\frac{dQ_f}{d\sigma_j} = \frac{\partial Q_f}{\partial Q_r} \frac{dQ_r}{d\sigma_j} + \frac{\partial Q_f}{\partial Q_s} \frac{dQ_s}{d\sigma_j},\ j = r, s, \tag{56}$$

where, using $\mathbf{1}_{\alpha_s \leq \alpha_1}$ to denote the indicator function that takes the value 1 if $\alpha_s \leq \alpha_1$ and 0 otherwise,

$$\frac{\partial Q_f}{\partial Q_r} = -\frac{\int_1 \frac{\partial p(t)}{\partial x(t)} \alpha(t) dt + \frac{\partial p_d}{\partial x_d} \int_d \alpha(t)\, dt + \mathbf{1}_{\alpha_s \leq \alpha_1} \cdot \frac{\partial p_s}{\partial x_s} \int_s \alpha(t)\, dt}{\int_1 \frac{\partial p(t)}{\partial x(t)} dt + \frac{\partial p_d}{\partial x_d} \int_d dt + \mathbf{1}_{\alpha_s \leq \alpha_1} \cdot \frac{\partial p_s}{\partial x_s} \int_s dt}, \tag{57}$$

$$\frac{\partial Q_f}{\partial Q_s} = -\frac{\frac{\partial p_d}{\partial x_d} \frac{1}{\eta_d} - \mathbf{1}_{\alpha_s \leq \alpha_1} \cdot \frac{\partial p_s}{\partial x_s} \frac{1}{\eta_s}}{\int_1 \frac{\partial p(t)}{\partial x(t)} dt + \frac{\partial p_d}{\partial x_d} \int_d dt + \mathbf{1}_{\alpha_s \leq \alpha_1} \cdot \frac{\partial p_s}{\partial x_s} \int_s dt}. \tag{58}$$

Expression (56) reflects that σ_r, σ_s affect fossil capacities only indirectly via their effects on renewable and storage capacities.

The above calculations have been necessary to evaluate the term $\int_{t_0}^{T} \frac{dY_f(t,\mathbf{Q})}{d\sigma_i} dt$ in the first-order conditions (38) for the three different cases. The expressions for optimal subsidies then follow straightforwardly. For *full usage of fossils during storage* ($\alpha_s \leq \alpha_1$), only the intermediate case 1 obtains and fossils always operate at full capacity so that

$$\int_{t_0}^{T} \frac{dY_f(t, \mathbf{Q})}{d\sigma_i} dt = \int_{d,1,s} dt \frac{dQ_f}{d\sigma_i}, \quad i = r, s. \tag{59}$$

Substitution of (59) together with (56) into the first-order conditions (38) and collecting terms with $\frac{dQ_r}{d\sigma_i}$ and $\frac{dQ_s}{d\sigma_i}$ yields

$$\sum_{j=r,s} \left(\sigma_j + \rho(\delta - \tau) \int_{d,1,s} dt \frac{\partial Q_f}{\partial Q_j} + Q_j \frac{\partial c_j}{\partial Q_j} \right) \frac{dQ_r}{d\sigma_i} = 0, \quad i = r, s. \tag{60}$$

In these two first-order conditions with respect to σ_r and σ_s, the term $\sum_{j=r,s}(\cdot)$ has the same value. Therefore, (60) is obviously satisfied if this term is equal to zero.[25] Substitution from (57) and (58) into (60) yields the optimal subsidies for $\alpha_s \leq \alpha_1$.

[25] For any given τ, this is the only solution if one abstracts from pathological cases.

Turning to the case of *partial usage of fossils during storage* ($\alpha_s \in (\alpha_1, \alpha_2]$), fossil capacities are only partly used during case 2 of the intermediate period so that (using Lemma 1 and 2)[26]

$$\int_{t_0}^{T} \frac{dY_f(t, \mathbf{Q})}{d\sigma_i} dt \quad = \quad \int_{d,1} dt \frac{dQ_f}{d\sigma_i} - \int_{2} \alpha(t)\, dt \frac{dQ_r}{d\sigma_i} - \int_{s} \alpha(t)\, dt \frac{dQ_r}{d\sigma_i} + \frac{1}{\eta_s}\frac{dQ_s}{d\sigma_i},\ i = r, s. \quad (61)$$

The first term on the right-hand side results from the full usage of fossils during destorage and case 1. The second and third term reflect that additional renewable capacities, which are triggered by the subsidies, substitute fossil production during case 2 and storage. The fourth term captures that additional storage capacities require more fossil production to be filled.

Finally, for *no usage of fossils during storage* ($\alpha_2 < \alpha_s$), the intermediate period extends to case 3, but fossils do not produce in this case, nor in the storage period that follows them. Therefore, subsidies have no effects on fossil production during these periods and expression (61) simplifies to

$$\int_{t_0}^{T} \frac{dY_f(t, \mathbf{Q})}{d\sigma_i} dt \quad = \quad \int_{d,1} dt \frac{dQ_f}{d\sigma_i} - \int_{2} \alpha(t)\, dt \frac{dQ_r}{d\sigma_i},\ i = r, s. \quad (62)$$

Substitution of (61) for $\alpha_s \in (\alpha_1, \alpha_2]$ and (62) for $\alpha_s > \alpha_2$ together with (56) into the first-order conditions (38), and collecting terms with $\frac{dQ_r}{d\sigma_i}$, $\frac{dQ_s}{d\sigma_i}$ yields the following two conditions:

$$\begin{aligned}
&\left(\sigma_r + \rho(\delta - \tau)\left(\int_{d,1} dt \frac{\partial Q_f}{\partial Q_r} - \int_{2,s} \alpha(t)\, dt\right) + Q_r \frac{\partial c_r}{\partial Q_r}\right)\frac{dQ_r}{d\sigma_i} \\
&\quad + \left(\sigma_s + \rho(\delta - \tau)\left(\int_{d,1} dt\, \frac{\partial Q_f}{\partial Q_s} + \frac{1}{\eta_s}\right) + Q_s \frac{\partial c_s}{\partial Q_s}\right)\frac{dQ_s}{d\sigma_i} \quad = \quad 0, \quad i = r, s, \qquad (63)
\end{aligned}$$

$$\begin{aligned}
&\left(\sigma_r + \rho(\delta - \tau)\left(\int_{d,1} dt \frac{\partial Q_f}{\partial Q_r} - \int_{2} \alpha(t)\, dt\right) + Q_r \frac{\partial c_r}{\partial Q_r}\right)\frac{dQ_r}{d\sigma_i} \\
&\quad + \left(\sigma_s + \rho(\delta - \tau)\left(\int_{d,1} dt\, \frac{\partial Q_f}{\partial Q_s}\right) + Q_s \frac{\partial c_s}{\partial Q_s}\right)\frac{dQ_s}{d\sigma_i} \quad = \quad 0, \quad i = r, s. \qquad (64)
\end{aligned}$$

The optimal subsidies for $\alpha_s \in (\alpha_1, \alpha_2]$ and $\alpha_s > \alpha_2$ follow after substitution from (57) and (58).

G Calibration for Section 7

We use German data to calibrate the model and make the following assumptions. 96 quarterly hours represent one cycle. Each representative quarterly hour depicts the weighted average of the hourly availability of solar PV (45%), wind offshore (5%), and wind onshore (50%) over an entire year ($\alpha_{\min} = 0.0976$, $\alpha_{\max} = 0.2918$), which roughly represents the German capacity mix. Demand is $x(t) = 75{,}000 - 375 \cdot p(t)$, where the sensitivity -375 reflects a price elasticity of demand of -0.25 at a reference price of 40 €/MWh (e.g., Thimmapuram and Kim, 2013).[27]

[26] The last two terms in (61) represent effects during storage, $-\int_s dt \frac{d(\alpha_s Q_r)}{d\sigma_i}$. In particular, $\frac{\partial \alpha_s}{\partial Q_f} = 0$ so that total differentiation of $\alpha_s Q_r$ and multiplication by $\int_s dt$ gives

$$\begin{aligned}
d(\alpha_s Q_r)\int_s dt \quad &= \quad \left[\left(\frac{\partial \alpha_s}{\partial Q_r} Q_r + \alpha_s\right) dQ_r + \frac{\partial \alpha_s}{\partial Q_s} Q_r dQ_s\right]\int_s dt \\
&= \quad \left(-\int_s (\alpha_s - \alpha(t))\, dt + \alpha_s \int_s dt\right) dQ_r - \frac{1}{\eta_s} dQ_s.
\end{aligned}$$

[27] See www.energy-charts.de/power_inst.htm for the German capacity mix and www.energy-charts.de/price.htm for load data.

We assume that fossil firms use a gas turbine technology and thus pay $c_f = 500,000$ €/MW for capacity. Natural gas prices are around 7.5 €/MWh, the efficiency of the fossil technology is assumed to be 50%, so that $k_f = 0.5 \cdot 7.5 = 15$ €/MWh are private production costs. On July 2, 2019, the CO_2 price in the EU ETS peaked at 29 €/ton, and we take this as the carbon tax. The emission factor of natural gas is 0.2358 tons CO_2/MWh (here MWh refers to the heat value of natural gas), yielding a carbon tax of $\tau = 0.5 \cdot 0.2358 \cdot 29 = 13.68$ €/MWh. For the social costs of carbon we take a value that is 50% higher, i.e., $\delta = 1.5 \cdot \tau = 20.51$ €/MWh.

Capacity costs of renewables and storage firms fall at the same rate, starting at $c_r = 800,000$ €/MW and $c_s = 80,000$ €/MWh, respectively. Actual costs of renewables are around $1,000,000$ (solar PV), $2,500,000$ (wind offshore), and $1,200,000$ €/MW (wind onshore) (IEA, 2015; Schröder, Kunz, Meiss, Mendelevitch, and Von Hirschhausen, 2013). However, renewables costs are expected to fall further, and thus the depicted costs seem to be a good guess. Conversion losses of storage operations are $\eta_d = 1.1$ and $\eta_s = 0.9$, reflecting total losses of 19%. There is no particular storage technology that fits our synthetic one (see IRENA, 2017; Nykvist and Nilsson, 2015; Schmidt et al., 2017, for different cost estimates). A good guess is pumped hydro with similar capacity costs (5,000 to 100,000 €/MWh, mean around 60,000 €/MWh) and efficiency losses (15 to 30%). Battery storage systems cost around 200,000 €/MWh, but costs are expected to fall to 75,000 €/MWh (efficiency losses of only 3%). Power-to-gas technologies face lower capacity costs (around 40,000 €/MWh in the future), but fundamentally higher higher conversion losses (40 to 70%).

Finally, we use a discount rate of 3% and lifetimes of 30 years to calculate ρ, but deviate from the theoretical model by abstracting from within-year discounting for parsimony. This yields $\rho = 365 \cdot \left(\frac{1}{r} - \frac{1}{r(1+r)^{30}}\right) = 5,430$ with $r = 0.03$.

We set up the program in GAMS as a welfare maximization problem. The assumption of linear demand makes the program quadratic with non-convex constraints from the three zero-profit conditions of fossil, renewable, and storage firms. We therefore use the solver IPOPT, which is powerful in solving non-convex programs and finding local maxima, but cannot ensure the global maximum. We therefore assist the solver by giving him the (linear) first-order conditions of firms production and capacity choices, and by constraining the solution space. The code is available upon request.