Diagnosing and Evaluating the Acquisition Process of Problem Solving Schemata in the Domain of Functional Programming

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Abstract

This paper describes an approach to model students´ knowledge growth from novice to expert within the framework of a help system, ABSYNT, in the domain of functional programming. The help system has expert knowledge about a large solution space. On the other hand, in order to provide learner-centered help there is a model of the students´ actual state of domain knowledge. The model is continuously updated based on the learner´s actions. It distinguishes between newly acquired and improved knowledge. *Newly acquired knowledge* is represented by augmenting the model with rules from the expert knowledge base. *Knowledge improvement* is represented by rule composition. In this way, the knowledge contained in the model is partially ordered from general rules to more specific schemas for solution fragments to specific cases (= example solutions).

The model is implemented but not yet actually used for help generation within the help system. This paper focuses on knowledge diagnosis as accomplished by the model, and on an empirical analysis of some of its predictions.

Keywords: Knowledge acquisition, knowledge optimization, schema identification, empirical validation of student models, analysis of time-based and correction-based data

1. Introduction

The problem of student modelling has become an important research topic especially within the context of help and tutoring systems (Anderson et al., 1987; Brown & Burton, 1982; Frasson & Gauthier, 1990; Kearsley, 1988; Sleeman, 1984; Sleeman & Brown, 1982; Wenger, 1987) because the design of such systems raises questions like: Which order is the best for a set of tasks to be worked on? Why is information useless to one person and helpful to another? How is help material to be designed? Advance in these questions seems to be possible only if the actual knowledge state of the learner can be diagnosed *online* in an efficient and valid way. This is difficult (Self, 1990; 1991) but necessary for a system in order to react adequately to the student´s activities. Furthermore, it has been well recognized that progress in student modelling depends much on understanding what the student is doing (and why). Thus detailed assumptions about problem solving, knowledge representation and acquisition processes are needed.

We face the student modelling problem within the context of a help system in the domain of functional programming: The ABSYNT Problem Solving Monitor. ABSYNT ("Abstract Syntax Trees") is a functional visual programming language designed to support the acquisition of basic

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functional programming knowledge. The ABSYNT Problem Solving Monitor provides help and proposals for the student while constructing ABSYNT programs to given tasks. In order to make the system´s actions adaptive to the student, we model the student´s knowledge. Our basic approach rests on three principles:

- To try to "understand what the student is doing", and why. This amounts to constructing a *theoretical framework* which is powerful enough to describe the continuous stream of hypothetical problem solving, knowledge acquisition and utilization events, and to explain the stream of observable actions and verbalizations of the student.

- To use a subset of this theoretical framework in order to construct a student model containing the actual hypothetical state of domain knowledge of the student. This *state model* must be (and can be) simpler than the theoretical framework because its job is *efficient online diagnosis of domain knowledge* based on the computer-assessable data provided by the student´s interactions with the system.

- To fill the gap between the theoretical framework and the state model by constructing an offline model of knowledge acquisition, knowledge modification, and problem solving processes. This *process model* provides hypothetical *reasons* for the changing knowledge states as represented in the state model.

In accordance with these principles, we persue a three-level approach:

- A theoretical framework of problem solving and learning serves as a base for interpreting and understanding the student´s actions and verbalizations. We call this framework *ISP-DL Theory* (Impasse - Success - Problem - Solving - Driven Learning Theory).

- An *internal model* (*IM*) diagnoses the actual domain knowledge of the learner at different states in the knowledge acquisition process (*state model*). It is designed to be an integrated part of the help system ("internal" to it) in order to provide user-centered feedback.

- An *external model* (*EM*) is designed to simulate the knowledge acquisition *processes* of learners on a level of detail not available to the IM (for example, including verbalizations). Thus the EM is not part of the help system ("external" to it) but supports the design of the IM.

Thus ISP-DL Theory, IM, and EM are designed to be mutually consistent but serve different purposes. This paper is concerned with the IM. It is organized as follows: First we will briefly describe the ISP-DL Theory, our help system, the ABSYNT problem solving monitor, and the domain of functional programming knowledge as incorporated in ABSYNT. Then the IM is described and illustrated in some detail. Empirical predictions and a first evaluation are presented. Finally we will discuss some possible extensions and the role of the IM for adaptive help generation.

2. The ISP-DL Knowledge Acquisition Theory

As indicated, the ISP-DL Theory is intended to describe the continuous flow of problem solving and learning of the student as it occurs in a sequence of, for example, programming sessions. In our view, existing approaches touch upon main aspects of this process but do not cover all of them. Consequently, the ISP-DL Theory is an attempt to integrate several approaches. Before describing it, we will briefly discuss three theoretical approaches relevant here:

- In van Lehn´s (1988; 1990; 1991b) theory of Impasse Driven Learning, the concept of an impasse is of central importance to the acquisition of new knowledge. Roughly, an impasse is a situation where "the architecture cannot decide what to do next given the knowledge and the situation that are its current focus of attention" (van Lehn, 1991b, p. 19). Impasses trigger problem solving processes which may lead to new information. Thus impasses are an important source for the acquisition of new knowledge, though probably not the only one (van Lehn, 1989; 1991b). Impasses are also situations where the learner is likely to actively look for and to accept *help* (van Lehn, 1988). There is also empirical evidence that uncertainty leads to active search for information (Lanzetta & Driscoll, 1968). But problem solving or trying to understand remedial information might as well lead to secondary impasses (Brown & van Lehn, 1980).
  The idea of impasse-driven learning is also found elsewhere. As an example from machine learning, Prodigy (Carbonell & Gil, 1987; Minton & Carbonell, 1987) acquires new domain knowledge and new heuristics in response to noticing differences between expected and obtained outcomes. As an example from memory research, scripts may be augmented with information about exceptions in response to mispredicted events (Lehnert, 1978; Schank, 1982). Refining hypotheses in the context of concept learning (i.e., Egan & Greeno, 1974) may be considered another instance.
  Impasse Driven Learning Theory is concerned about *conditions* for problem solving, using help, and thereby acquiring new knowledge. It is not concerned about optimizing knowledge already acquired. "Knowledge compilation ... is not the kind of learning that the theory describes" (van Lehn, 1988, p. 32). Thus Impasse Driven Learning Theory covers an importatnt part of the processes we are interested in, but not all of them.

- In SOAR (Laird, Rosenbloom & Newell, 1986; 1987; Rosenbloom et al., 1991) the concept of impasse driven learning is elaborated by different types of impasses and weak heuristics performed in response to them. Impasses trigger the creation of subgoals and heuristic search in corresponding problem spaces. If a solution is found, a chunk is created acting as a new operator in the original problem space.
  In SOAR all learning is triggered by impasses. But these impases are more fine-grained than in van Lehn´s theory. Since our intention is to describe and understand students´actions and verbalizations, we are interested in coarse-grained impasses corresponding to observable behavior. On this level of analysis, it seems questionable whether all knowledge acquisition events can reasonably be described as resulting from impasses (van Lehn, 1989; 1991b). For example, existing knowledge may be deductively improved as a result of its successful application without changing the problem space.

- ACT* (Anderson, 1983; 1986; 1989) focuses on the success-driven optimization of already existing knowledge by knowledge compilation but pays less attention to the problem where new knowledge comes from. This is a main topic of PUPS (Anderson & Thompson, 1986; Anderson, 1987; 1989) which provides mechanisms for the inductive acquisition of rules from the perception of causal relationships and from analogy. But conditions for knowledge acquisition events (like impasses) is less focused on.

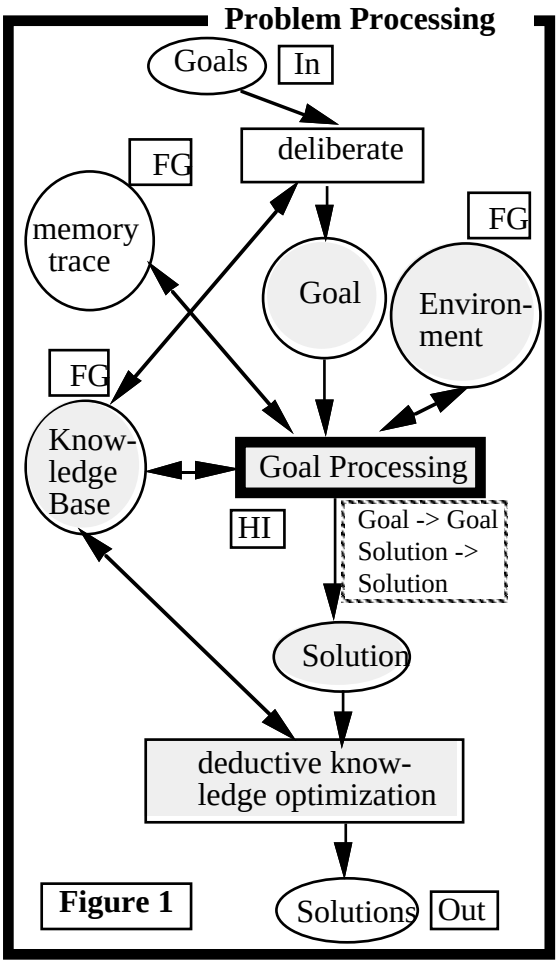
We think that for our purposes it is necessary to cover problem solving, impasse-driven learning, and success-driven learning as well. Thus ISP-DL Theory incorporates the following aspects:

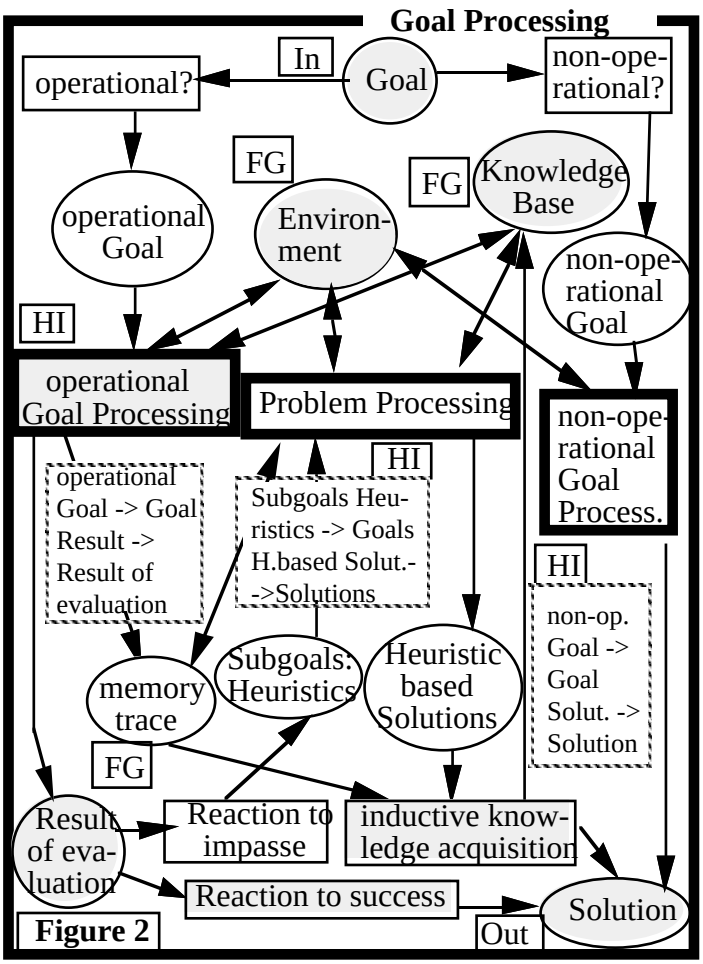
- The distinction of different problem solving phases (according to Gollwitzer, 1990): *Deliberating* with the result of choosing a goal, *planning* a solution to it, *executing* the plan and *evaluating* the result.

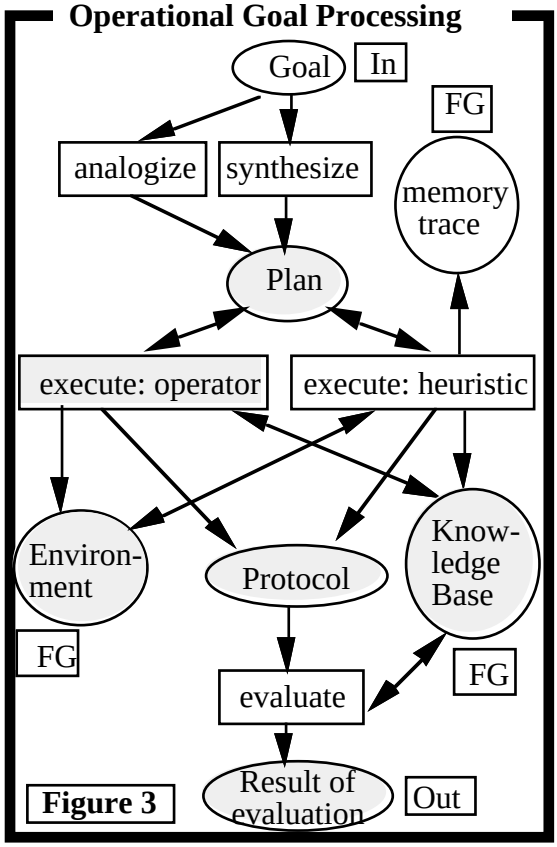
- The *impasse-driven acquisition of new knowledge*. In response to impasses, the problem solver applies weak heuristics, like asking questions, looking for help, etc.(Laird, Newell & Rosenbloom, 1987; van Lehn, 1988; 1989; 1990; 1991b). Thus *new* knowledge may be *acquired*.

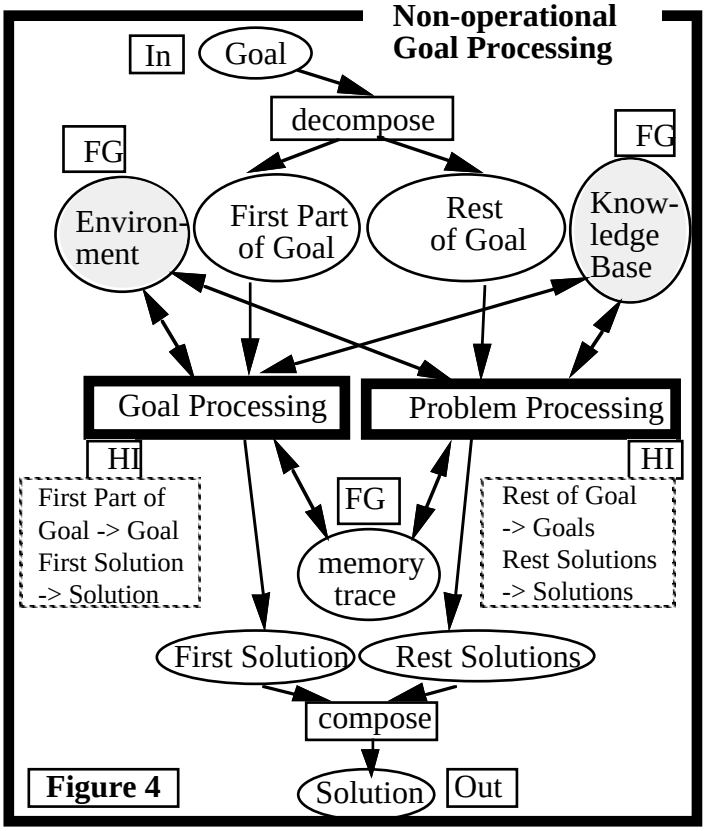
- The *success-driven improvement of acquired knowledge*. *Successfully used* knowledge is *improved* so it can be used more effectively. More specifically, by *rule composition* (Anderson, 1983; 1986; Lewis, 1987; Neves & Anderson, 1981; Vere, 1977), the number of control decisions and subgoals to be set is reduced. In our approach, composition is based on the resolution and unfolding method (Hogger, 1990).

We describe the ISP-DL Theory by *hierarchical higher Petri nets* (Huber et al., 1990), though alternative modelling formalisms are possible, e.g., *stream* communication (Gregory, 1987). Petri nets show temporal constraints on the order of processing steps more clearly than a purely verbal presentation. Thus they emphasize empirical predictions. The whole process is divided into 4 recursive subprocesses (*pages*): "Problem Processing", "Goal Processing", "Nonoperational Goal processing" and "Operational Goal Processing" (Figures 1-4). *Places* (circles/ellipses) represent states (e.g., the content of data memories), *transitions* (rectangles) represent events or process steps.









Figures 1 - 4: The ISP-DL Theory of problem solving and learning

Places may contain tokens which represent mental objects (goals, memory traces, heuristics etc.) or real objects (eg. a solution or a behaviour protocol). Places can be marked with tags (*In* for entering, *Out* for exiting place, *FG* for global fusion set). An FG tagged place is common to several nets (eg. the Knowledge Base). Transitions can be tagged with HI (HI for hierarchical invocation transition). This means that the process is continued in the called subnet. The dotted boxes show which places are corresponding in the calling net and in the called net. Shaded transitions and places are taken into account by the IM (see below).

Problem Solving is started in the page "*Problem Processing*" (Figure 1). The problem solver (PS) strives for one goal to choose out of the set of goals: "*deliberate*".

A goal may be viewed as a set of facts about the environment which the problem solver wants to become true (Newell, 1982). A goal can be expressed as a *predicative description* which is to be achieved by a problem solution. For example, the goal to create a program which tests if a natural number is even, "even(n)", can be expressed by the description: "funct even = (nat n) bool: exists ((nat k) 2 * k = n)". The "even" problem can be implemented by a function with the same name, one parameter "n" which has the type "*natural number*", the output type of the function is a *boolean* truth value, and the body of the function has to meet the declarative specification: "There exists a natural number k such that 2 * k = n". The goal is achieved when a program is created which satisfies this description.

The goal is processed in the page "*Goal Processing*" (Figure 2). If the PS comes up with a solution, the used knowledge is optimized: *deductive knowledge optimization*. When the PS encounters a similar problem, the solution time will be shorter. The net is left when there are no tokens in "*Goals*", "*Goal*" and "*Solutions*".

In the page "Goal Processsing" (Figure 2) the PS checks whether his set of problem solving operators is sufficient for a solution: "*operational?"/"non-operational?*".

An operational goal is processed according to the page "*Operational Goal Processing*" (Figure 3). A plan is *synthesized* by applying problem solving operators, or it is created by *analogical* reasoning. The plan is a partially ordered sequence or hierarchy of domain-specific problem solving goals (or of domain-unspecific heuristic goals, this will be explained in a moment). In either case, the goals in the plan are *executed*, using domain-specific or heuristic operators. Execution leads to a problem solving *protocol* which is used in combination with the knowledge base to *evaluate* the outcome. The *result of the evaluation* generates an impasse or a success. The result of the evaluation is transferred back to the page "*Goal Processing*".

Within the page "Goal Processing", impasses may arise at different points. For example, the "synthesize" process may fail to proceed with the plan because of missing planning knowledge or insufficient control knowledge to make a decision. An impasse might also arise during the "execution" process if there are no operators or heuristics to execute a particular plan fragment.

The *reaction* of the PS to *success* is: leave "*Goal Processing*" with a *solution*. The reaction to an *impasse* is the creation of subgoals to use weak heuristics for problem solving. Now there is a recursive call to "Problem Processing". "Goal Processing" and "Operational Goal Processing" are

called again. This time, within Operational Goal Processing a plan to use heuristics is synthesized and executed. (Simple examples for these weak heuristics are to use a dictionary, to find an expert to consult, and so on.) A memory trace of the situation which led to the impasse is kept. If the use of heuristics is successful, the result is *twofold*:

- The heuristically based solution is transferred back further to the instance of the page "Goal Processing" where the impasse arose. Now the impasse is solved. The obtained solution is related to the memory trace of the impasse situation. Thus within "*Goal Processing*" new *domain-specific* problem solving operators are inductively *acquired.*

- The obtained heuristically based solution is transferred back to "Problem Processing". Thus in "*Problem Processing*" the *domain-unspecific* heuristic knowledge is deductively *optimized*. So next time the PS encounters an impasse, he or she will be more skilled and efficient in using a dictionary, finding someone to consult, etc.

Finally, a non-operational goal is processed according to the page "*Non-operational Goal Processing*" (Figure 4). The problem is decomposed and the subsolutions are composed to a final solution.

It is possible and necessary to refine the theory´s transitions and places. For our purpose this simple theory is sufficient. Important for the rest of the paper are the theoretically and empirically validated statements:

- *New knowledge is acquired only at impasse time after the successful application of weak heuristics and on the basis of memory traces.*

- *Information is helpful only in impasses and if it is synchronized with the knowledge state of the PS.*

3. The ABSYNT Problem Solving Monitor

The visual language ABSYNT is based on ideas stated in an introductory computer science textbook (Bauer & Goos, 1982). ABSYNT is a tree representation of pure LISP without the list data structure (but we currently incorporate it) and is aimed at supporting the acquisition of basic functional programming skills, including abstraction and recursive systems. The motivation and analysis of ABSYNT with respect to properties of visual languages is described in Möbus & Thole, 1989. The ABSYNT Problem Solving Monitor provides an *iconic programming environment* (Chang, 1990). Its main components are a visual editor, trace, and a *help component*: a *hypotheses testing environment.*

In the editor (Figure 5) ABSYNT programs can be constructed. There is a head window and a body window. The left part of Figure 5 shows the tool bar of the editor: The bucket is for deleting nodes and links. The hand is for moving, the pen for naming, and the line for connecting nodes. Next, there is a constant, parameter and "higher", self-defined operator node (to be named by the learner, using the pen tool). Constant and parameter nodes are the *leaves* of ABSYNT trees. Then several primitive operator nodes follow ("if", "+", "-", "*", ...). Editing is done by selecting nodes with the mouse and placing them in the windows, and by linking, moving, naming, or deleting them.

Nodes and links can be created *independently*: If a link is created before the to-be-linked nodes are edited, then shadows are automatically created at the link ends. They serve as place holders for nodes to be edited later. Shadows may also be created by clicking into a free region of a window. In Figure 5, a program is actually under development by a student. There are subtrees not yet linked and nodes not yet named or completely unspecified (shaded areas). The upper part of Figure 5 shows the Start window for calling programs. This is also where the visual trace starts if selected by the student. In the visual trace, each computational step is made visible by representing computation goals and results within the upper and lower region of operator nodes, and within the lower region of parameter nodes (see Möbus & Schröder, 1990).

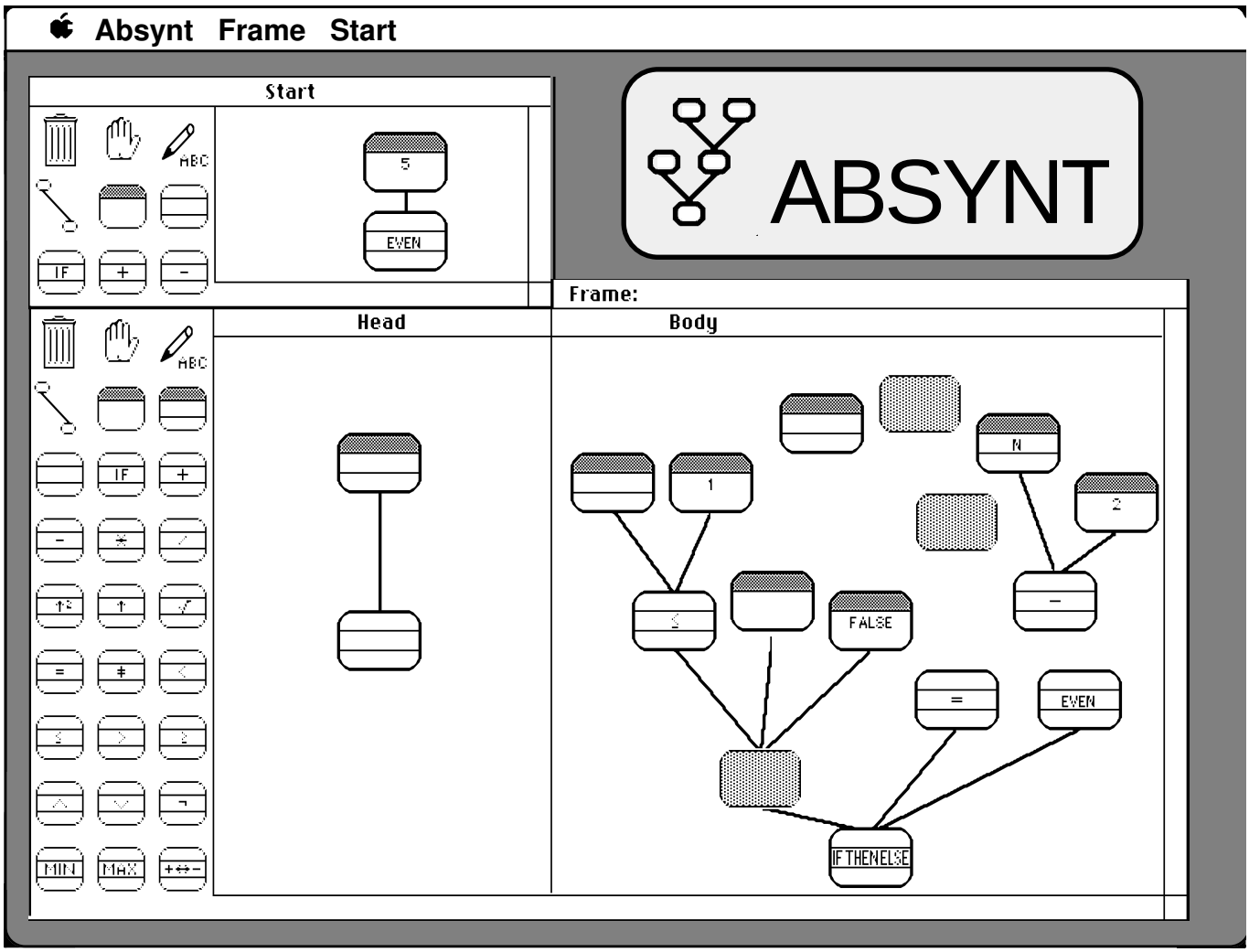


Figure 5: A snapshot of the visual editor of ABSYNT

In the *hypotheses testing environment* (Figure 6), the PS may state hypotheses (bold parts of the program in the upper worksheet in Figure 6) about the correctness of programs or parts thereof for given programming tasks. The hypothesis is: "It is possible to embed the boldly marked fragment of the program in a correct solution to the current task!". The PS then selects the current task from a menu, and the system analyzes the hypothesis. If the hypothesis can be confirmed the PS is shown a copy of the hypothesis. If this information is not sufficient to resolve the impasse, the PS may ask for

more information (completion proposals). If the hypothesis cannot be confirmed the PS receives the message that the hypothesis cannot be completed to a solution known by the system.

The upper part of Figure 6 shows a solution proposal to the "even" problem just constructed by a student: "Construct a program that determines whether a number is even!" This solution does not terminate for odd arguments. Despite of that the *hypothesis* (bold program fragment in the upper part of Figure 6) is embeddable in a correct solution. So the hypothesis is returned as feedback to the student (thin program fragment in the middle part of Figure 6). The student then may ask for a completion proposal generated by the system. In the example the system completes the hypothesis

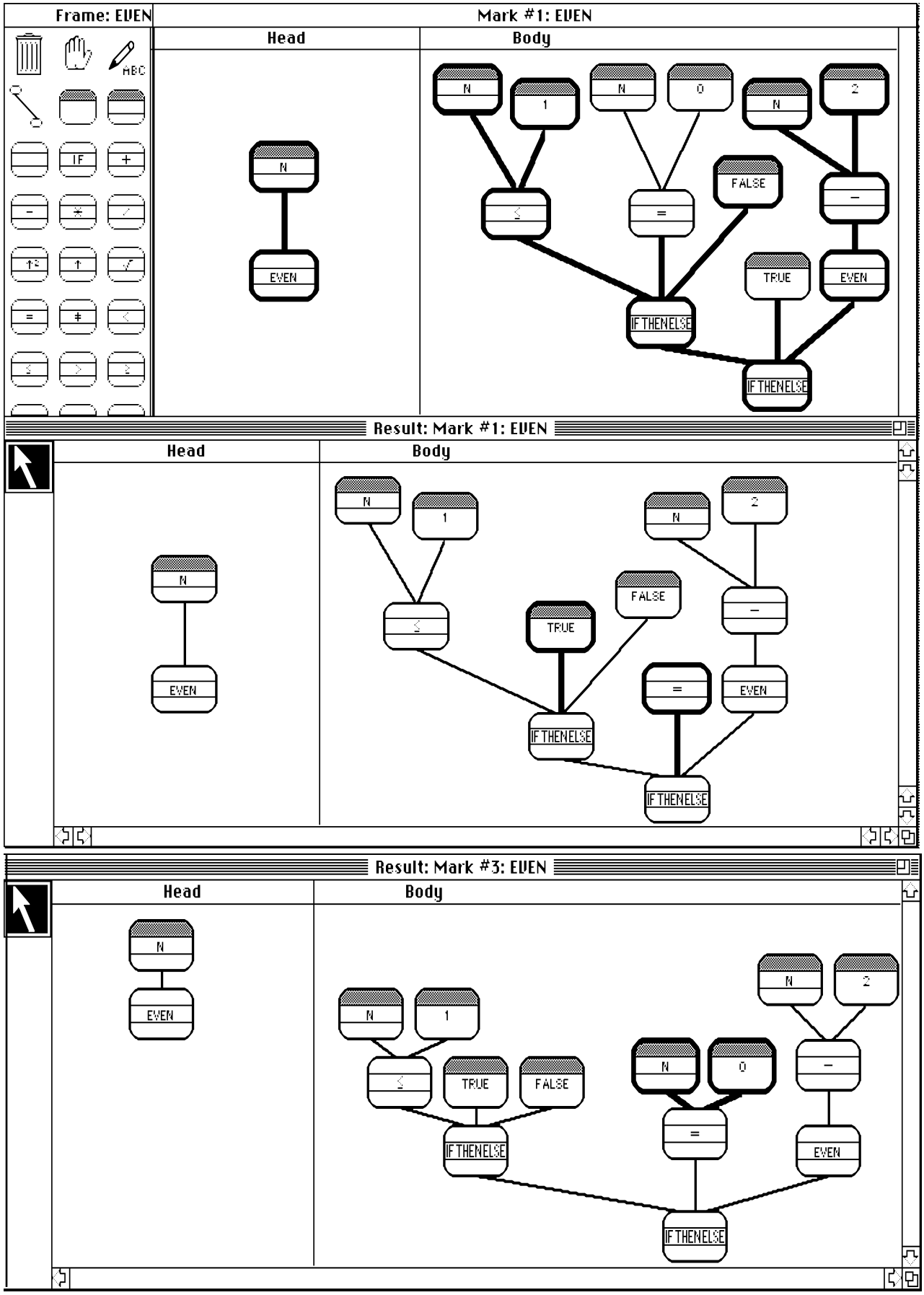


Figure 6: Snapshot of the ABSYNT Hypotheses Testing Environment

successively with the constant "true" and with the "="-operator (bold program fragments in the middle part of Figure 6). Internally, the system has generated a complete solution visible in the lower part of Figure 6. So the student´s solution in the upper part of Figure 6 may be corrected by an interchange of program parts.

The hypotheses testing environment is the most significant aspect where the ABSYNT Problem Solving Monitor differs from other systems designed to support the acquisition of functional programming knowledge, like the LISP tutor (Anderson & Swarecki, 1986; Anderson, Conrad & Corbett, 1989; Corbett & Anderson, 1992), the SCENT advisor (Greer, 1992; Greer, McCalla & Mark, 1989), and the ELM system (Weber, 1988; 1989). One reason for the hypotheses testing approach is that in programming a bug usually *cannot be absolutely localized*, and there is a variety of ways to debug a wrong solution. Hypotheses testing leaves the decision which parts of a buggy solution proposal to keep to the PS and thereby provides a rich data source about the PS´s knowledge state. Single subject sessions with the ABSYNT Problem Solving Monitor revealed that hypotheses testing was heavily used. It was almost the only means of debugging wrong solution proposals, despite the fact that the subjects had also the visual trace available. This is partly due to the fact that in contrast to the trace, hypotheses testing does not require a complete ABSYNT program solution.

The answers to the learner´s hypotheses are generated by rules defining a *goals-means-relation (GMR)*. These rules may be viewed as "pure" expert domain knowledge not influenced by learning. Thus we will call this set of rules EXPERT in the remainder of the paper. Currently, EXPERT contains about 650 rules and analyzes and synthesizes several millions of solutions for 40 tasks (Möbus, 1990; 1991; Möbus & Thole, 1990). One of them is the "even" task just introduced; more tasks will be presented later (see Figure 14). We think that such a large solution space is necessary because we observed that especially novices often construct unusual solutions due to local repairs. (This is exemplified by the clumsy-looking student proposal in the upper part of Figure 6.)

The completions shown in the middle part of Figure 6 (bold program fragments) and the complete solution in the lower part of Figure 6 were generated by EXPERT rules. EXPERT analyzes and synthesizes solution proposals but is not *adaptive* to the learner´s knowledge. Usually EXPERT is able to generate a large set of *possible* completions. Thus the main function of the *IM* (internal student model), which rules are derived from EXPERT, is to *select* a completion from this set which is maximally *consistent* with the learner´s current knowledge state. This should minimize the learner´s surprise to feedback and completion proposals.

4. GMR Rules

This section describes the goals-means-relation GMR. The set of GMR rules may be split in two ways: *rule type* (simple, composed) vs. *database* of the rules (EXPERT, POSS, IM).

- There are three kinds of *simple rules*: *goal elaboration rules*, *rules implementing one ABSYNT node*, and *rules implementing ABSYNT program heads*.

- *Composite rules* are created by merging at least two successive rules parsing a solution. Composites may be produced from simple rules and composites. A composite is called a *schema* if it contains at least one pair of variables which can be bound to a goal tree and a corresponding ABSYNT program subtree. But if a composite is fully instantiated (i.e., its variables can only be bound to node names or node values), then it is called a *case*.

The other way to partition the set GMR is the *data base* of the rules. As stated, EXPERT contains the expert domain knowledge. The sets IM and POSS will be described below.

Figure 7 shows examples for simple rules depicted in their visual representations. Each rule has a *rule head* (left hand side, pointed to by the arrow) and a *rule body* (right hand side, where the arrow is pointing from). The rule head contains a *goals-means-pair* where the goal is contained in the ellipse and the means (implementation of the goal) is contained in the rectangle. The rule body contains one goals-means-pair or a conjunction of pairs, or a primitive predicate (is_parm, is_const).

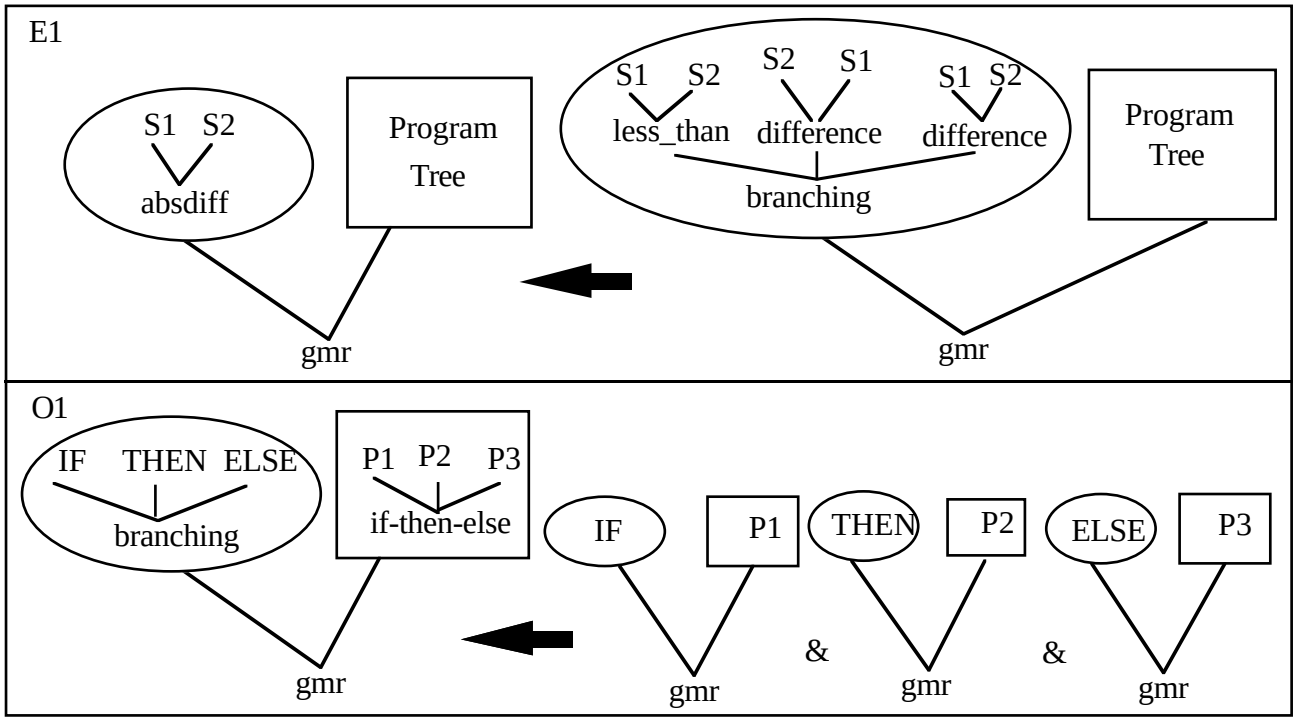


Figure 7: A goal elaboration rule (E1) and a rule (O1) implementing the ABSYNT node "if-then-else"

The first rule of Figure 7, E1, is a goal elaboration rule. It can be read:

If (*rule head*):
your main goal is "absdiff" with two subgoals S1 and S2,
then leave space for a program tree yet to be implemented, and (*rule body*):
If in the next planning step you create the new goal "branching" with the three subgoals
"less_than (S1, S2)", "difference (S2, S1)", and "difference (S1, S2)",
then the program tree solving this new goal will also be the solution for the main goal"

O1 in Figure 7 is an example of a simple rule implementing one ABSYNT node (operator, parameter, or constant):

If (*rule head*):
your main goal is "branching" with three subgoals (IF, THEN, ELSE),

then *implement* an "if-then-else"-node (or "if-"-node) with three links leaving from its input, and leave space above these links for three program trees P1, P2, P3 yet to be implemented; and (*rule body*):
if in the next planning step you persue the goal IF,
then its solution P1 will also be at P1 in the solution of the main goal, and
if in the next planning step you persue the goal THEN,
then its solution P2 will also be at P2 in the solution of the main goal, and
if in the next planning step you persue the goal ELSE,
then its solution P3 will also be at P3 in the solution of the main goal.

5. Composition of Rules

In our theory, composites represent improved sped-up knowledge. Together with the simple rules, they constitute a partial order from simple rules ("micro rules") to solution schemata to specific cases representing solution examples for tasks. In this section we will define rule composition.

If we view the rules as Horn clauses (Kowalski, 1979), then the composite RIJ of two rules RI and RJ can be described by the inference rule:

RI: (F <- P & C) RJ: (P' <- A)

RIJ: (F <- A & C)σ

The two clauses above the line resolve to the resolvent below the line. A, C are conjunctions of atomic formulas. P, P', and F are atomic formulas. σ is the most general unifier of P and P'. RIJ is the result of unfolding RI and RJ - a sound operation (Hogger, 1990).

For example we can compose the *schema* C7 (Figure 8) out of the set of simple rules {O1, O5, L1, L2}, where:

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O1:  gmr(branching(IF,THEN,ELSE),if-pop(P1,P2,P3)):-
          gmr(IF,P1),gmr(THEN,P2),gmr(ELSE,P3).
O5:  gmr(equal(S1,S2), eq-pop(P1,P2)):- gmr(S1,P1),gmr(S2,P2).
L1:  gmr(parm(P), P-pl):- is_parm(P).
L2:  gmr(const(C), C-cl):- is_const(C).
C7:  gmr(branching(equal(parm(Y),const(C)),parm(X),ELSE),
                                   if-pop(eq-pop(Y-pl,C-cl),X-pl,P)):-
          is_parm(Y),is_const(C),is_parm(X),gmr(ELSE,P).
```

where:

if-pop	=	primitive ABSYNT operator "if-then-else" (or "if")
eq-pop	=	primitive ABSYNT operator "="
P-pl, X-pl, Y-pl	=	unnamed ABSYNT parameter leaves
C-cl	=	empty ABSYNT constant leaf

We also can describe the composition of node implementing rules RI and RJ with a shorthand notation:

$$RIJ = RI_k \bullet RJ$$

The index k denotes the place k in the goal tree of the head of RI. A place k is the k-th variable leaf numbered from left to right (e.g.: $O1_3$ = ELSE). The semantics of "•" can be described in three

steps. First, the variable in place k in the goal term in the head of RI is substituted by the goal term in the head of RJ. Second the call term P in the body of RI which contains the to be substituted variable unifies with the head of RJ and is replaced by the body of RJ. Third the unifier σ is applied to the term resulting from the second step, leading to the composed rule RIJ. Thus the variables effected by the unification in step two are replaced by their bindings.

For example $O1_2 \bullet L1$ = gmr(branching(IF, parm(P), ELSE), if-pop(P1,P-pl,P3)):-gmr(IF,P1), is_parm(P), gmr(ELSE, P3). C7 can be composed from the rule set {O1, O5, L1, L2} in 16 different ways. Two possibilities are:

$C7 = (O1_2 \bullet L1)_1 \bullet ((O5_2 \bullet L2)_1 \bullet L1)$
$C7 = (((O1_1 \bullet O5)_3 \bullet L1)_2 \bullet L2)_1 \bullet L1$

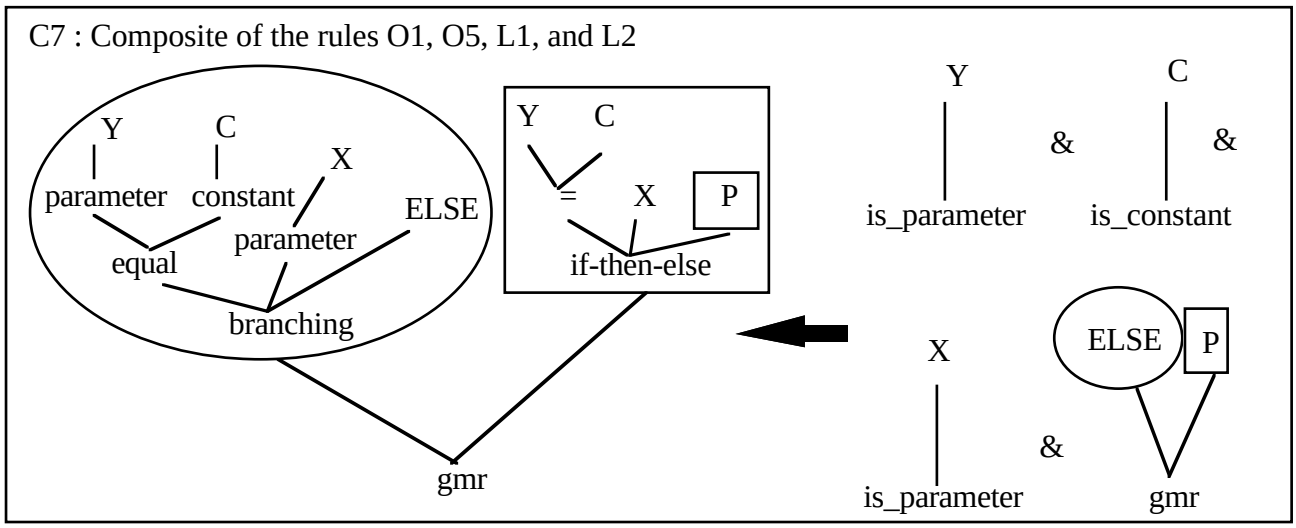


Figure 8: The composite C7

6. Empirical Constraints of Simple Rules, Chains, Schemata and Cases

Rules, rule chains and schemata give rise to different *empirical predictions*. The purpose of this section is twofold:

- To introduce hypotheses about the application of novice and expert knowledge, viewed as simple GMR rules and composites. These hypotheses will be used in the Internal Model.
- To show which specific predictions follow from these hypotheses.

Any approach designed to represent changing knowledge states must mirror the shift from novice to expert. In general, novices work *sequentially*, set more subgoals, and need more control decisions, while experts work in *parallel*, set less subgoals, and need less control decisions (Chase & Simon, 1973; Elio & Scharf, 1990; Gugerty & Olson, 1986; Simon & Simon, 1978). Here this difference is reflected in the partial order from simple rules to schemata to specific cases.

In order to demonstrate this difference, it is necessary to specify hypotheses about the problem solving behavior. According to the ISP-DL Theory, a plan is synthesized from a goal, and

execution of operators leads to a protocol of actions and verbalizations (Figure 3). Thus with respect to the theory we make a distinction between the problem solving phases of *planning* and *execution*: A *plan synthesizer* or *"planner" synthesizes* plans, and an *operator executor* or *"coder" executes* operators to implement the plans. The coder has domain specific knowledge (GMR rules) for implementing ABSYNT trees, but no planning knowledge. The coder also has very limited execution knowledge: pattern matching without unification (except for parameter and higher operator names, and constant values). More complex processes are left to the planner whose job is to guide the coder, based on domain-specific planning knowledge and on weak heuristics (to be specified by the External Model, as stated earlier).

For illustration of a hypothetical interaction sequence between planner and coder, we assume that the goal "branching (equal (parm(y), const(0)), parm(x), ELSE)" is to be implemented, and that the coder has knowledge about the set of simple GMR rules {O1, O5, L1, L2}. Figure 9 shows how the interaction might proceed: At time t_0, the planner delivers the goal. The coder has no rule for it so he rejects the goal. So the planner chops the goal into subgoals. Next, he may present the subgoal "parm(y)" to the coder. The coder now has a rule, L1, instantiates it to L1', and edits an ABSYNT parameter node with the name "y". Next, the planner delivers the subgoal "parm(x)". The planner uses L1 again, leading to the instantiation L1", and programs a parameter x. Then the planner comes up with "const(0)". The coder uses L2, applying L2' and programming a constant node 0. Next, the subgoal "equal(S1,S2)" is given. The planner instantiates O5 to O5' and creates a "=" node with two open links: their upper ends are shadows (place holders for nodes). After time t_j, the planner tells the coder that "equal(S1,S2)" has "parm(y)" as its first subgoal. So the coder connects the first input link of the "=" node to the parameter y. Next, the planner tells the coder that "equal(S1,S2)" has "const(0)" as its second subgoal, so the coder connects the second input link of the "=" node to the constant 0. Thus the coder has to rearrange the position of the nodes and/or the orientation of the links. This is symbolized by the hand in Figure 9. Next, the planner comes up with the "branching(IF,THEN,ELSE)" subgoal. The coder implements it, instantiating O1 to O1'. After time t_m, the planner tells the coder that "branching(IF,THEN,ELSE)" has "parm(x)" as its second subgoal and "equal(S1, S2)" as its first subgoal. So the coder connects the second and first input link of the "if-then-else" node to the parameter x and to the "=" node, respectively. Again, the position of links and/or nodes on the screen may have to be rearranged. Now the goal is solved.

Thus the planner does not know about the coder´s knowledge, and vice versa. There is no fixed order of application of GMR rules. The order solely depends on how the goals are delivered to the coder by the planner. In the example the coder created the sequence of rule instantiations (L1', L1", L2', O5', O1') depending on the goals delivered by the planner.

In contrast to this sequence, if the same goal "branching (equal (parm(y), const(0)), parm(x), ELSE)" is given and the coder knows the schema C7, then the interaction shown in Figure 10 will be produced. Again, at time t_0 the planner delivers the goal. This time the coder instantiates C7 to C7' and implements the ABSYNT tree contained in C7' without requiring subgoals and linking instructions from the planner.

If we compare the first interaction (Figure 9) where the coder knows {O1, O5, L1, L2} with the second one (Figure 10) where the coder knows C7, we observe:

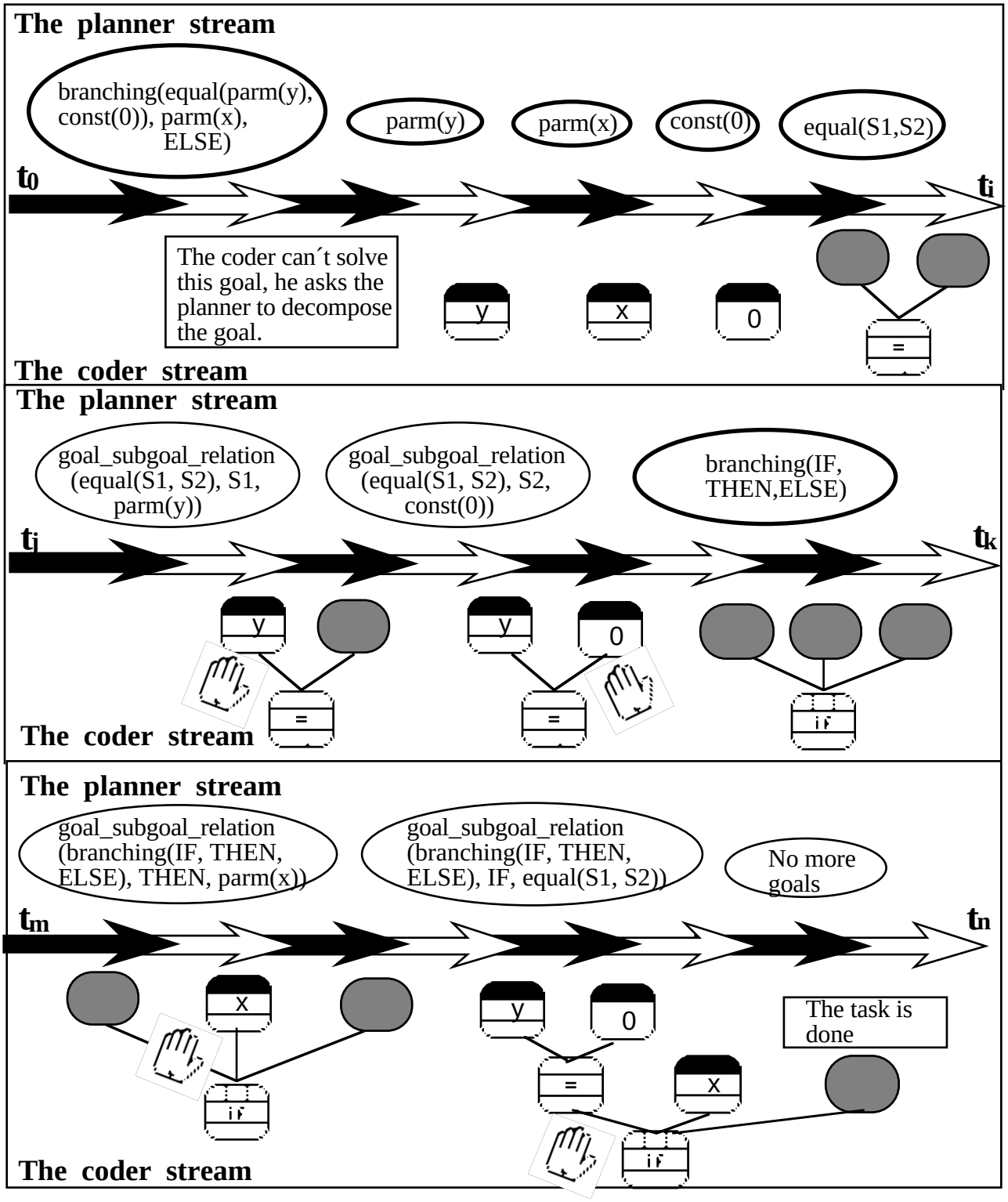


Figure 9: Squence of interactions between planner and coder while solving the goal "branching (equal (parm(y), const(0)), parm(x), ELSE)" with the set {O1, O5, L1, L2} of simple rules

- In the first sequence the coder implements five program fragments corresponding to the subgoals delivered by the planner. In the second sequence the coder implements just one program tree corresponding to the goal.

- In the first sequence the planner gives explicit information about linking program fragments, and the coder rearranges program fragments accordingly, if necessary. In the second sequence there is no such information.

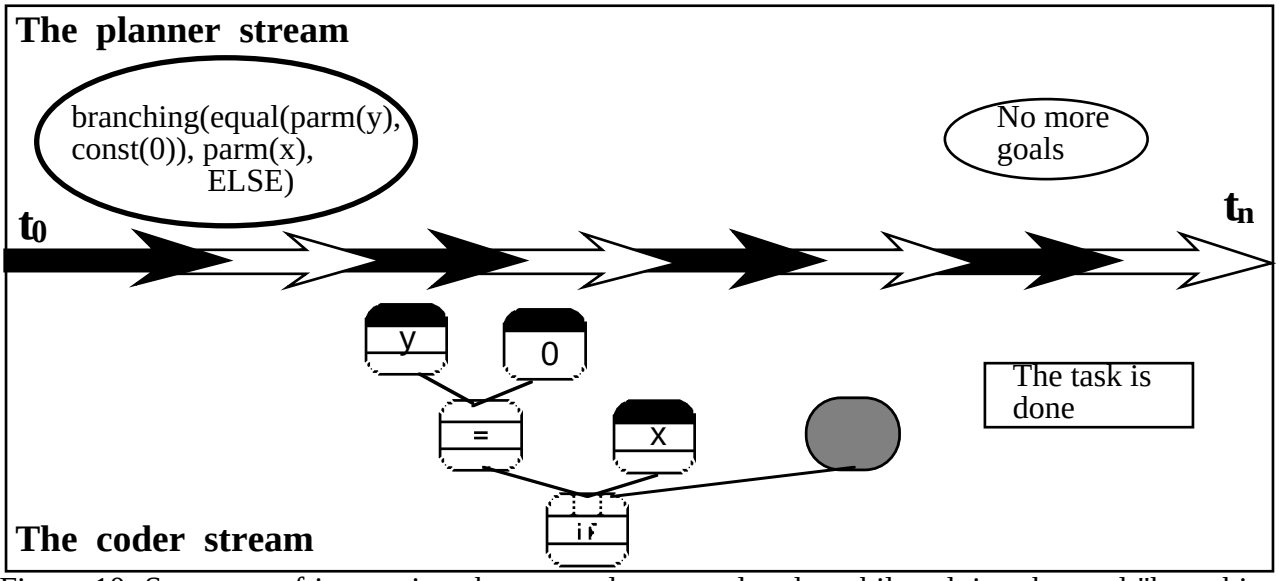


Figure 10: Sequence of interactions between planner and coder while solving the goal "branching (equal (parm(y), const(0)), parm(x), ELSE)" with the schema C7

In order to enable *empirical predictions*, we associate the following empirical claims with these observations:

- *Implementation* of ABSYNT program fragments:
  If the coder applies a certain GMR rule, then exactly the ABSYNT program fragment contained in it is implemented in an uninterrupted sequence of programming actions (like positioning a node, drawing a link, etc.). We do not postulate order constrains *within* this sequence, but we expect the sequence not to be interrupted by programming actions stemming from *different* rule instantiations.

- *Verbalization* of goals:
  Following the theoretically motivated distinction of a planner and a coder, selecting goals and subgoals for implementation by the coder is an act of planning involving control decisions. So is seems reasonable that at these decision points the selected goals may be verbalized (Ericsson & Simon, 1984). The verbalizations explained by the selection of a certain GMR rule may be intermixed with the rule´s programming actions, but not with verbalizations and actions stemming from different rule instantiations.

- *Correction* of positions:
  If the just implemented program fragment solves a dangling call or calls for another fragment already implemented, then it is to be connected with this existing fragment. Now corrective programming actions are likely: lengthening links, changing their orientation, and moving nodes.

If we compare the application of a single composite to the application of a set of simple rules (like C7 vs. {O1, O5, L1, L2}), then the following empirical consequences are assumed to result:

- *Implementation* of ABSYNT program fragments (*no-interleaving hypothesis*):
  For the set of simple rules, the order of rule applications is indeterminate, but the programming actions described by each rule should be continuous. *Actions of different rule instantiations should not interleave*. In contrast, when applying the composite there are no order constraints on the programming actions at all since just one rule is applied.

- *Verbalization* of goals (*verbalization hypothesis*):
  In the example, if the coder´s knowledge contains C7 the planner has to make one control decision. If the coder knows only {O1, O5, L1, L2}, the planner has to make at least five control decisions (depending on how the goal is decomposed). Thus we expect that applying composites is accompanied by *less goal verbalizations* than applying corresponding sets of simple rules.

- *Correction* of positions (*rearrangement hypothesis*):
  In case of the composite there are no open GMR calls to be implemented, and there are no to-be-linked program fragments left by earlier rule applications. Thus we expect that applying composites leads to *less position corrections* of ABSYNT nodes and links than applying the corresponding sets of simple rules.

- Performance *time* (*time hypothesis*):
  Planning, selecting, and verbalizing goals, and correcting positions of nodes and links are internal or external actions that are expected to need time (i.e. Rosenbloom & Newell, 1987). Thus we expect that applying composites is *faster* than applying the corresponding sets of simple rules.

These relationships are illustrated in Figure 11 (suppressing the location information for composites) for the rule set {O1, O5, L1, L2}, the composite C7 which may be generated from it, and different sets in between, containing composites and simple rules. The rule sets are organized in a partial order which reflects the *degree of predictability of the order* of programming actions, the *degree of verbalization, position corrections, and performance time.*

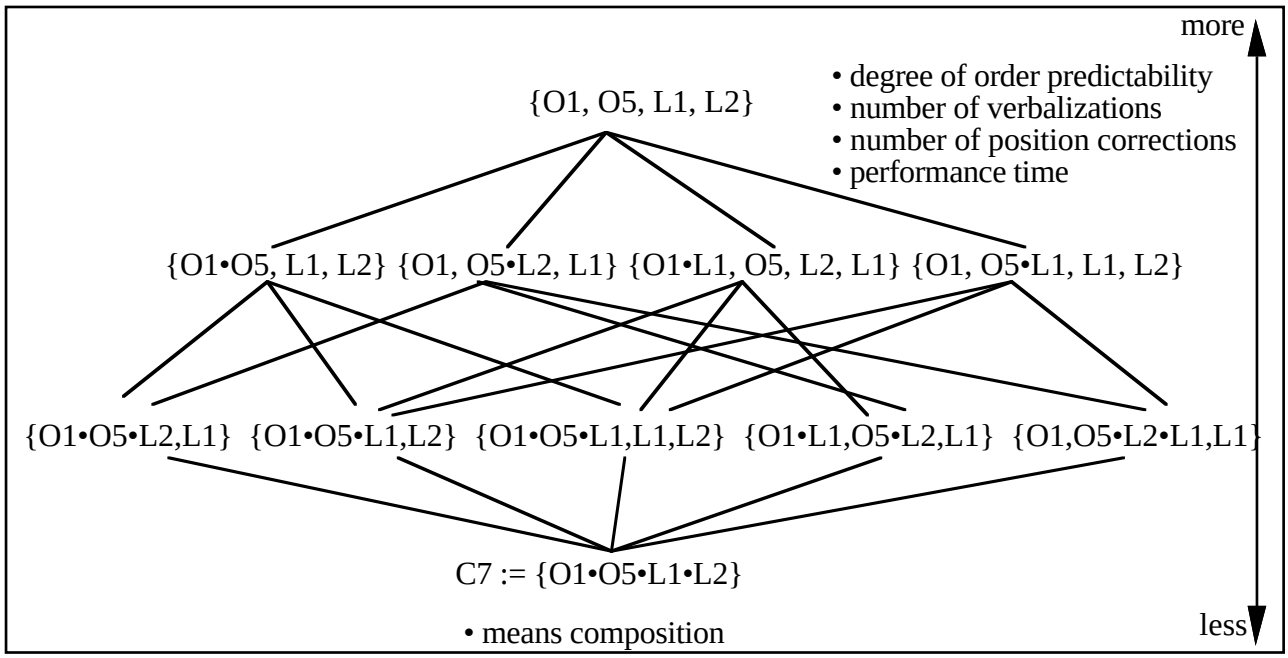


Figure 11: Rule sets partially ordered according to expected degree of order predictability, number of verbalizations, position corrections, and performance time

For example, if the rule set {O1, O5, L1, L2} is applied to the goal "branching (equal (parm(y), const(0)), parm(x), ELSE)", the planner has to chop this goal tree because the coder´s knowledge contained in the set {O1, O5, L1, L2} is not sufficient to implement this highly structured goal. If the goal tree is chopped to the stream of goals and goal-subgoal-relations

(branching(IF, THEN, ELSE),

equal(S1, S2),
goal_subgoal_relation(branching(IF, THEN, ELSE), IF, equal(S1, S2)),
parm(y),
goal_subgoal_relation(equal(S1, S2), S1, parm(y)),
parm(x),
goal_subgoal_relation(branching(IF, THEN, ELSE), THEN, parm(x)),
const(0),
goal_subgoal_relation(equal(S1, S2), S2, const(0))),

then the stream of *event sets* (events(O1') < events(O5') < events(connect(O1', 1, O5')) < events(L1') < events(connect(O5', 1, L1')) < events(L1'') < events(connect(O1', 2, L1'')) < events(L2') < events(connect(O5', 2, L2'))) should be observed empirically, where

- events(O1') = {verb(branching(IF, THEN, ELSE), act(if-then-else), act(link(if-then-else, 1)), act(link(if-then-else, 2)), act(link(if-then-else, 3))}
- events(O5') = {verb(equal(S1, S2)), act(=), act(link(=, 1)), act(link(=, 2))}
- events(connect(O1',1,O5')) = {verb(connect(branching(IF,THEN,ELSE), IF, equal(S1,S2))), act(connect(link(if-then-else), 1, =))}
- events(L1') = {verb(parm(y)), act(parameter(y))}
- events(connect(O5',1,L1')) = {verb(connect(equal(S1, S2), S1, parm(y))), act(connect(link(=), 1, parameter(y)))}
- events(L1'') = {verb(parm(x)), act(parameter(x))}
- events(connect(O1',2,L1'')) = {verb(connect(branching(IF,THEN,ELSE), THEN, parm(x))), act(connect(link(if-then-else), 2, parameter(x)))}
- events(L2') = {verb(const(0)), act(constant(0))}
- events(connect(O5',2, L2')) = {verb(connect(equal(S1, S2), S2, const(0))), act(connect(link(=), 2, constant(0)))}
- A < B means that the events in set A are followed by the events in set B.

The empirical meaning of the terms is:

- verb(Goal): The Goal is *possibly* verbalized.
- verb(connect(Goal1, S, Goal2): It is *possibly* verbalized that the subgoal S of Goal1 is Goal2.
- act(Node): The Node is *necessarily* implemented in ABSYNT in a free region or on a link shadow.
- act(link(Node, I)): An ABSYNT link entering the I-th input of Node is *necessarily* implemented. Its other end is connected to another node or left as a shadow to be filled later.
- act(connect(link(N1),I,N2)): The ABSYNT link entering the I-th input of node N1 is connected to node N2. (That is, N2 is dragged onto the shadow at the upper end of the link, and/or the link is lengthened to N2.)

The planner may deliver the stream of goals and goal-subgoal-relations in a different order, like the one depicted in Figure 9. Then the order of the empirical event sets should change accordingly. But in any case, the actions and verbalizations *within* each event set should occur in an *uninterrupted sequence*. In contrast, there is no order predictability for the actions and verbalizations corresponding to the *schema* C7, and there is no information about goal-subgoal-relations. Just one set of events can be predicted:

- events(C7') = {verb(branching(IF, THEN, ELSE), verb(equal(S1, S2)), verb(parm(x)), verb(parm(y)), verb(const(0)), act(if-then-else), act(=), act(parameter(y)), act(parameter(x)), act(constant(0)), act(link(if-then-else, 1)), act(link(if-then-else, 2)),

act(link(if-then-else, 3)), act(link(=, 1)), act(link(=, 2))}

We started to investigate some of these predictions empirically (see below). In addition, the no-interleaving hypothesis and the time hypothesis are used in the construction of the Internal Model to be described now.

7. The Internal Model (IM)

The IM is a set of domain specific knowledge (simple GMR rules and composites) which are utilized and continuously updated. As stated earlier, the IM covers the subset of the ISP-DL Theory shaded in Figures 1 to 4. So before describing it in detail, we will sketch it in terms of the ISP-DL Theory.

- *Concerning Figure 1*: The PS is faced with a programming task (*goal*) and constructs a solution proposal (*solution*). The solution is parsed, using the *knowledge base* (rules in the IM and - as far as needed - in EXPERT). Subsequently, the rules just used for parsing are *optimized* by composition.
  Since these new composites may be based on EXPERT rules, they are not directly inserted into the IM: According to ISP-DL Theory, a rule can only be improved after it is successfully applied. This implies for the IM that it cannot at the same time be augmented by a new simple rule (from EXPERT) and by composites built from the same simple rule. For this reason, in addition to the IM there is a set POSS of possible candidates for future composites of the IM. Composites of the rules used for parsing a solution proposal are generated and kept in POSS as candidates. Only those surviving a later test are moved into the IM.

- *Concerning Figure 2*: If parsing the solution is possible solely with rules in the IM, then the IM is considered as sufficient to construct the solution, and "Goal Processing" is terminated ("*reaction to success*"). But if parsing the solution requires additional EXPERT rules, then the IM may be augmented by these (simple) rules ("*inductive knowledge acquisition*").
  Thus, in accordance with ISP-DL-Theory, the IM contains *simple rules* representing newly acquired but not yet improved knowledge, and *composites* representing various degrees of expertise.

- *Concerning Figure 3*: The parse tree represents the student´s hypothetical solution *plan*, which *execution* led to a *protocol*: the sequence of programming actions, verbalizations, and corrections exhibited by the student. We call that part of the protocol consisting only of the student´s programming actions (creating nodes and links, naming nodes) the student´s *action sequence*. The action sequence is used to evaluate the parse rules:

  - Since knowledge improvement should result in sped-up performance (*time hypothesis*), a composite is moved from POSS to IM only if the PS shows a *speedup from an earlier to a later action sequence* where both sequences can be produced by the composite.
  - The IM contains only GMR rules (simple rules and composites) which proved to be *plausible* with respect to an action sequence at least once. This is defined now. With respect to some action sequence, GMR rules form four subsets:
    1. Rules not containing any program fragments ("goal elaboration rules") are *nondecisive* with respect to the action sequence. (But verbalizations can be related to the goal elaboration rules; Möbus & Thole, 1990).
    2. Rules whose head contains a program fragment which is part of the final result produced by the action sequence, and which was programmed in a *noninterrupted*, temporally continuous subsequence (see the *no-interleaving hypothesis*). These rules are *plausible* with respect to the action sequence.

3. Rules also containing a program fragment which is part of the final result of the action sequence, but this fragment corresponds only to the result of a *non*continuous action subsequence *interrupted* by other action steps. These rules are *implausible* with respect to the action sequence.
4. Rules whose head contains a program fragment which is not part of the final result produced by the action sequence. These rules are *irrelevant* to the action sequence.

- A *credit* scheme rewards the usefulness of the rules in the IM. The credit of a rule is the total number of action steps explained by this rule in the problem solving process of the PS. It is the product of the length of the action sequence explained by the rule and the number of its successful applications. Thus the credit depends on the empirical evidence gathered for a rule.

During the knowledge acquisition process the IM is utilized and continuously updated according to a processing cycle shown in Figure 12:

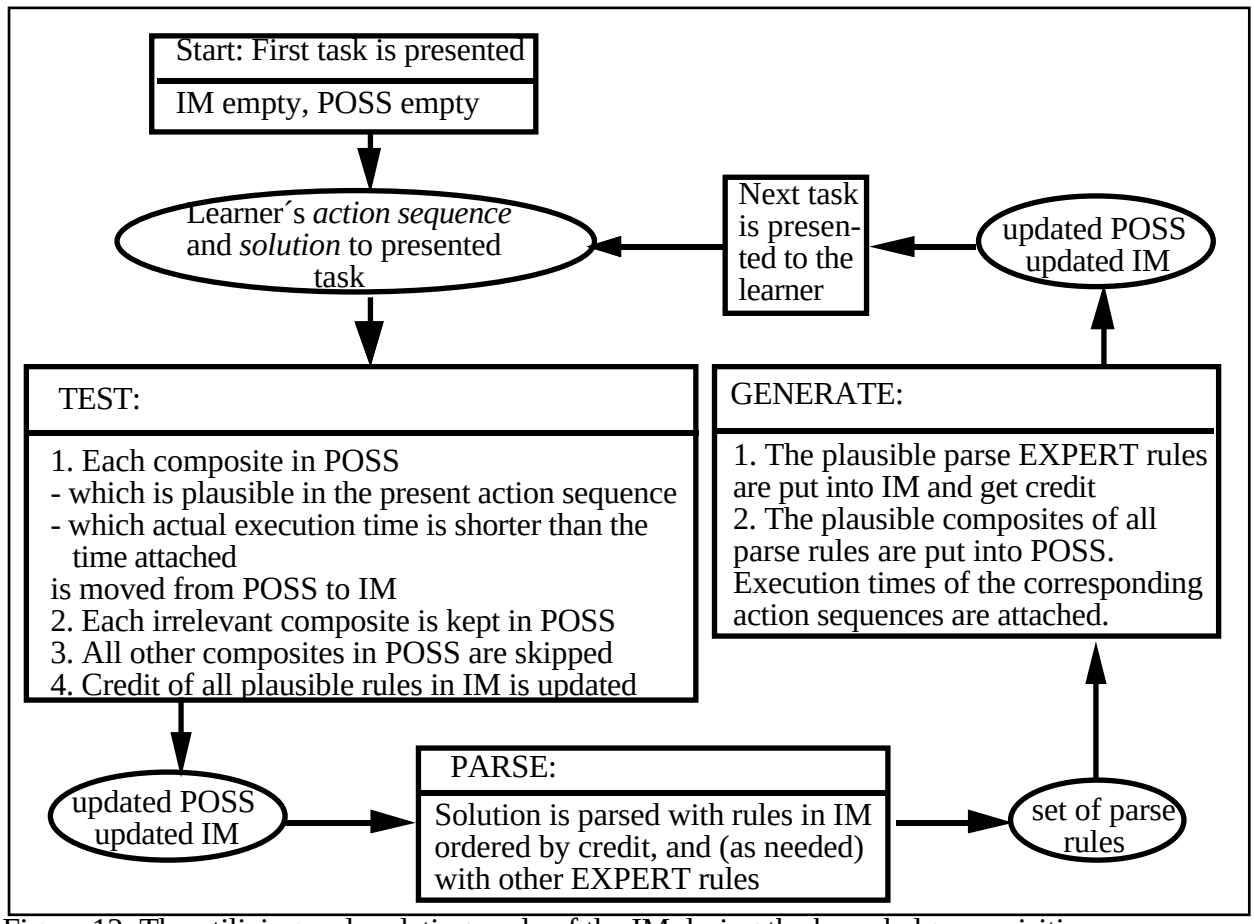


Figure 12: The utilizing and updating cycle of the IM during the knowledge acquisition process

- *Start* (Top of Figure 12): The first programming task is presented. Initially, both sets IM and POSS are empty.

- Now the learner solves the first task presented. Thus an *action sequence* is produced, leading to a *solution* to the task. The action sequence is saved in a log file.

- *First Test:* IM and POSS are empty, so nothing happens.

- *First Parse:* The learner´s ABSYNT program solution to the actual task is parsed with the EXPERT rules, leading to a set of parse rules.

- *First Generate*: The EXPERT rules just used for parsing are compared to the action sequence. The *plausible* parse EXPERT rules are put into the IM and get credit. These rules are hypothesized as newly acquired by the PS while solving the first task.
  Next, the composites of all parse rules are created and compared to the action sequence. The plausible composites are kept in POSS. These rules are hypothesized as newly created as a result of success-driven learning, but not yet actually used. Thus they are candidates of improved knowledge useful for future tasks. For each plausible composite, the time needed by the PS to perform the corresponding action sequence is attached.
  So the Generate phase results in an updated POSS and IM.

- Now the next task is presented to the PS. The PS creates an ABSYNT action sequence and solution to it.

- *Second Test*: Each composite in POSS is checked if
  a) it is plausible with respect to the action sequence, and
  b) the time needed by the PS to perform the respective continuous action sequence is shorter than the time attached to the composite. This means that the PS performs the action set *faster* than the previous corresponding action set which led to the creation of the composite.

  The composites meeting these requirements are put into the IM. Composites irrelevant to the action sequence of the solution just created are left in POSS. They might prove as useful composites on future tasks. All other composites violate the two requirements. They are skipped: that is, composites implausible to the actual sequence, or composites which predict a more speedy action sequence than observed. This means that the PS performs the action set *slower* than the previous corresponding action set which led to the creation of the composite. This slow-down is inconsistent with our model assumption that the PS prefers composites to simple rules, thus the composite is not transfered to the IM but skipped. Finally, the credits of all rules in the IM which are plausible with respect to the present action sequence are updated. Thus the second test leads to an updated POSS and IM.

- *Second Parse*: Now the solution of the second task is parsed with the rules of the IM ordered by their credits. As far as needed, EXPERT rules are also used for parsing.

- *Second Generate*: The plausibility of EXPERT rules which have just been used for parsing is checked. The plausible EXPERT parse rules are again put into the IM and get credit. As in the first Generate Phase, they are hypothesized as the newly acquired knowledge in response to impasses on the task just performed. Furthermore, the composites of all actual parse rules are created. The plausible composites are put into POSS, they will be tested on the next test phase. Again the time needed for the corresponding action sequence is stored with each composite.

8. Illustrations of the IM

To illustrate, Figure 13 shows a continuous fragment of the action sequence of a PS, Subject 2 (S2), on a programming task. Again we will restrict our attention to the rules O1, O5, L1, L2, and C7 (see Figures 7 and 8). When S2 performs the sequence of Figure 13, O1, L1 and L2 are already in the IM from earlier tasks. O5 is not yet in the IM but only in the set of EXPERT rules. C7 has not yet been created.

After S2 has solved the task, the *Test Phase* (Figure 12) starts. Since the only composite we look at here (C7) has not been created, we only consider the fourth subphase: Credit updating. O1 is *implausible* with respect to Figure 13 because the actions corresponding to the rule head of O1 are not continuous but *interrupted*. They are performed at 11:15:52, 11:15:58, 11:16:46, and 11:16:55

(Figure 13). Thus the action sequence corresponding to the rule head of O1 is interrupted at 11:16:42 and 11:16:50.

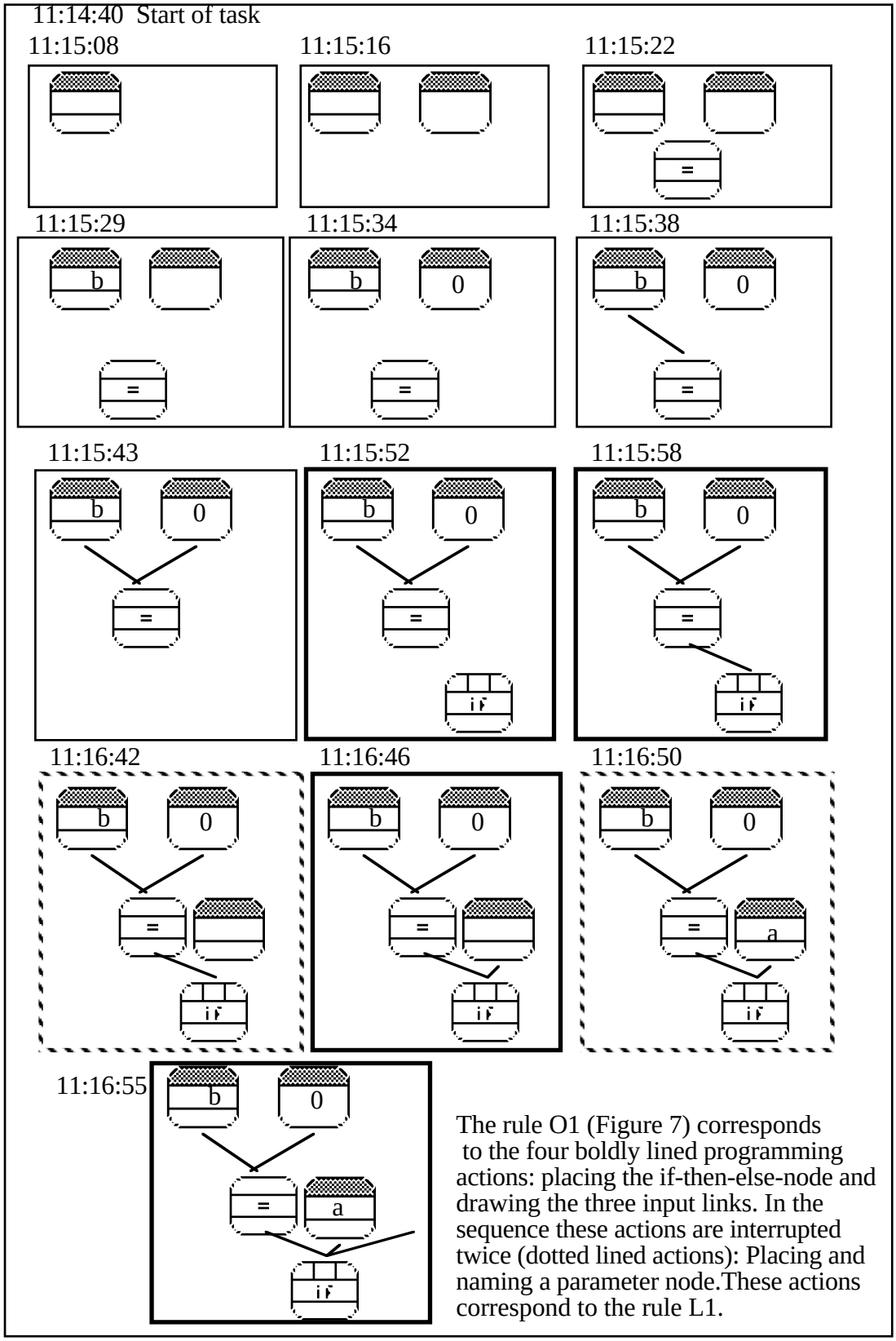


Figure 13: A continuous fragment of a sequence of programming actions of Subject S2

L1 and L2 are also implausible. Actions corresponding to L1 are performed the first time at 11:15:08 and 11:15:29. Thus this sequence is interrupted at 11:15:16 and 11:15:22. L1-like actions

are shown a second time by the PS at 11:16:42 and 11:16:50. These are interrupted, too. Actions corresponding to L2 are performed at 11:15:16 and 11:15:34, with interruptions at 11:15:22 and 11:15:29. So since O1, L1, and L2 are implausible, their credits are not changed.

Now S2´s solution is *parsed* with rules in the IM and, as needed, with additional EXPERT rules (Figure 12). O1, O5, L1, and L2 are among the parse rules in this case, as no other rules have a higher credit and are able to parse the solution.

After the Parse Phase, the *Generate Phase* (Figure 12) starts. O5 is an EXPERT rule used for parsing. But O5 is implausible, since its corresponding actions were performed at 11:15:22, 11:15:38, and 11:15:43, with interruptions at 11:15:29 and 11:15:34. So O5 is not put into the IM. Then the composites of the parse rules are formed. C7 (Figure 8) is a composite formed from O1, O5, L1 and L2. This composite is plausible because it describes the uninterrupted sequence of programming actions from 11:15:08 to 11:16:55 (see Figure 13) - despite the fact that its components O1, O5, L1, and L2 are all implausible. Starting from the beginning of the task (at 11:14:40), the time for this action sequence is 135 seconds. Thus the composite C7 is stored in POSS with "135 seconds" attached to it.

After S2 has solved the next task, the now following Test phase reveals that C7 is plausible again. The corresponding action sequence (not depicted) was performed in 92 seconds, which is less than 135. So C7 is moved into the IM and gets a credit of 13 since it describes 13 programming steps (see Figure 13). This credit will be incremented by 13 each time the composite is plausible again.

What does the IM look like after several tasks are solved? Figure 14 depicts the body trees of the solutions of 6 ABSYNT programming tasks: "diffmaxmin" (subtraction of the smaller from the larger of two numbers), "quot" (division of the larger by the smaller of two numbers), "abs" (absolute value of a number), "absdiff" (like "diffmaxmin": absolute difference of two numbers), addaddone (expressing addition by "+ 1"), diffdiffone (expressing subtraction by "- 1"). Some of the programming actions leading to these solutions are labeled with the time when they were performed. For example, the "<"-node in the solution to the task "diffmaxmin" was programmed at 9:08:06. The link between the "<"-node and the "if-then-else"-node was created at 9:07:20. The times of the actions of writing a value or name into a node are written in *italics*.

After solving the last task of this sequence, "diffdiffone", the IM contains simple ("micro") rules, schemata, and cases. They can be ordered as a specialization graph, as shown in Figure 15. The circled numbers are the credits. Each composite in Figure 15 is connected to the rules it is built from. For example, the "(less_than & if-then-else) & parameter & constant" composite in Figure 15 is:

```
gmr(branching (less_than (parm (Y), const (C)), parm (X), Else),
                                    if-pop (lt-pop (Y-pl, C-cl), X-pl, P)) :-
                        is_parm (Y), is_const (C), is_parm (X), gmr (Else, P).
```

("lt-pop" is the primitive ABSYNT operator "<".)

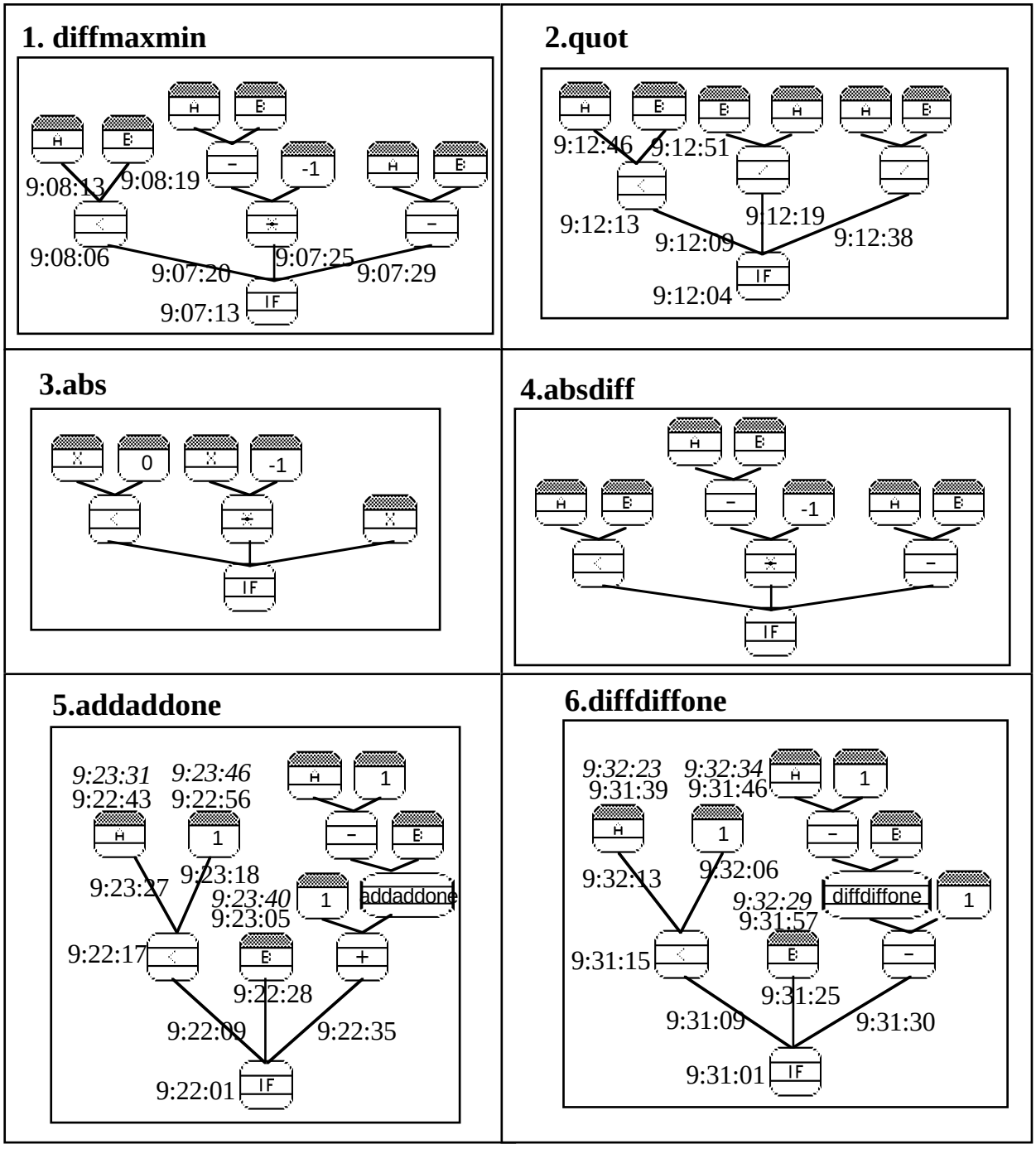


Figure 14: Simulated action streams and solution proposals to 6 ABSYNT programming tasks

According to Figure 15, this composite is the result of composing the "less_than & if-then-else" composite with the with the parameter node rule L1 and the constant node rule L2 presented earlier:

"less_than & if-then-else" composite:
gmr(branching (less_than (S1, S2), Then, Else), if-pop (lt-pop (P1, P2), P3, P4)) :-
gmr (S1, P1), gmr (S2, P2), gmr (Then, P3), gmr (Else, P4).
L1: gmr(parm(P), P-pl):- is_parm(P).
L2: gmr(const(C), C-cl):- is_const(C).

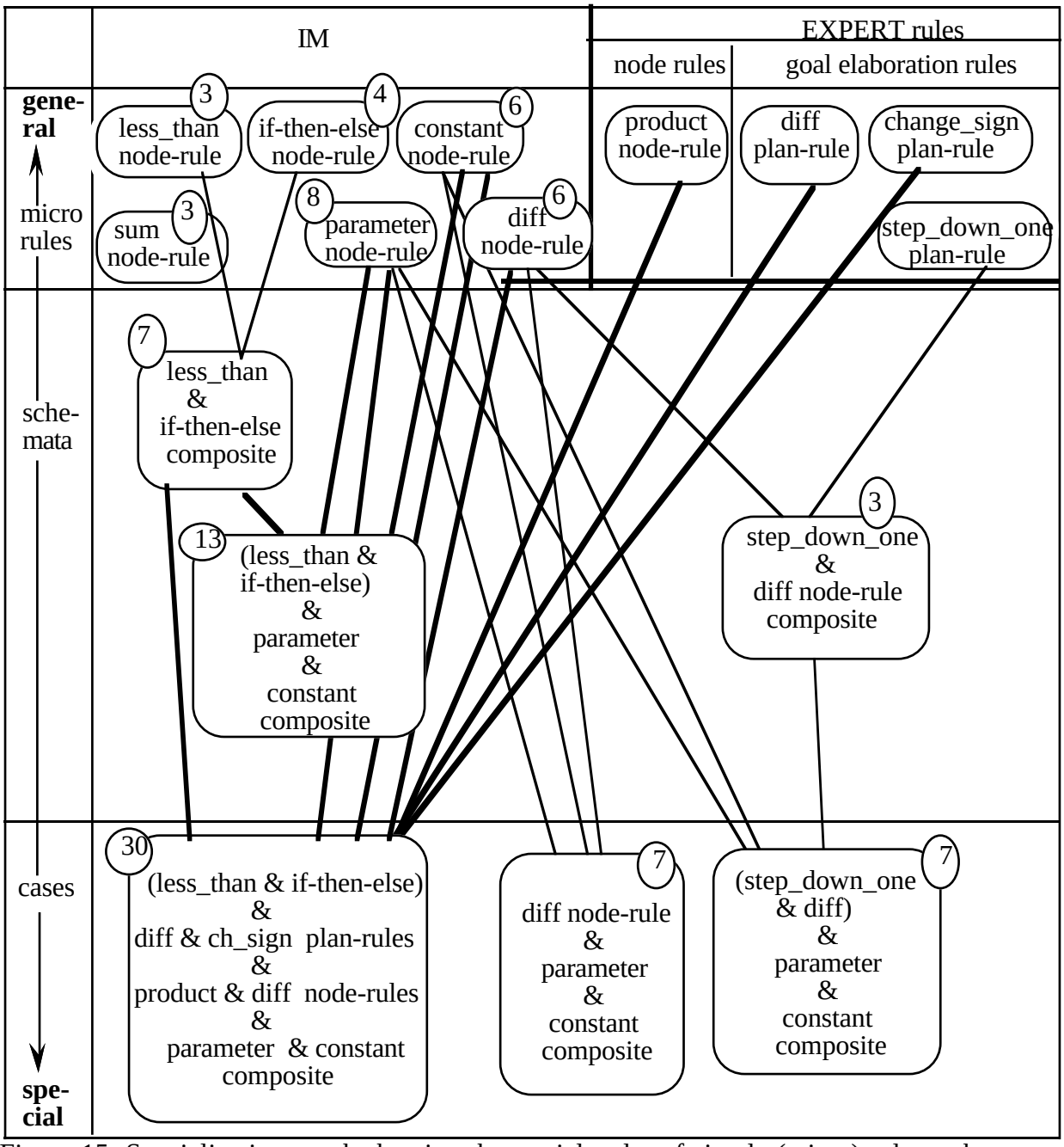


Figure 15: Specialization graph showing the partial order of simple (micro) rules, schemata, and cases built up in to the artificial programming sequence of Figure 14

This composition can be expressed by

"(less_than & if-then-else) & parameter & constant" composite =

(("less_than & if-then-else" composite$_1$ • L1)$_1$ • L2)$_1$ • L1$_1$.

A few examples will demonstrate how the IM in Figure 15 develops for the simulated programming sequence of Figure 14:

- Initially the IM is empty, so the solution to the first task (diffmaxmin) is parsed with EXPERT micro rules. The if-then-else-node rule (rule O1 shown earlier) and the less_than node rule are among the parse rules of the solution of the first task. The times attached to the solution in Figure 14 show that these rules are plausible. For example, the "if-then-else"-node and the three links leaving it were programmed in a continuous uninterrupted sequence (four programming actions from 9:07:13 to 9:07:29). The same is true for the "<"-node and the two

links leaving it (three programming actions from 9:08:06 to 9:08:19). So these two rules get into the IM and get the credits 4 and 3, respectively.

- Among the composites built from the parse rules of the solution to diffmaxmin, there is a schema, the "less_than & if-then-else composite". It is also plausible so it is moved into POSS. The action sequence explained by this composite starts at 9:07:13 and ends at 9:08:19, so the time "66 seconds" is attached to it.

- After solving "quot", the "less_than & if-then-else composite" is plausible again. Additionally, the corresponding action sequence is faster than 66 seconds (from 9:12:04 to 9:12:51, which is 47 seconds). So this composite is moved into the IM and gets a credit of 7 since it describes 7 programming actions.

- Another example is the "(less_than & if-then-else) & parameter & constant composite". The corresponding 13 actions are performed at the task "addaddone" in a continuous sequence (from 9:22:01 to 9:23:46, which is 105 seconds). Thus this schema is plausible and is put into POSS. On the next task, diffdiffone, this composite is plausible again, and the corresponding action sequence is sped up (from 9:31:01 to 9:32:34, which is 93 seconds). So the schema gets part of the IM with a credit of 13.

Figure 15 also shows that composites may be in the IM but not the micro rules they origin from. For example, the product node rule is not part of the IM but has been used for creating a case which is in the IM.

9. An Empirical Analysis of the IM

The IM represents the actual hypothetical knowledge of the PS. In this section we will investigate the no-interleaving hypothesis stating that the programming actions described by a rule in the IM are performed in a continuous uninterrupted temporal sequence. We will also take a look at some verbalizations, position corrections, and performance times. The analysis is based on the programming actions performed by a single subject, S2, solving seven consecutive nonrecursive ABSYNT programming tasks. The IM was run offline based on the action sequences exhibited by S2.

- *Material and procedure*. In a "getting-started" phase, S2 constructed an ABSYNT Start tree for each primitive ABSYNT operator node, and reconstructed given programs. The purpose of this phase was to introduce S2 to the ABSYNT interface and language. Then she solved the following tasks: "diffmaxmin", "interval" (program that tests if a number lies between 1 and 2), "absdiff", "quot", "quotzero" (like quot, but preventing division by zero), "abs", and "volume" (program that computes the difference between the volume of a cube and a sphere, where the diameter of the sphere is equal to the length of the edge of the cube).

- *Creating subsequent states of the IM*. Subsequent states of the IM were created by creating an initial state of the IM and then running it on S2´s solution sequence. We created an initial IM based on the following assumption: Since the subject was introduced to all ABSYNT nodes before she worked on the first programming task, "diffmaxmin", it seemed reasonable to put the primitive node rules, the constant node rule, and the parameter node rule into the IM. Then the IM was run on the sequence of solutions from "diffmaxmin" to "volume" constructed by S2. This produced a sequence of seven subsequent states of the IM.

- *Analyzing S2´s protocol*. The protocol of S2´s solutions to the seven programming tasks (S2´s complete *subject trace*) was analyzed according to the following categories of actions and verbalizations:

- placing a node
- naming a parameter, constant, or higher operator node
- creating a link
- deleting a node or a link
- replacing a node by another node, or changing a parameter name or a constant value
- correcting the position of a node or a link
- verbalizing a goal to place, name, or replace a node, or to create a link
- verbalizing uncertainty ("maybe I should ...") or negations ("I don´t know whether ...")

The actions and verbalizations of S2 while working in the Hypotheses Testing Environment were not included in this analysis because our hypotheses are not aimed at this activity.

- *Predicting action and verbalization sequences.* Based on
  - the state of the IM right before each task, and
  - S2´s action and verbalization sequence leading to a solution of this task,

  the following predictions for this action and verbalization sequence are possible:

  • *Sets* containing actions of placing and naming nodes, creating links, and verbalizing respective goals *(model trace).* Each set corresponds to the application of one IM rule. Thus the model trace consists of sets where each set contains actions and verbalizations expected to occur in a continuous uninterrupted sequence within S2´s subject trace (*no-interleaving hypothesis*).

  • *Position corrections*. If the position of a node is corrected, the IM rule explaining the corrected node should *not* explain the nodes connected to this node. Rather, these linked nodes should be explained by different rules. If the position of a link is corrected, the IM rule explaining it should not explain the node at the upper end of this link (*rearrangement hypothesis*).

  • *Performance times*. An action sequence explained by a composite should be shorter than the earlier action sequence which led to the creation of the composite (*time hypothesis*).

- *Evaluation of the subject trace with respect to the model trace*. S2´s subject trace was compared to the model trace in the following way: For each pair of actions / verbalizations a "+" was denoted if
  - this pair was explained by the same IM rule (both actions / verbalizations are contained within one set of the model trace), and
  - this pair was *subsequent* in the subject trace.

  For each pair of actions / verbalizations a "-" was denoted if
  - this pair was explained by the same IM rule, and
  - this pair was *interrupted* by some action(s) not explained by this IM rule.

  Thus "+" denote correspondencies to the predictions, and "-" denote contradictions.

  Figures 16 and 17 show the state of the IM, S2´s solution, subject trace, model trace, and correspondencies / contradictions for two consecutive tasks of the sequence, "absdiff" and "quot". Figure 16 shows:
  a) a subset of the rules in the IM after solving the task "interval" (second task of the sequence). Here only the rule names are given. The actual rules are shown in Appendix B.
  b) S2´s solution to "absdiff", which is the third task in the sequence.
  c) S2´s subject trace of the solution to "absdiff".
  d) The predicted model trace, given this subject trace and the state of the IM.
  e) The cases corresponding (+) and contradicting (-) to these predictions.
  Figure 17 shows the same information for the next task, "quot".

- *Results*

  • *Comparison of model trace and subject trace*. For S2´s complete subject trace (for all seven tasks), there were 84 "+" and 52 "-". Since more "+" should lead to longer and thus fewer runs than an equal distribution of "+" and "-", we applied the Runs-test. There were 46 runs, significantly less than to be expected by chance ($p < 0.001$).

• *Position corrections*. S2´s complete subject trace contained six position corrections. One of them occurs in the subject trace of "quot" (Figure 17c). There were three node corrections of parameters and constants. They were explained by different rules than the nodes connected to them. There were also three corrections of operator nodes and one of their input links. (In Figure 17c, the "if-then-else" node and its first input link are rearranged.) They were also explained by different rules than the node at the upper end of the respective link. (In Figure 17c, the "if-then-else" node and the "≤" node are explained by different IM rules.) So all position corrections are consistent with the rearrangement hypothesis.
• *Performance time*. Only two action sequences of S2´s complete subject trace are explained by composites. One of them shows a speedup (from 387 to 211 seconds) which is consistend with the time hypothesis, but the other one shows a slowdown (from 26 to 34 seconds).

- *Discussion*

The results indicate that the IM adequately describes a considerable portion of the protocol of S2´s actions and verbalizations with respect to the no-interleaving and rearrangement hypotheses. There were only two action sequences relevant to the time hypothesis. We will briefly discuss five points:
• There is another observation about time. It is concerned with the action sequences of S2´s subject trace which completely correspond to a set of the model trace. (For example, in Figure 17c, placing operator node $/_1$ and creating its left and right link is such a sequence since it is uninterrupted, as expected by the set {place($/_1$), link($/_1$,1,parameter a_2), link($/_1$,2,parameter b_2)} of the model trace). The complete subject trace contains 24 such sequences. For 19 of them their first action takes more time than each of the other actions. This is exactly what we would expect since the coder has to look for and select a rule before executing its *first* action.
• Another observation is that with respect to the no-interleaving hypothesis, a large portion of the discrepancies ("-") seems to be caused by parameters and constants. Tabel 1 shows the distribution of "+" and "-" across different types of rules in the IM:

	Parameter node rule	Constant node rule	Primitive operator node rules	Composites
"+" cases	3	4	51	26
"-" cases	28	7	15	2

Thus the parameter node rule, for example, is responsible for 3 "+" and for 28 "-": S2 usually does not place and name a parameter node in sequence. The same seems true for the constant node rule. Obviously, given that this result will be reproduced with other subjects, it should be possible to enhance the IM by splitting the parameter node rule (and the constant node rule as well) into two new rules: One for positioning and one for naming a parameter node. Then the current parameter node rule would be considered as a *composite* of these two new rules.
• As already noted, by the end of the last task ("volume"), there were only two composites in the IM. The virtually created programming sequence shown in the preceding section led to six composites (three schemata, three cases: Figure 15) after solving six tasks, and even more composites would have been possible. Thus according to the IM, subject S2 does not make much use of her own previous solutions but does much problem solving. This conclusion is supported by an inspection of the solutions of S2 to the seven tasks. For example, she solves "diffmaxmin" by "maximum of a and b minus minimum of a and b", but she solves the essentially identical task "absdiff" by "if b less than a then a minus b else (a minus b) times -1". Subsequently, the task "quot" is solved in yet another way by interchanging parameters. Thus the diversity in solution approaches is reflected in the IM by the fact that it contains only few composites.
• Finally, what about impasses? Based on S2´s IM we cannot predict impasses because
- the IM currently contains *only* implementation knowledge ("the coder´s knowledge") but no planning knowledge. (We work on extending the IM in this way.)
- the IM contains *sufficient* implementation knowledge because, as stated, it contains all primitive node rules and parameter and constant rule from the beginning.
So there should be no impasses based on insufficient implementation knowledge. Consequently, all impasses in the protocol should be attributable to insufficient planning knowledge. If we propose verbalizations of uncertainty and negative comments as one

empirical criterion for an impasse (similar to van Lehn, 1989; 1991b), then the protocol contains five impasses (without the hypotheses testing episodes). In three of these cases S2 considers different implementations ("if-then-else" or a logic operator; ">" or "<", and so on) and is uncertain about them. Thus there appears to be a planning problem. In a fourth case the impasse arises because S2 thinks that the solution just created will deliver a wrong result for a critical input value. In response to this, S2 switches parameter names. This does not seem to be an implementation problem either.
We are working on extending the ABSYNT Problem Solving Monitor and the IM by a planning level (see below). Then it should be possible also to predict impasses based on missing planning knowledge.

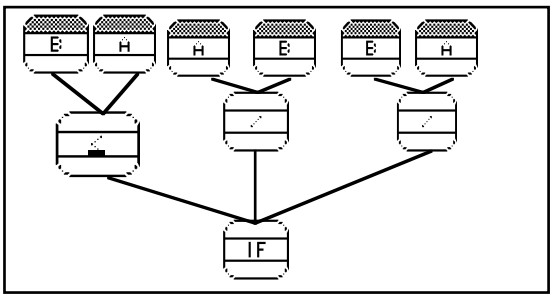
a) Subset of the rules in the IM before S2 solves the task "quot" (after solving "absdiff")

if-then-else node-rule
less_equal node rule

quotient node rule
sum node rule
differencenode rule

parameter node rule
constant node rule

b) S2´s solution to the task "quot" (program body)



e) correspondencies (+) and contradictions (-) between subject trace and model trace

c) subject trace

d) model trace

	c) subject trace	d) model trace
-	place parameter b1	{place parameter b1, name parameter b1}
	place parameter a1	{place parameter a1, name parameter a1}
-	place operator ≤	{place operator ≤, create left link into ≤, create right link into ≤}
+	create left link into ≤	
+	create right link into ≤	
	name parameter b1	
	name parameter a1	
	place op. if-then-else	{place op. if-then-else, create left link into if-then-else, create middle link into if-then-else, create right link into if-then-else}
+	create left link into if-then-else	
-	place parameter a2	{place parameter a2, name parameter a2}
	place parameter b2	{place parameter b2, name parameter b2}
-	place operator /1	{place operator /1, create left link into /1, create right link into /1}
+	create left link into /1	
+	create right link into /1	
	name parameter a2	
	name parameter b2	
	correct position of if-then-else and its first input link	{correct position of if-then-else and its first input link}
	create middle link into if-then-else	
-	place parameter b3	{place parameter b3, name parameter b3}
	place parameter a3	{place parameter a3, name parameter a3}
-	place operator /2	{place operator /2, create left link into /2, create right link into /2}
+	create left link into /2	
+	create right link into /2	
	create right link into if-then-else	
	name parameter b3	
	name parameter a3	

Figure 17: a) subset of the IM before solving "quot", b) S2´s solution to "quot", c) subject trace, d) model trace, e) correspondencies (+) and contradictions (-)

11. Discussion

We presented an approach to online diagnosis of students´ knowledge which is aimed at meeting the following requirements:

- to be based on a theoretical framework on problem solving and learning,
- to be computationally effective and empirically valid,
- to support adaptive help generation.

We will now discuss how far the IM meets these requirements and how we plan to improve it.

Foundation on a theoretical framework. In section 7 we showed how in our view the IM is related to the ISP-DL Theory. We tried to motivate the features of the IM by the theory. But still many aspects of the theory remain uncovered by the IM. Two of them are:

- *Generalization* of knowledge. Our observations from single-subject sessions with ABSYNT indicated use of previous solutions and positive transfer especially for recursive tasks. Thus composites in the IM should be generalized. *Generalization of composites* may be viewed as another way of knowledge optimization (e.g., Anderson, 1983; Wolff, 1987) in response to the successful utilization of knowledge (Figure 1). Additionally, generalized knowledge should also result from *analogizing* as an alternative to synthesizing a plan (Figure 3).
- Synthesizing a *plan*. Currently the IM takes only account of the implementation level, but there is no representation of planning knowledge within the IM.

We will sketch our current work on these two aspects:

- Concerning *generalization*, we will consider a simple example. We suppose that

  a) The two fragments shown in Figure 18 were programmed on two consecutive tasks

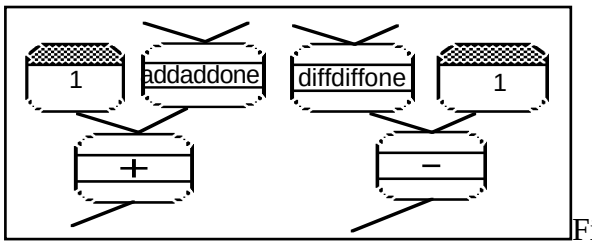


Figure 18: Two ABSYNT fragments

  b) The following two corresponding composites were plausible and thus moved into POSS:

```
C1: gmr (sum (const(C), addaddone (S1, S2)), add-pop (C-cl, Addaddone -hop (P1, P2))) :-
                    is_const (C), gmr (S1, P1), gmr (S2, P2).
C2: gmr (diff (diffdiffone (S1, S2), const(C)), sub-pop (Diffdiffone-hop (P1, P2), C-cl)) :-
                    gmr (S1, P1), gmr (S2, P2), is_const (C).
```

("add-pop" is the primitive ABSYNT operator "+",
"sub-pop" is the primitive ABSYNT operator "-",
"Addaddone-hop" and "Diffdiffone-hop" are self-defined "higher" ABSYNT operators with names given by the user.)

  Furthermore, C1 was composed from the node rules:

```
O2: gmr(sum(S1, S2), add-pop(P1, P2)) :- gmr(S1, P1), gmr(S2, P2).
L2: gmr(const(C), C-cl) :- is_const(C).
O3: gmr(addaddone(S1, S2), Addaddone-hop(P1, P2)) :-
                    gmr(S1, P1), gmr(S2, P2).
```

The composite C1 can be described by the formula $(O2_1 \bullet L2)_1 \bullet O3$.
In order to obtain a generalization of these two composites, first the two solution fragments have to be syntactically aligned by goal elaboration rules. For example, by using the goal elaboration rule

E2: gmr (sum (S1, S2), P) :- gmr (sum (S2, S1), P).

expressing commutativity of addition, together with O2, L2, and O3 the program fragment on the left of Figure 18 can be generated. This syntactically aligned program fragment corresponds to the composite $C1_{ex}$ ("ex" for exchange):

$C1_{ex}$: gmr (sum (addaddone (S1, S2), const(C)), add-pop(Addaddone-hop(P1, P2),C-cl)):-
gmr (S1, P1), gmr (S2, P2), is_const (C).

which is based on the same node rules as C1 and can be described by $(O2_1 \bullet O3)_1 \bullet L2$.

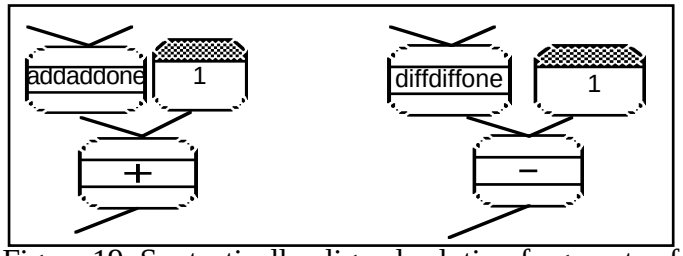


Figure 19: Syntactically aligned solution fragments of Figure 18

Now a new generalized rule G_{msg} can be created from C2 and $C1_{ex}$ by replacing the different goals and operators corresponding to the two program fragments (Figure 19) by variables. The possible values of the new variables are restricted by constraints. These constraints are built from the constants and their relations of the two original rules C2 and $C1_{ex}$.:

G_{msg}: gmr (Goal_1 (Goal_2 (S1, S2), const(C)),
Op_Name_1-pop (Op_Name_2-hop (P1, P2), C-cl)) :-
constraints([on(Goal_1, [sum, diff]),
on (Goal_2, [diffdiffone, addaddone]),
on(Op_Name_1, [add, sub]),
gmr(Goal_1(_,_), Op_Name_1(_,_)),
gmr(Goal_2(_,_), Op_Name_2(_,_))]),
gmr (S1, P1), gmr (S2, P2), is_const (C).

This is an example for a most specific generalization ("G_{msg}"). The rule G_{msg} is not able to parse or to generate similar problems. For example if the root goal is the goal to program a product the rule G_{msg} will fail, because the constraints are not satisfied. If the problem solver has no knowledge to program a product then there will be an impasse. One way to overcome this impasse would be to extend the constraints of the rule G_{msg} accordingly by inserting the "product" goal into the list [sum, diff] and the "mult" node into the list [add, sub].

It is also possible to generate another rule G_{mgg} from C2 and $C1_{ex}$. This most general generalization of the constraints differs from the example above by the missing variable restrictions:

G_{mgg}: gmr (Goal_1 (Goal_2 (S1, S2), const(C)),
Op_Name_1-pop (Op_Name_2-hop (P1, P2), C-cl)) :-
gmr (S1, P1), gmr (S2, P2), is_const (C).

This rule is an overgeneralization so it may produce errors, that is impasses. Remedial information (i.e. error feedback to hypotheses) may lead to a stepwise restriction of the variables by contraints.

- As mentioned, introducing a *planning level* is another topic of our current research. Currently the learner´s hypothetical solution plan is the parse tree of the solution. It is reconstructed retrospectively by the system after the solution is complete. We want the learner to be able to construct plans with an extension of the ABSYNT language by new goal nodes so that *mixed* ABSYNT programs containing operator nodes and goal nodes will be possible. The learner will be able to test hypotheses and to receive error and completion feedback at this *planning* level even if the learner has no idea yet about the implementation. Thus the learner may first *plan* a goal tree for the task at hand, test hypotheses about it, and debug it, if necessary. Afterwards the learner may *implement* the goals by replacing them with operator nodes or subtrees.

  For the user´s point of view, the benefit of using goal nodes will be that hypotheses testing will be possible at the *planning stage*, not just at the implementation stage. From a psychological point of view, the benefit is that *objective* data about the planning process can be obtained in addition to the verbalizations. Finally, from a help system design point of view, the benefit is that in addition to hypotheses testing it will be possible to offer *planning rules* as help to the learner. The planning rules will be visual representations of GMR goal elaboration rules.

Computational feasibility and empirical validity. A current problem with the IM is that composites are first generated, based on the parse rules of a solution, and then tested for plausibility. Generation of composites can be time-consuming for very complicated ABSYNT program solutions. It is possible to change this situation by generating composites only for program fragments which were created by the student in temporal sequence. In this way many composites which would not pass the plausibility test would not be created in the first place.

Another problem is that the IM currently does not deal with program modifications performed by the student, like deleting and replacing nodes and links. Despite of these shortcomings, we think that it is possible to extend the IM in appropriate ways. As we have also shown, it is possible to put the IM to empirical test and to draw conclusions for its improvement. For example, the study described above suggested changing simple parameter node rules. Some more testable hypotheses will be presented below. Thus advance towards an empirically validated knowledge diagnosis seems possible.

Adaptive help generation. The ultimate goal of the IM is to provide *adaptive* help or, more generally, to have an impact on the user-system-interaction in a way that takes account of the individual. In the ABSYNT Problem Solving Monitor, the need for the IM is very clear:

- There is a large solution space (the system is able to analyze and generate many solutions to given tasks) which is necessary because we want to be able to take care of novices´ often unusual or unnecessarily complicated solutions (see Figure 6).

- Because of the large solution space, there is usually a large amount of completion proposals that can be generated by the system. So the problem is which one to select. The task of the IM is to enable *user-centered* selection.

But as indicated, the role of the IM will not be restricted to the completion of ABSYNT nodes. Extending completion to the planning level and offering visual planning rules as help will impose additional demands to the IM. Additionally, the IM does more than just help selection. The information provided to the student may be varied in several ways, and this gives rise to empirical

predictions which in turn might support or weaken the IM. Figure 22 illustrates how information intended as help can be varied, and what can be predicted. Basically, when the student is caught in an impasse and asks for a completion proposal, according to the IM there are two possible situations:

- The student has knowledge how to proceed but does not make use of it. Thus with respect to the interaction of planning and coding described earlier, there is a *planning* problem.
- The student lacks domain-specific implementation knowledge, there is a *coding* problem.

The latter situation is depicted in Figure 22: The student just performed some programming actions, then gets stuck, and asks for completion proposals. According to the IM, there is a knowledge gap on the coding level, and after filling it the student would be able to proceed (shaded part of the horizontal arrow in the upper right of Figure 22). Now there are several possibilities to react to the gap: The information provided might vary in *grain size* and *amount* (on the left of Figure 22).

- *Grain size* concerns the rules underlying the completion proposal. If the grain size is *fine*, then the completion proposal may rest on a chain of simple rules which covers the gap. In this case the completion proposal may consist of an ABSYNT subtree with an explanation of each programming step needed to construct this subtree, where the explanation is based on the goal structure of the chain of simple rules. If the grain size is *coarse*, then the completion proposal may rest on a single composite (to take the other extreme). Thus the same subtree may be provided, but without an explanation.
- *Amount* concerns the relation between the completion and the gap. The completion proposal might *exactly fill* the gap, so subsequently the student can proceed by relying on her / his own knowledge. Alternatively, the completion proposal may contain *too much* information (more than necessary) or *not enough* information (the gap is not completely covered).

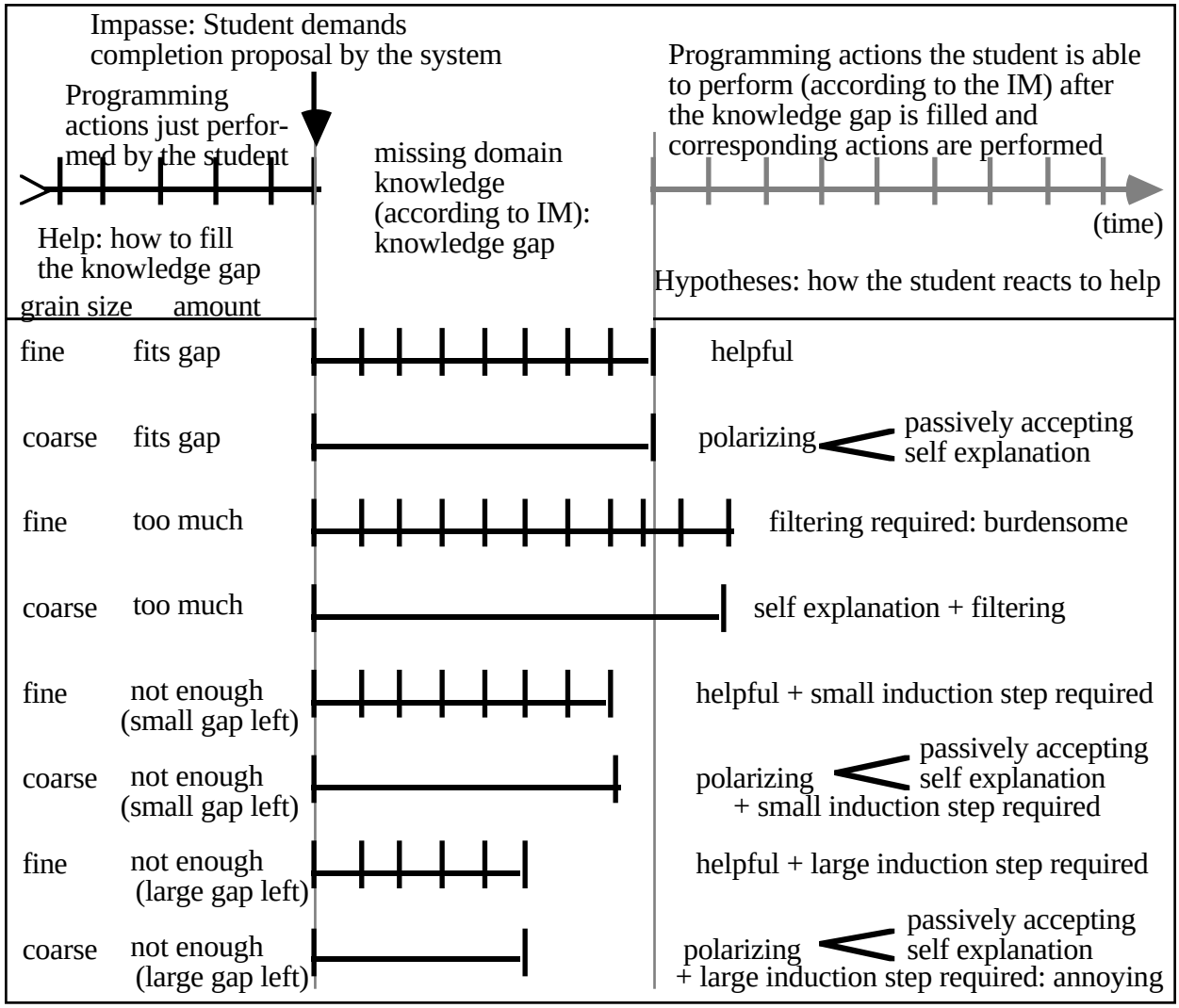


Figure 22: Types of information possibly provided in response to a knowledge gap diagnosed by the IM, and hypotheses conserning the student´s reaction to this information

On the left and middle part of Figure 22, the different combinations of grain size and amount of information are shown. They lead to different hypotheses (on the right of Figure 22). We will describe some of them:

- If the information is fine-grained and exactly fills the gap (first row in Figure 22), then we would expect that the student considers this information as *helpful*.
- If the information is coarse-grained and exactly fills the gap (second row), then the student misses explanations. So s/he might either *passively accept* what is being offered, or engage in *self-explanation* (van Lehn, 1991a).
- If the information is fine-grained but exceeds the knowledge gap (third row), then the student has to"*filter*" the content relevant to the current situation. This might be experienced as *burdonsome*.
- If the information leaves a small knowledge gap (fifth and sixth row), then the student might try to induce one new simple rule and thereby cover the rest of the gap. (This situation seems similar to the induction of one subprocedure at a time by van Lehn´s (1987) SIERRA program.)

- Finally, the last case to be considered here is that there is a large gap left, and the information offered is too coarse (last row). The student should experience such an information as very inadequate to his current problem. Thus she or he should feel annoyed or even upset.

There remains much work, of course, to work out these hypotheses and put them to empirical test. But we think we have shown that the IM is an empirically fruitful approach to knowledge diagnosis and adaptive help generation which is testable and also touches upon further important research problems, like motivation and emotion.

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