Universität Rostock

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Chair of Mechatronics

Sliding Mode Control for Uncertain Thermal SOFC Models with Physical Actuator Constraints

SWIM 2012: Small Workshop on Interval Methods

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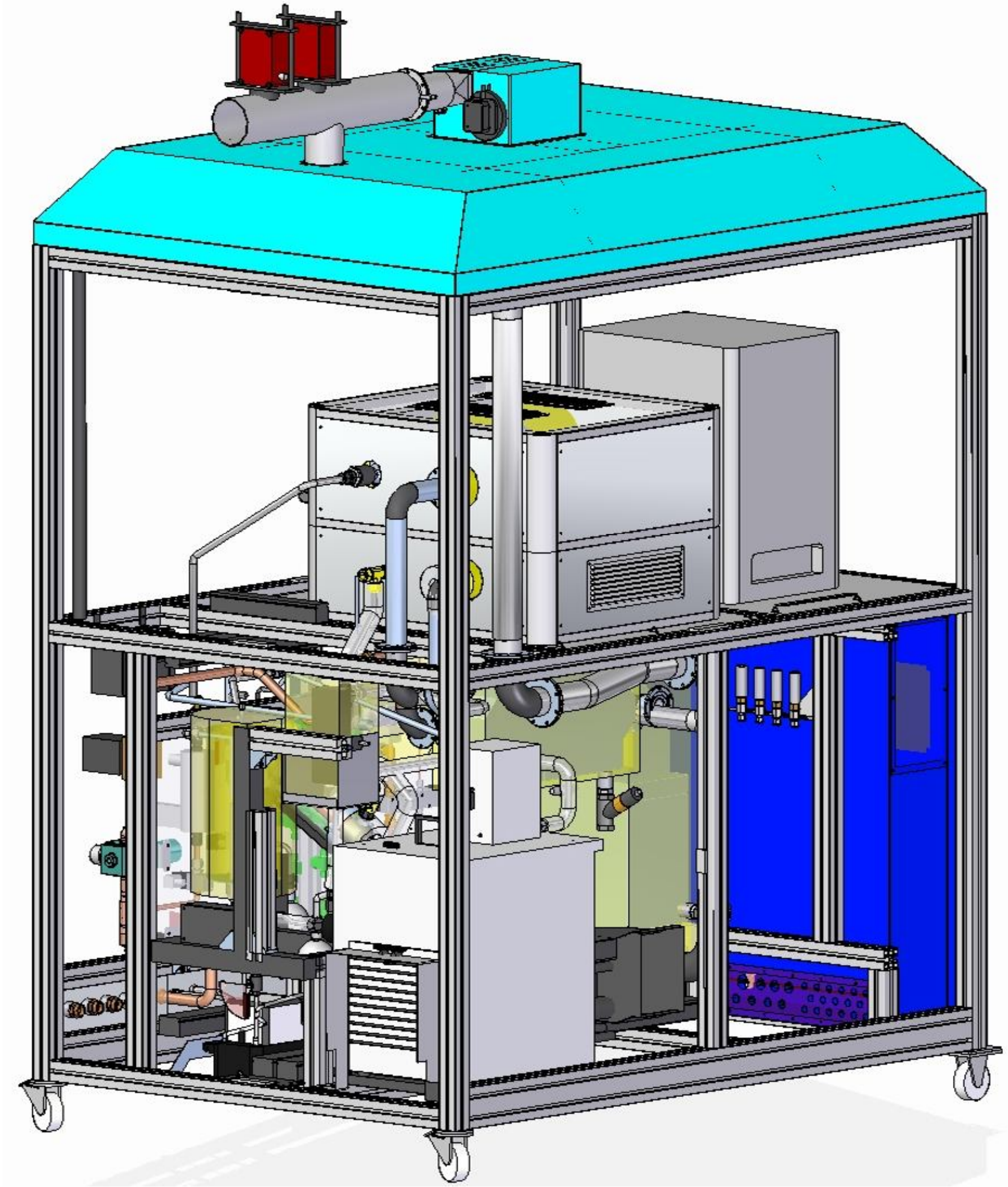
Contents

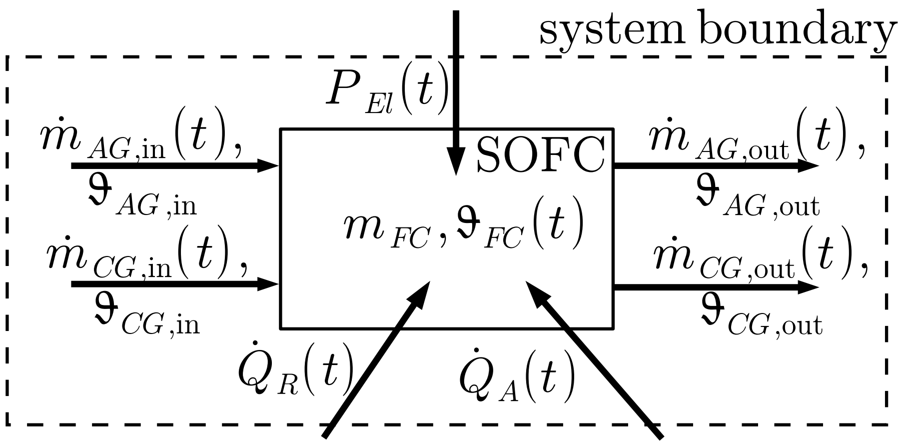
Working principles

- Fluid supply (fuel gas, air)
- Independent preheaters for fuel gas and air
- Stack module containing fuel cells in electric series connection
- Variable electric load as a disturbance



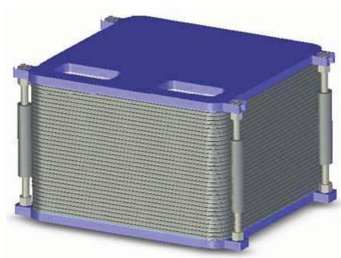
Modeling Approach

Energy balance of the SOFC stack module



- Control-oriented modeling of a SOFC stack module for the derivation of control and observer strategies

- Integral balancing of an instationary energy conversion process in the whole stack module as well as in individual finite volume elements



- Impact of the variation of the internal energy on the local temperature distribution in the stack module

Modeling Approach

- Relation between the variation of the internal energy and the stack temperature for constant material parameters c_{FC} and m_{FC}

$$\frac{dE_{FC}(t)}{dt} = c_{FC} \cdot m_{FC} \cdot \frac{d\vartheta_{FC}(t)}{dt}$$

- Modeling of the effects on the internal energy

$$\begin{aligned}\frac{dE_{FC}(t)}{dt} =& C_{AG}(\vartheta_{FC}, t)\left(\vartheta_{AG,in}(t) - \vartheta_{FC}(t)\right) \\ &+ C_{CG}(\vartheta_{FC}, t)\left(\vartheta_{CG,in}(t) - \vartheta_{FC}(t)\right) \\ &+ \dot{Q}_R(t) + P_{El}(t) + \dot{Q}_A(t)\end{aligned}$$

- Reaction heat flow of the hydrogen oxidation reaction

$$\dot{Q}_R = \frac{\Delta_R H(\vartheta_{FC}) \cdot \dot{m}_{H_2}(t)}{M_{H_2}}$$

Modeling Approach

- Relation between the variation of the internal energy and the stack temperature for constant material parameters c_{FC} and m_{FC}

$$
\begin{aligned}
c_{FC} \cdot m_{FC} \cdot \frac{d\vartheta_{FC}(t)}{dt} =& C_{AG}(\vartheta_{FC}, t)\left(\vartheta_{AG,in}(t) - \vartheta_{FC}(t)\right) \\
&+ C_{CG}(\vartheta_{FC}, t)\left(\vartheta_{CG,in}(t) - \vartheta_{FC}(t)\right) \\
&+ \dot{Q}_R(t) + P_{El}(t) + \dot{Q}_A(t)
\end{aligned}
$$

- Heat transfer including linearized heat radiation to ambient media

$$
\dot{Q}_A = \frac{1}{R_A}\left(\vartheta_A - \vartheta_{FC}\right)
$$

- Ohmic loss effects in the stack material

$$
P_{El}(t) = R_{El} I^2(t)
$$

Modeling Approach

- Relation between the variation of the internal energy and the stack temperature for constant material parameters c_{FC} and m_{FC}

$$
\begin{aligned}
c_{FC} \cdot m_{FC} \cdot \frac{d\vartheta_{FC}(t)}{dt} =& C_{AG}(\vartheta_{FC}, t) \left(\vartheta_{AG,in}(t) - \vartheta_{FC}(t)\right) \\
&+ C_{CG}(\vartheta_{FC}, t) \left(\vartheta_{CG,in}(t) - \vartheta_{FC}(t)\right) \\
&+ \dot{Q}_R(t) + P_{El}(t) + \dot{Q}_A(t)
\end{aligned}
$$

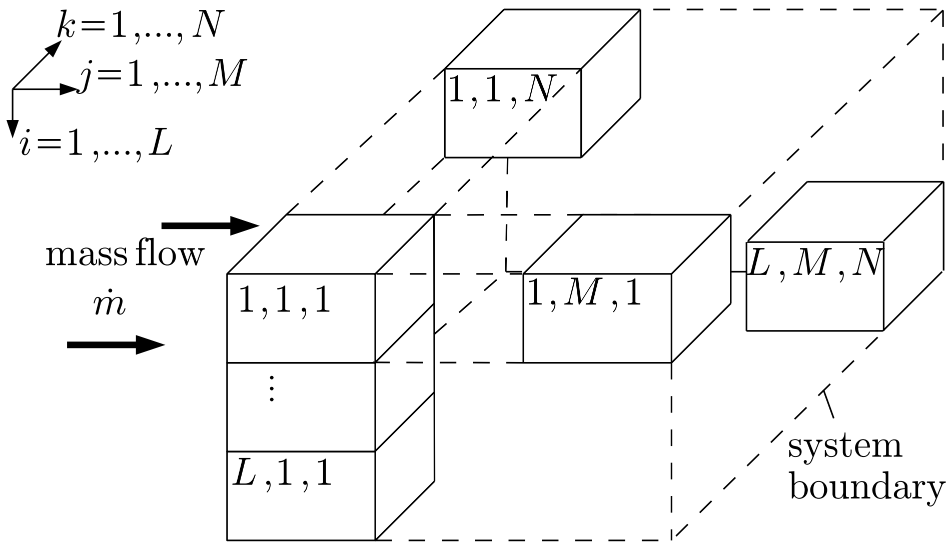
- Anode gas: Heat capacity approximated by 2nd-order polynomials for c_χ with $\chi \in \{H_2, N_2, H_2O\}$

$$
\begin{aligned}
C_{AG}(\vartheta_{FC}, t) =& c_{H_2}(\vartheta_{FC}) \dot{m}_{H_2}(t) \\
&+ c_{N_2}(\vartheta_{FC}) \dot{m}_{N_2}(t) + c_{H_2O}(\vartheta_{FC}) \dot{m}_{H_2O}(t)
\end{aligned}
$$

- Cathode gas: Heat capacity approximated with 2nd-order polynomials for c_{CG}

$$
C_{CG}(\vartheta_{FC}, t) = c_{CG}(\vartheta_{FC}) \cdot \dot{m}_{CG}(t)
$$

Semi-Discretization: The Finite Volume Method



- Semi-discretization into $n_x = L \cdot M \cdot N$ finite volume elements to describe the internal temperature distributions

- Local energy balances lead to a set of n_x coupled ODEs represented by a state vector $x^T = [\vartheta_{1,1,1}, \ldots, \vartheta_{L,M,N}] \in \mathbb{R}^{n_x}$

- System boundary includes the thermal stack insulation

Semi-Discretization: The Finite Volume Method

- ODE for the local temperature distribution in a SOFC stack module

$$
\begin{aligned}
c_{i,j,k} \cdot m_{i,j,k} \cdot \dot{\vartheta}_{i,j,k}(t) = {} & C_{AG,i,j,k}(\vartheta_{i,j,k}, t)\Big(\vartheta_{i,j-1,k}(t) - \vartheta_{i,j,k}(t)\Big) \\
& + C_{CG,i,j,k}(\vartheta_{i,j,k}, t)\Big(\vartheta_{i,j-1,k}(t) - \vartheta_{i,j,k}(t)\Big) \\
& + \dot{Q}_{\eta,i,j,k}(t) + \dot{Q}_{R,i,j,k}(t) + P_{El,i,j,k}(t)
\end{aligned}
$$

- Modeling of local temperature-dependent and time-varying influence factors

Heat flow: $\dot{Q}_{\eta,i,j,k}(t) = \sum_{\eta \in \mathcal{N}} \frac{1}{R_{th,\eta}^{i,j,k}} \left(\vartheta_\eta(t) - \vartheta_{i,j,k}(t)\right)$

Reaction heat flow: $\dot{Q}_{R,i,j,k}(t) = \frac{\Delta_R H_{i,j,k}(\vartheta_{i,j,k}) \cdot \dot{m}_{H_2,i,j,k}(t)}{M_{H_2}}$

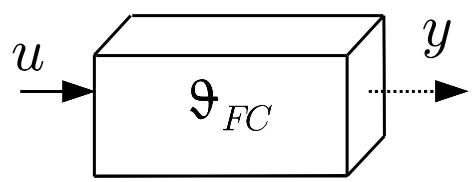
Ohmic losses: $P_{El,i,j,k}(t) = R_{El,i,j,k} I_{i,j,k}^2(t)$

Semi-Discretization: The Finite Volume Method

Case 1: Semi-discretization into a single finite volume element leads to the global energy balance described before

- State variable x and output variable y

$$x(t) = \vartheta_{FC}(t)$$
$$y(t) = h(x) = \vartheta_{FC}(t)$$



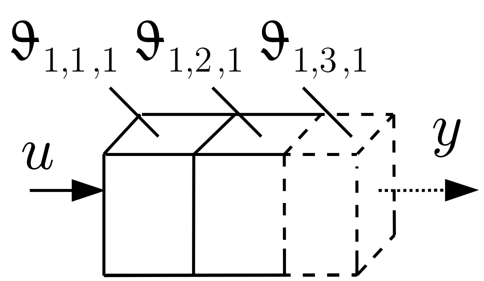
- Nonlinear ordinary differential equation

$$\dot{\vartheta}_{FC} = \Phi\left(\vartheta_{FC}(t), u(t)\right)$$

Case 2: Semi-discretization into three finite volume elements oriented in the direction of mass flow

- State vector x and output variable y

$$x(t) = \left[\vartheta_{111}(t),\ \vartheta_{121}(t),\ \vartheta_{131}(t)\right]^T$$
$$y(t) = h(x) = \vartheta_{131}(t)$$



- Set of coupled nonlinear ordinary differential equations

$$\dot{x}(t) = \Phi\left(x(t), u(t)\right)$$

State Equations — Simplification for Control Synthesis

- Input-affine description of the nonlinear thermal subsystem

$$\begin{aligned}\dot{x}(t) =& f\big(x(t)\big)+g\big(x(t)\big)\cdot u(t), \;\; x \in \mathbb{R}^{n_x}\\ y(t) =& h(x(t)), \;\; y \in \mathbb{R}^{n_y}\\ u(t) =& \dot{m}_{CG}\cdot \Delta\vartheta(t)\end{aligned}$$

- Realization of the temperature difference $\Delta\vartheta$ in the control input by an underlying controller for the preheating devices

$$\Delta\vartheta(t) := \begin{cases}\vartheta_{CG}(t)-\vartheta_{FC}(t) & \text{for} \;\; x(t)=\vartheta_{FC}(t)\\ \vartheta_{CG}(t)-\vartheta_{111}(t) & \text{for} \;\; x(t)=\left[\vartheta_{111}(t),\; \vartheta_{121}(t),\; \vartheta_{131}(t)\right]^T\end{cases}$$

- Exact input-output linearization with relative degree δ (Computation of the Lie-Derivatives of y)

$$\frac{d^i y}{dt^i} = L_f^i h(x) = L_f\left(L_f^{i-1}h(x)\right), \quad i=0,...,\delta-1$$

- Relative degree δ determines the smallest order that explicitly depends on the input u

Modeling Approach — Transformation of the State-Space

- Nonlinear transformation of the state equations with the relative degree $\delta = n_x$ according to

$$z^T = [z_1 \ z_2 \ z_3] = [y \ \dot{y} \ \ddot{y}] = \left[h(x) \ L_f h(x) \ L_f^2 h(x)\right]$$

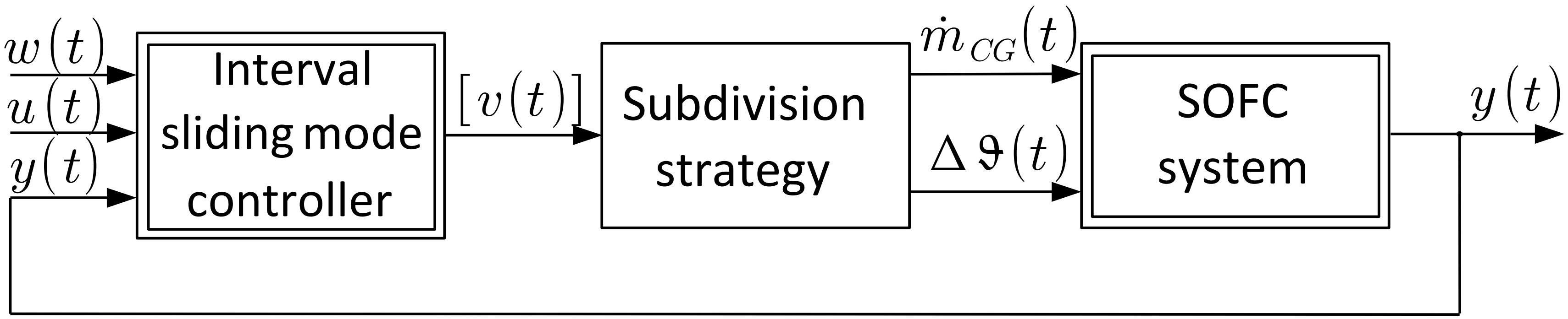
- Nonlinear controller normal form (NCNF)

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} L_f h(x) \\ L_f^2 h(x) \\ L_f^3 h(x) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ L_g L_f^2 h(x) \end{bmatrix} u$$

- Feedback linearizing control law for sufficiently small variations of the mass flow used for the heat-up phase of the SOFC

$$u := \frac{-L_f^3 h(x) - \alpha_0 h(x) - \alpha_1 L_f h(x) - \alpha_2 L_f^2 h(x) + \mu(t)}{L_g L_f h(x)}$$

Robust Sliding Mode Control



- Rejection of disturbances in the neighborhood of a desired operating point by means of sliding mode control regarding physical actuator constraints

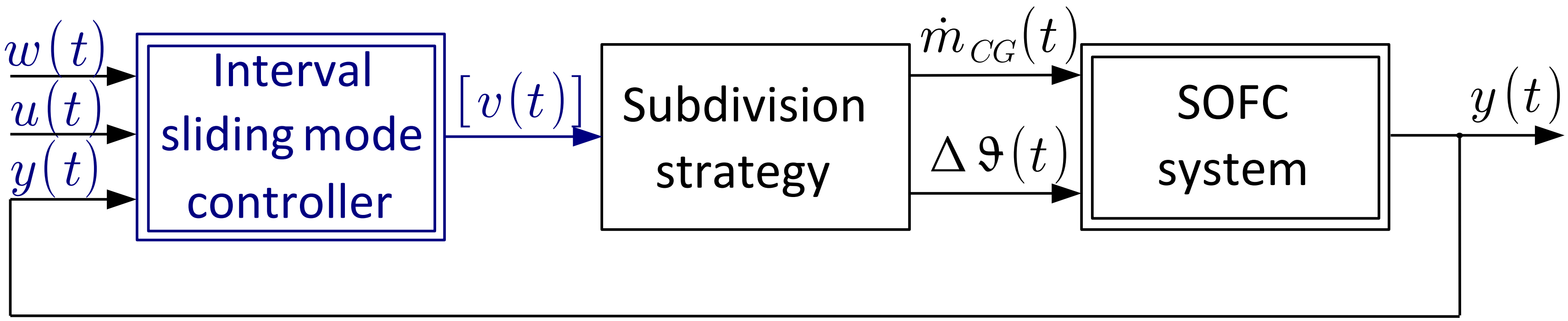
- Online application of interval analysis to account for uncertainties in measurements and for state reconstruction errors

- Quality criterion for choosing adequate values for $\dot{m}_{CG}$ and $\Delta\vartheta$ to manipulate the enthalpy flow of the cathode gas

- Online subdivision strategy allows for converting the interval-based controller output $[v(t)]$ into a point-valued system input $u(t)$

- Guarantee of asymptotical stability inspite of system uncertainties

Robust Sliding Mode Control



- Mathematical model of the SOFC system in an input-affine description is extended by a bounded disturbance $d \in [d]$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \\ \tilde{a}(z,p,d) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \tilde{b}(z,p) \end{bmatrix} v$$

- Disturbance influences the system according to $\tilde{a} = L_f^3 h + d$

- Identification of interval parameters $p \in [p]$

Robust Sliding Mode Control

- Definition of an asymptotically stable sliding surface $s(\tilde{z}) = 0$ with the tracking error $\tilde{z}_1^{(j)} = z_1^{(j)} - z_{1,d}^{(j)}$

$$s(\tilde{z}) = \tilde{z}_1^{(2)} + \alpha_1 \tilde{z}_1^{(1)} + \alpha_0 \tilde{z}_1^{(0)} = 0$$

  and the output z_1 and its time derivatives $z_1^{(j)}$, $j = 1, ..., \delta - 1 = n_x - 1$

- Stabilization of the motion towards the sliding surface by a suitable Lyapunov function V

$$V = \frac{1}{2}s^2 > 0 \quad \text{for} \quad s \neq 0, \quad \text{and its time derivative} \quad \dot{V} = s\dot{s}$$

- The condition $\dot{V} = s\dot{s} \leq 0$ for the time derivative of the Lyapunov function is fulfilled with

$$s\dot{s} \leq -\eta s \ \mathrm{sign}\{s\} \quad \text{which is guaranteed for}$$

$$\dot{s} + \eta \cdot \mathrm{sign}\{s\} = -\beta \cdot \mathrm{sign}\{s\}, \quad \eta, \beta > 0$$

- Control input v is obtained from

$$\tilde{a}(z,p,d) + \tilde{b}(z,p)v - z_{1,d}^{(3)} + \alpha_1 \tilde{z}_1^{(2)} + \alpha_0 \tilde{z}_1^{(1)} = -(\beta + \eta) \cdot \mathrm{sign}\{s\}$$

Robust Sliding Mode Control

- Control law for the disturbance rejection in the thermal subsystem

$$[v] := \left[\frac{-\tilde{a}\left(z, [p], [d]\right) + z_{1,d}^{(3)} - \alpha_1 \tilde{z}_1^{(2)} - \alpha_0 \tilde{z}_1^{(1)}}{\tilde{b}\left(z, [p]\right)} - \frac{1}{\tilde{b}\left(z, [p]\right)} \underbrace{(\eta + \beta)}_{=: \tilde{\eta} > 0} \cdot \mathrm{sign}\{s\} \right] \Bigg|_{\substack{p \in [p] \\ d \in [d]}}$$

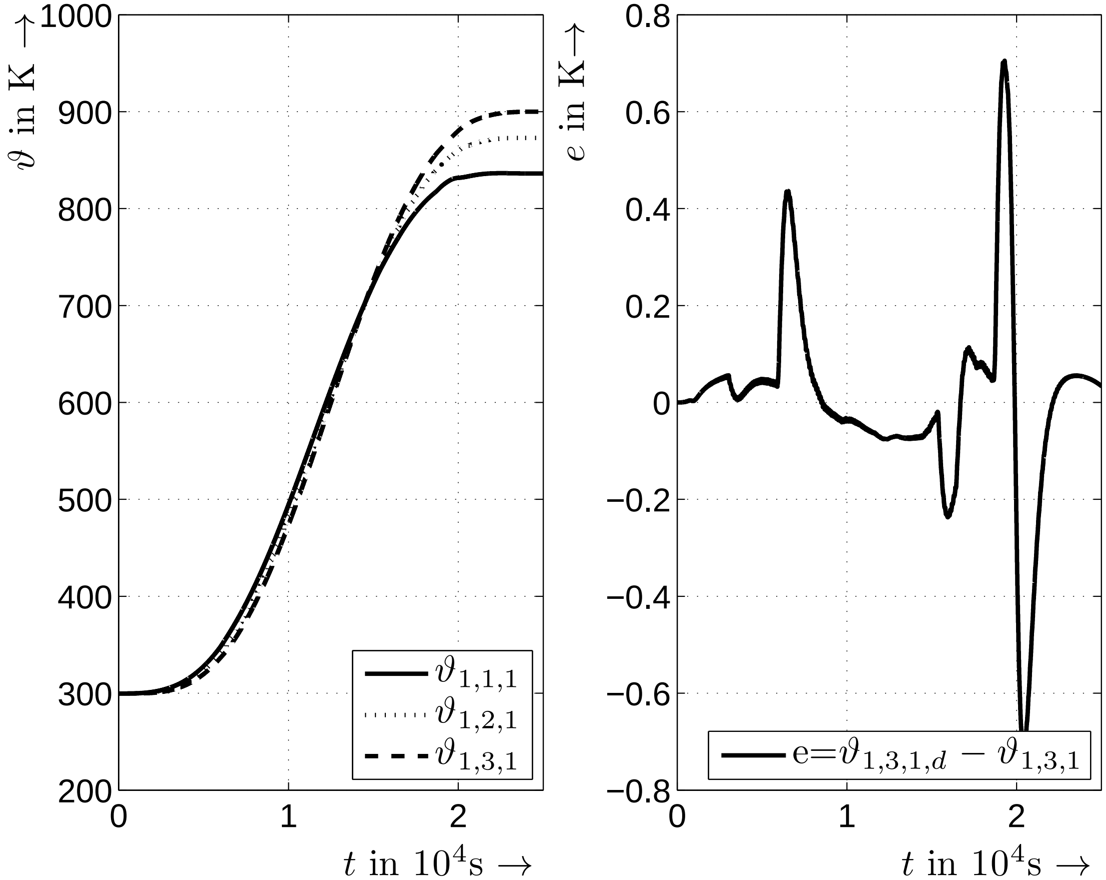
- Appropriate choice of the switching amplitude $\tilde{\eta}$ in the case of control design for interval parameters $p \in [p]$ and interval disturbance $d \in [d]$

- Controller output for a guaranteed stabilization of the thermal SOFC system

$$v := \begin{cases} \sup\{[v]\} & \text{for} \quad s \geq 0 \\ \inf\{[v]\} & \text{for} \quad s < 0 \end{cases}$$

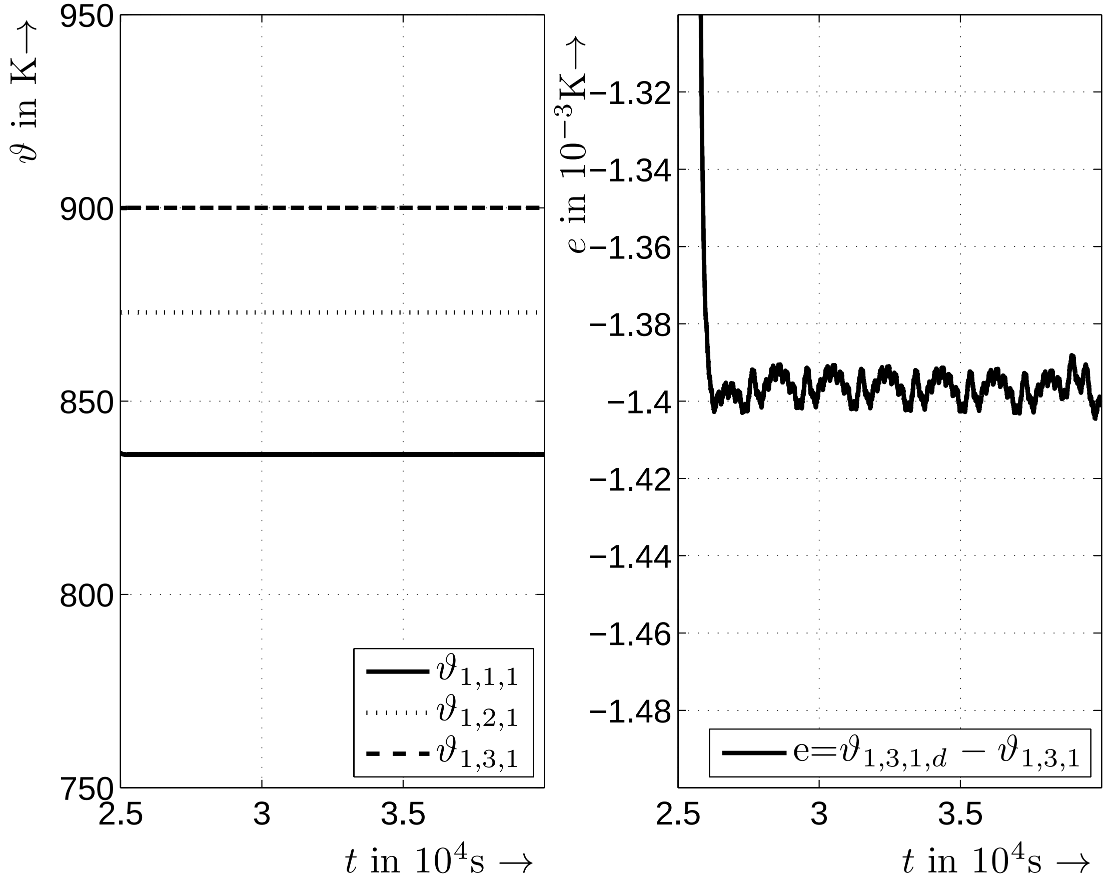
Robust Sliding Mode Control

- Instationary heating phase of the SOFC stack module using an exact linearizing control law to reach a desired operating point



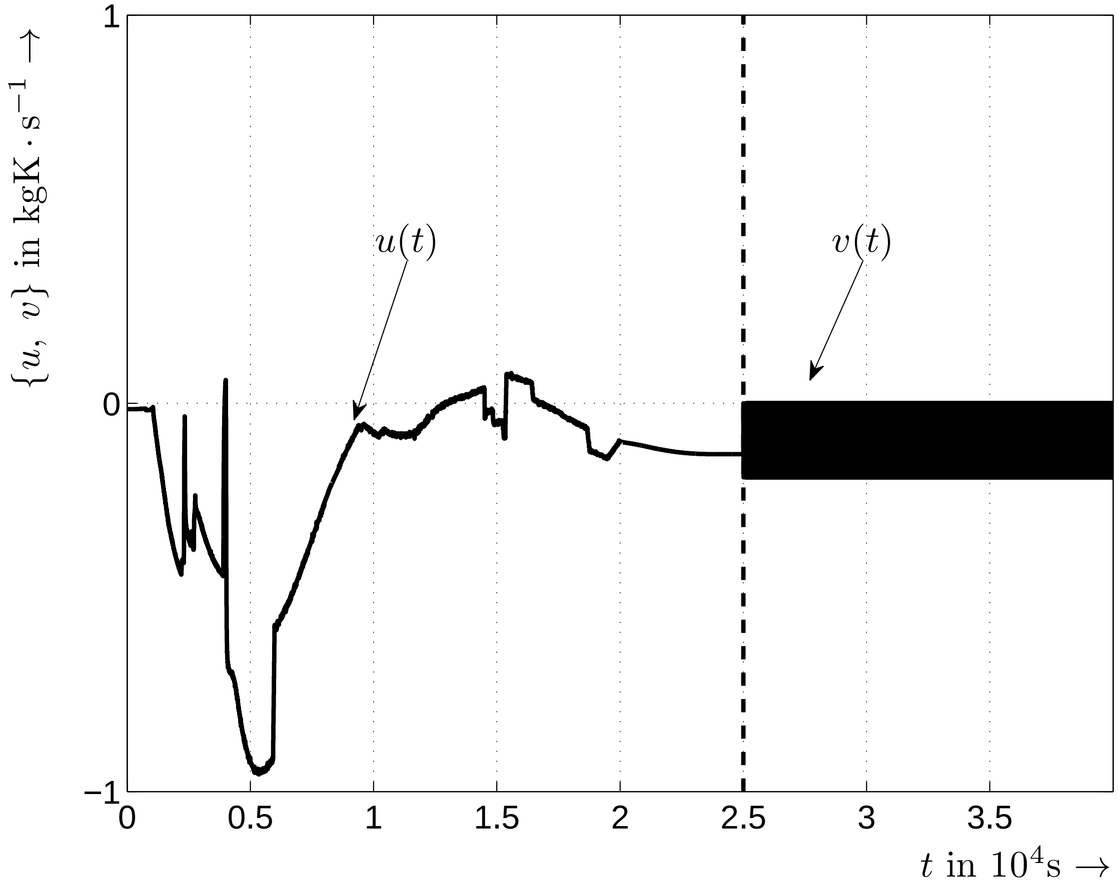
Robust Sliding Mode Control

- Switching to the interval-based sliding mode control law in the point of time $t = 2.5 \cdot 10^4$ s
- **Objective:** Rejection of disturbances and stabilization of desired operating points accounting for parameter uncertainties and bounded state enclosures



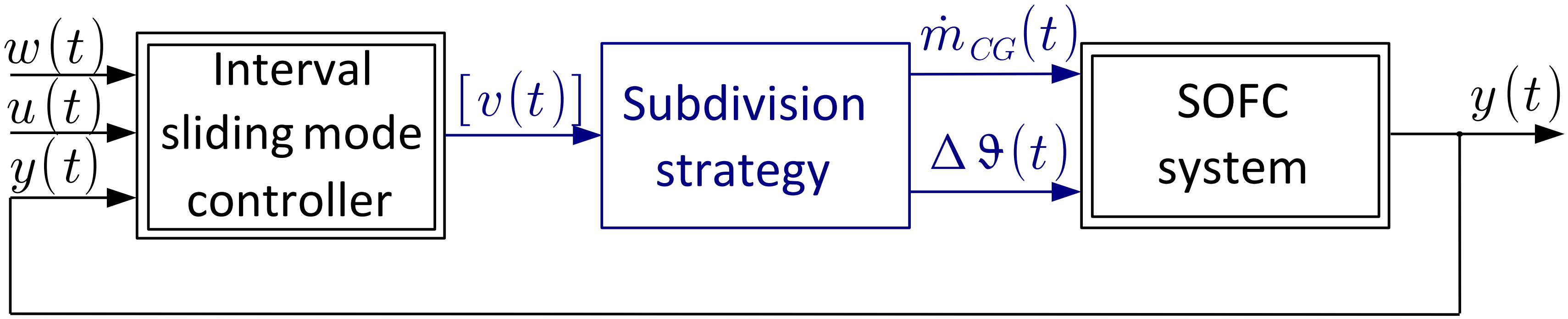
Robust Sliding Mode Control

- Output of exact linearizing feedback control law $u(t)$ with switching to the output of the interval-based sliding mode controller $v(t)$ at the point of time $t = 2.5 \cdot 10^4$s



Problem: Adequate setting of the SOFC system input $u = \dot{m}_{CG}\Delta\vartheta$ with an available sliding mode controller output $v(t)$

Robust Sliding Mode Control



- Subdivision strategy to determine appropriate control inputs $\dot{m}_{CG}$ and $\Delta\vartheta$ corresponding to $[v(t)]$
- The product of the mass flow $\dot{m}_{CG}$ and of the temperature difference $\Delta\vartheta$ determines the system input

$$u := (\dot{m}_{CG} \cdot \Delta\vartheta)$$

- Operating ranges of the actuators are defined by bounded intervals

Implementation of the Interval-Based Control Law in Simulations

- A **splitting procedure** is employed in each time step k starting with the initial interval box described by $\left[\dot{m}_{CG}^{<0>}\right]$ and $\left[\Delta\vartheta^{<0>}\right]$ which is identical to the physical actuator constraints

- Multi-sectioning of the input interval vector $\left[\left[\dot{m}_{CG}^{<l>}\right]\ ;\ \left[\Delta\vartheta^{<l>}\right]\right]^T$ into the four interval boxes the mass flow and temperature difference in the time step k

$$\left[\left[\dot{m}_{CG}^{<l>}\right]\ ;\ \left[\Delta\vartheta^{<l>}\right]\right]^T := \begin{bmatrix} \left[\inf\left(\left[\dot{m}_{CG}^{<l>}\right]\right)\ ;\ \mathrm{mid}\left(\left[\dot{m}_{CG}^{<l>}\right]\right)\right] \\ \left[\inf\left(\left[\Delta\vartheta^{<l>}\right]\right)\ ;\ \mathrm{mid}\left(\left[\Delta\vartheta^{<l>}\right]\right)\right] \end{bmatrix}$$

$$\left[\left[\dot{m}_{CG}^{<L+1>}\right]\ ;\ \left[\Delta\vartheta^{<L+1>}\right]\right]^T := \begin{bmatrix} \left[\mathrm{mid}\left(\left[\dot{m}_{CG}^{<l>}\right]\right)\ ;\ \sup\left(\left[\dot{m}_{CG}^{<l>}\right]\right)\right] \\ \left[\inf\left(\left[\Delta\vartheta^{<l>}\right]\right)\ ;\ \mathrm{mid}\left(\left[\Delta\vartheta^{<l>}\right]\right)\right] \end{bmatrix}$$

$$\left[\left[\dot{m}_{CG}^{<L+2>}\right]\ ;\ \left[\Delta\vartheta^{<L+2>}\right]\right]^T := \begin{bmatrix} \left[\inf\left(\left[\dot{m}_{CG}^{<l>}\right]\right)\ ;\ \mathrm{mid}\left(\left[\dot{m}_{CG}^{<l>}\right]\right)\right] \\ \left[\mathrm{mid}\left(\left[\Delta\vartheta^{<l>}\right]\right)\ ;\ \sup\left(\left[\Delta\vartheta^{<l>}\right]\right)\right] \end{bmatrix}$$

$$\left[\left[\dot{m}_{CG}^{<L+3>}\right]\ ;\ \left[\Delta\vartheta^{<L+3>}\right]\right]^T := \begin{bmatrix} \left[\mathrm{mid}\left(\left[\dot{m}_{CG}^{<l>}\right]\right)\ ;\ \sup\left(\left[\dot{m}_{CG}^{<l>}\right]\right)\right] \\ \left[\mathrm{mid}\left(\left[\Delta\vartheta^{<l>}\right]\right)\ ;\ \sup\left(\left[\Delta\vartheta^{<l>}\right]\right)\right] \end{bmatrix}$$

Implementation of the Interval-Based Control Law in Simulations

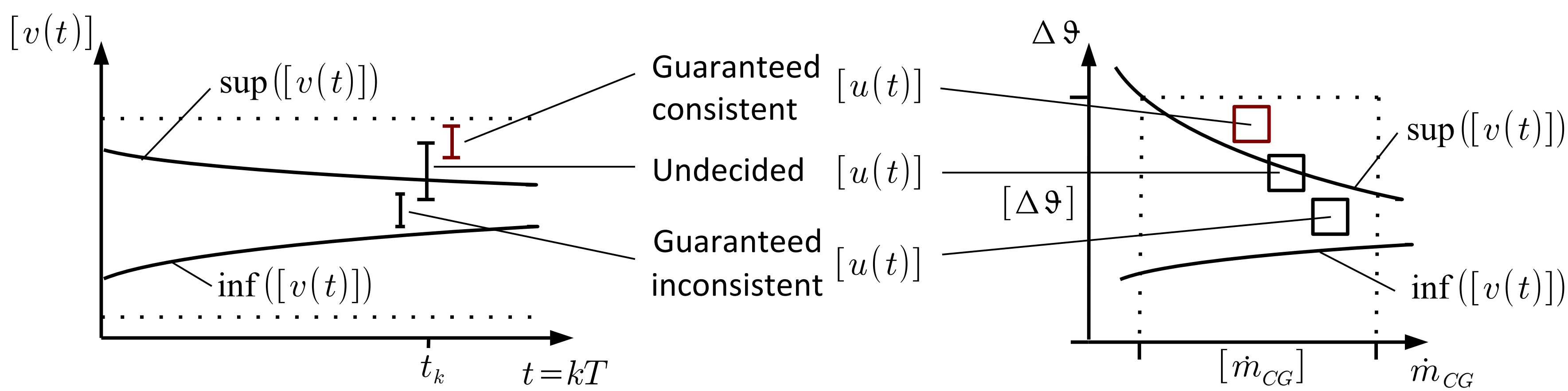
- A **validity test** of $\left[u^{<l>}\right] = \left[\dot{m}_{CG}^{<l>}\right]\left[\Delta\vartheta^{<l>}\right]$ is performed according to the controller output $[v]$ for classifying **guaranteed consistent**, **undecided** and **guaranteed inconsistent** input intervals

- Consistency of $\left[u^{<l>}\right]$ in $[v]$ is proven if

$$\sup\{[v]\} < \inf\{\left[u^{<l>}\right]\} \quad \text{for} \quad s \geq 0$$

$$\inf\{[v]\} > \sup\{\left[u^{<l>}\right]\} \quad \text{for} \quad s < 0$$

- Illustration of the consistency test for $s > 0$ bounded by actuator constraints (dashed lines)



Implementation of the Interval-Based Control Law in Simulations

- Compositions of $u(t)$ are assessed for l subdivided intervals in each time step k
- Optimal interval box of $[\dot{m}_{CG}]$ and $[\Delta\vartheta]$ is detected with the quality criterion

$$\left[J_k^{<l>}\right] = \kappa_1 \left(\left[\Delta\vartheta_k^{<l>}\right] - [\Delta\vartheta_{nom}]\right)^2 + \kappa_2 \left(\left[\Delta\vartheta_k^{<l>}\right]\right)^2 + \kappa_3 \left(\left[\dot{m}_{CG,k}^{<l>}\right] - [\dot{m}_{nom}]\right)^2$$

- The minimization of $J_{opt} = \min\left(\inf\left(\left[J_k^{<l>}\right]\right)\right)$ yields

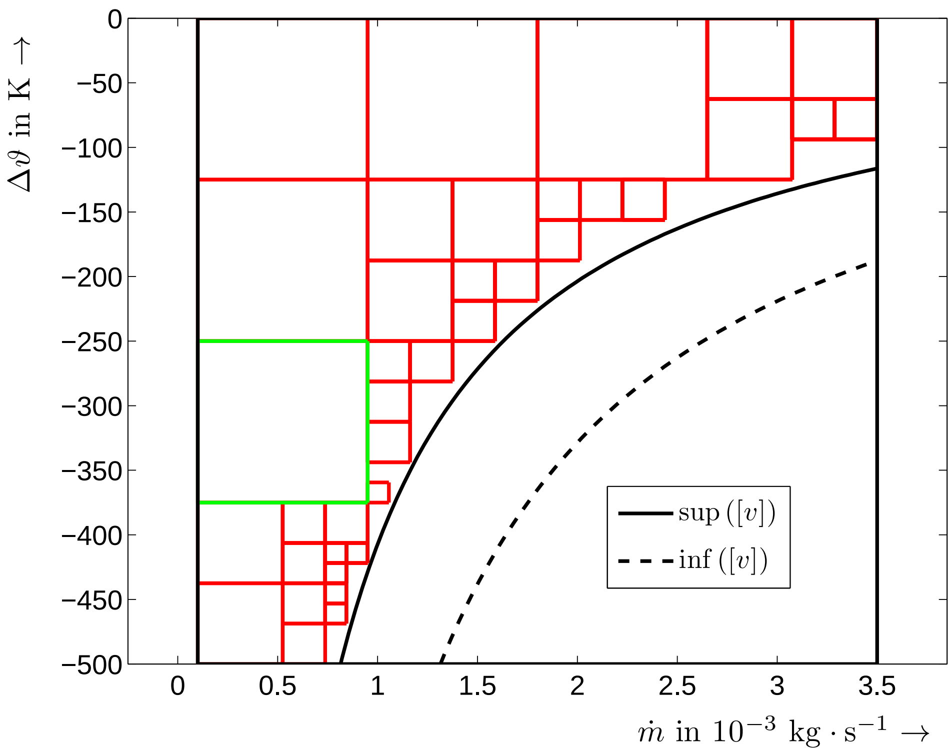
$$\left[\dot{m}_{CG}^{<opt>}\right] \quad \text{and} \quad \left[\Delta\vartheta^{<opt>}\right]$$

- The guaranteed stabilizing control output for the SOFC system with $v \geq \sup([v])$ is determined by

$$u(t) = \operatorname{mid}\left(\left[\dot{m}_{CG}^{<opt>}\right] \cdot \left[\Delta\vartheta^{<opt>}\right]\right)$$

Implementation of the Interval-Based Control Law in Simulations

- Depiction of the optimal system input with reference to the nominal values for $[\dot{m}_{nom}]$ and $[\Delta\vartheta_{nom}]$
- Cooling process with a value $s > 0$ in the sliding mode control design



Conclusions and Outlook

Conclusion

- Nonlinear modeling of the thermal subsystem of SOFCs including uncertainties in the parameterization and the system states
- Design of an interval-based sliding mode controller that is capable to cope with bounded uncertainties in a desired operating point
- Optimal adjustment of the enthalpy flow as a control input of the system employing a subdivision strategy regarding actuator constraints

Dötschel, Thomas; Rauh, Andreas; Aschemann, Harald: *Reliable Control and Disturbance Rejection for the Thermal Behavior of Solid Oxide Fuel Cell Systems*, presented at MATHMOD 2012, Vienna, Austria, 2012. to appear on IFAC-PapersOnLine.net

Conclusions and Outlook

Outlook

- Proof of the robustness in case of a switching output y where the remaining system dynamics have to be enclosed in state intervals
- Implementation of the presented approaches in the SOFC system available at the Chair of Mechatronics at the University of Rostock
- Translation of the software routines in INTLAB into the real-time capable C-XSC implementation

Rauh, Andreas; Aschemann, Harald: *Interval-Based Sliding Mode Control and State Estimation for Uncertain Systems*, IEEE Intl. Conference on Methods and Models in Automation and Robotics MMAR 2012, Miedzyzdroje, Poland, 2012.