Properties and Estimations of Parametric AE Solution Sets

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Consider linear systems $A(p)x = b(p)$ whose input data are linear functions of uncertain parameters varying within given intervals, $p_i \in [p_i]$, $i = 1, \ldots, k$. Such systems are common in many engineering analysis or design problems, control engineering, robust Monte Carlo simulations, etc., where there are complicated dependencies between the model parameters which are uncertain. We are interested in the parametric AE solution sets, which are defined by universally and existentially quantified parameters, and the former precede the latter. For two disjoint sets $\mathcal{E}$ and $\mathcal{A}$, such that $\mathcal{E} \cup \mathcal{A} = \{1, \ldots, k\}$,

$$\begin{aligned}\Sigma^p_{AE} &= \Sigma(A(p), b(p), [p]) \\ &:= \{x \in \mathbf{R}^n \mid (\forall p_{\mathcal{A}} \in [p_{\mathcal{A}}])(\exists p_{\mathcal{E}} \in [p_{\mathcal{E}}])(A(p)x = b(p))\}.\end{aligned}$$

In this talk we present the explicit description of some parametric AE solution sets together with some important inclusion relations between such solution sets. In the special cases of parametric tolerable and controllable solution sets we discuss some newly proven properties of these solution sets. Numerical examples accompanied by graphic representations illustrate the solution sets and their properties. Some methods for inner and outer estimations of parametric AE solution sets will be also presented. Special discussion on the properties of these methods for estimating the parametric tolerable and controllable solution sets will be provided.

As an application of the parametric AE solution set estimations we consider parametric AE solution sets of the interval Lyapunov matrix equation of small size. The most common approach in solving matrix equations is to transform the matrix equation into a corresponding linear system. The state matrix in the Lyapunov matrix equation can be either interval or can have linear uncertainty structure. In both cases the parametric approach is applicable if we want sharp estimations.