Rigorous Computation with Function Enclosures in Chebyshev Basis ⋆

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When rigorously computing with a real function, a truncated Taylor series polynomial approximation is commonly used to replace the actual function. One of the applications of such a rigorous function enclosure lies in verified algorithms for integration of non-linear ordinary differential equations [4].

In this paper, we present a multi-variable function enclosure using the Chebyshev series approximation and the remainder term stored as an interval. Since the Chebyshev series converge faster for all analytic functions compared to the Taylor series, our function enclosures approximate real functions with tighter remainder intervals.

In existing work [1,2], only operations with functions in one variable are described. In [1], the function approximation is stored in the form of function values in the Chebyshev nodes. The authors use non-rigorous methods to compute coefficients of Chebyshev polynomials and no enclosure of the exact function value is guaranteed. On the other hand, the authors in [2] use rigorous methods, but only addition, multiplication and composition of one variable functions are presented.

We present an efficient algorithm for rigorous addition, substraction, multiplication, division, composition, integration and derivative of multi-variable Chebyshev function enclosures. Our publicly available implementation [3] supports function enclosures based on both Taylor and Chebyshev truncated series and allows their comparison. Computational experiments with the initial value problem of ordinary differential equations show that the approach is competitive with the best publicly available verified solvers.

References

1. Z. Battles and L. N. Trefethen. An extension of MATLAB to continuous functions and operators. *SIAM J. Sci. Comput.*, 25:1743–1770, May 2004.
2. N. Brisebarre and M. Joldeş. Chebyshev interpolation polynomial-based tools for rigorous computing. In *Proceedings of the 2010 International Symposium on Symbolic and Algebraic Computation*, ISSAC '10, pages 147–154, New York, NY, USA, 2010. ACM.
3. T. Dzetkulič. ODEINTEGRATOR. `http://odeintegrator.sourceforge.net`, 2012. Software package.
4. K. Makino and M. Berz. Rigorous Integration of Flows and ODEs using Taylor Models. *Symbolic Numeric Computation*, pages 79–84, 2009.

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