Interval Particle Filter for LiDAR-Based Object Tracking

Towards Robust Tracking via Probabilistic and Interval Methods

Mohamed FNADI and Régis LHERBIER

LISIC – UR4491 – Université du Littoral Côte d'Opale

mohamed.fnadi@univ-littoral.fr

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Research Context and Objectives

Floating debris and marine waste

Small vessels and navigation hazards

Scientific Challenges

- **Diverse targets**: Various floating objects (debris, nets, vessels) with different properties
- **Harsh environments**: Dynamic marine conditions with waves and weather effects
- **Perception Sensors Uncertainties**: Noisy LiDAR measurements

Research Context and Objectives

Floating debris and marine waste

Small vessels and navigation hazards

Research Objectives

- Develop **robust LiDAR-based detection** for floating objects in marine environments
- Implement **interval-based tracking** for reliable trajectory estimation of floating objects

LiDAR-based Detection and Tracking of Floating Objects

LiDAR Data → Floating Object Detection (not treated in work)

Ouster OS1-128

Velodyne HDL-32

Floating Object Detection
(not treated in this work)

↓

Interval Particle Filter
Robust Tracking

↓

Target Trajectography

Outline

LiDAR-Based Detection: Principles and Models

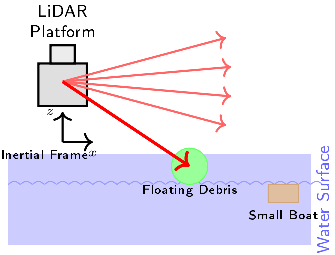
LiDAR Measurement Parameters

- **Distance** (r): Radial distance to object barycenter
- **Orientation** (ϕ): Relative orientation between LiDAR and object frame

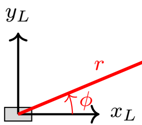
Key Assumptions

- Object detection available at each sampling interval
- LiDAR sensor position known and fixed in inertial frame
- Nonlinear uncertain system dynamics and observation models
- Measurements subject to bounded uncertainties:
$$[\mathbf{Y}_k] = [[\boldsymbol{r}_k]\ ,\ [\boldsymbol{\phi}_k]]$$
$$= [[r_k - \Delta r_k, r_k + \Delta r_k]\ ,\ [\phi_k - \Delta\phi_k, \phi_k + \Delta\phi_k]]$$

Marine Detection Scenario



Coordinate System



Bayes Filter: Prediction and Correction

Prediction Relation: Chapman-Kolmogorov equation

$$
\begin{aligned}
\texttt{pred}(\boldsymbol{x}_k) &= p(\boldsymbol{x}_k|\boldsymbol{y}_{0:k-1}) \\
&= \int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) \cdot p(\boldsymbol{x}_{k-1}|\boldsymbol{y}_{0:k-1})\, d\boldsymbol{x}_{k-1} \quad \text{(total prob.)} \\
&= \int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) \cdot \texttt{bel}(\boldsymbol{x}_{k-1})\, d\boldsymbol{x}_{k-1} \quad \text{(def. of bel)}
\end{aligned}
$$

Upadte

$$
\begin{aligned}
\texttt{bel}(\boldsymbol{x}_k) &= p(\boldsymbol{x}_k|\boldsymbol{y}_{0:k}) \\
&= p(\boldsymbol{x}_k|\boldsymbol{y}_k, \boldsymbol{y}_{0:k-1}) \quad \text{(Markov)} \\
&= \frac{1}{p(\boldsymbol{y}_k|\boldsymbol{y}_{0:k-1})}\, p(\boldsymbol{y}_k|\boldsymbol{x}_k) \cdot p(\boldsymbol{x}_k|\boldsymbol{y}_{0:k-1}) \quad \text{(Bayes)} \\
&= \frac{p(\boldsymbol{y}_k|\boldsymbol{x}_k) \cdot \texttt{pred}(\boldsymbol{x}_k)}{p(\boldsymbol{y}_k|\boldsymbol{y}_{0:k-1})} \quad \text{(def. of pred)}
\end{aligned}
$$

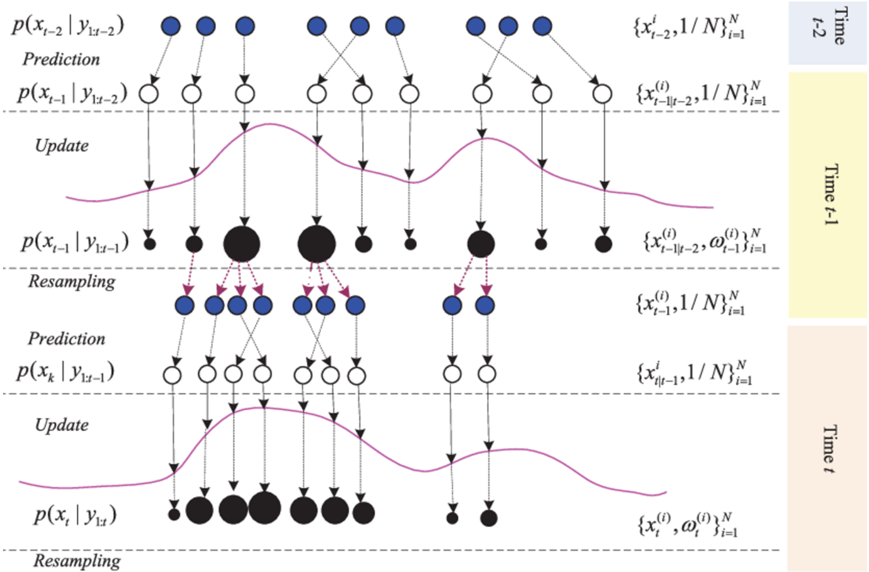
Bayesian Filter Algorithm

Algorithm 1: Bayesian Filter

Input: $\texttt{bel}(\mathbf{x}_{k-1})$, u_k, y_k
Output: $\texttt{bel}(\mathbf{x}_k)$
for *each* $\mathbf{x}_k$ **do**
  $\texttt{pred}(\mathbf{x}_k) \leftarrow \int p(\mathbf{x}_k|\mathbf{x}_{k-1}, u_k) \cdot \texttt{bel}(\mathbf{x}_{k-1})\, d\mathbf{x}_{k-1}$
  $\texttt{bel}(\mathbf{x}_k) \leftarrow h \cdot p(y_k|\mathbf{x}_k) \cdot \texttt{pred}(\mathbf{x}_k)$
end
return $\texttt{bel}(\mathbf{x}_k)$

Difficult to implement exactly because there isn't always an analytical solution for the integral (line 4) and product (line 5)

Classical Particle Filter



Classical Particle Filter Algorithm

Algorithm 2: Classical Particle Filter

Input: $X_i(k)$, $w_i(k)$ (particles and weights at time k)
Output: $X_i(k+1)$, $w_i(k+1)$ (particles and weights at time $k+1$)

```
while explore do
    y(k+1) = measure();                                         LiDAR sensor observation
    for each particle i from 1 to N do
        X_i(k+1) ← f(X_i(k), u(k), σ)                           prediction using noisy model
        w_i(k+1) ← p(y(k+1) | X_i(k+1))                         weight update
    end for
    for each particle i from 1 to N do
        w_i(k+1) = w_i(k+1) / Σ_{j=1}^{N} w_j(k+1)              Normalization
        N_eff ← 1 / Σ_{i=1}^{N} w_i^2(k+1)                      calculation of N_eff
    end for
    if N_eff < N_threshold then
        for each particle i from 1 to N do
            X_i(k+1) ← resample(X_i(k+1), w_i(k+1))             resampling
            w_i(k+1) ← 1/N                                      weight normalization
        end for
    end if
    k ← k+1
end while
```

Where:

$$w_i(k+1) = \frac{w_i(k+1)}{\sum_{j=1}^{N} w_j(k+1)}$$

$$N_{\text{eff}} \leftarrow \frac{1}{\sum_{i=1}^{N} w_i^2(k+1)}$$

Why Interval Particle Filter?

Limitations of Classical Approaches

- **Kalman Filter**: Optimal for linear Gaussian systems but limited in complex marine environments
- **Particle Filter (PF)**: Effective for linear and nonlinear non-Gaussian noise but requires accurate noise distribution models
- **Set-Membership Methods**: Provide guaranteed bounds but often yield overly conservative estimates

Interval Particle Filter Advantage

Hybrid approach combining the probabilistic robustness of particle filters with the guaranteed bounded-error properties of interval analysis

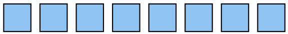
F. Abdallah, A. Gning, and P. Bonnifait. Box particle filtering for nonlinear state estimation using interval analysis. *Automatica*, 44(3), pp. 807–815, 2008.

IPF Algorithm Overview

Step 1: Initialization

❶ Initialization

$$\left\{ \left[\mathbf{x}_0^{(\eta)} \right],\ \omega_0^{(\eta)} = \frac{1}{N_p} \right\}_{\eta=1}^{N_p}$$



N_p *initial particle boxes*

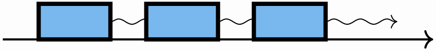
- Particle initialization with bounded boxes
- Uniform distribution of initial weights
- Each particle represents an initial state hypothesis of the target

IPF Algorithm Overview

Step 2: Prediction

❷ Prediction

$$\left\{ \left[\mathbf{x}_{k+1}^{(\eta)}\right] = [\mathbf{f}] \left(\left[\mathbf{x}_{k}^{(\eta)}\right], [\mathbf{u}_k], \left[\mathbf{w}_{k}^{(\eta)}\right] \right) \right\}_{\eta=1}^{N_p}$$



Propagation with system uncertainty

- Particle propagation via the target dynamic model – inclusion function $[\mathbf{f}]$
- Incorporation of system uncertainties
- Expansion of uncertainty intervals using inclusion function $[\mathbf{f}]$

❸ Forward-Backward Contraction

Forward propagation: $\left\{\left[\mathbf{y}_{k+1}^{(\eta)}\right] = [\mathbf{g}]\left(\left[\mathbf{x}_{k+1}^{(\eta)}\right]\right)\right\}_{\eta=1}^{N_p}$

Observation error contraction: $\left\{\left[\boldsymbol{v}_{k+1}^{(\eta)}\right] = \left[\boldsymbol{v}_{k+1}^{(\eta)}\right] \cap \left(\color{red}{[\mathbf{y}_{k+1}]} - \left[\mathbf{y}_{k+1}^{(\eta)}\right]\right)\right\}_{\eta=1}^{N_p}$

Measurement update: $\left\{\left[\mathbf{y}_{k+1}^{(\eta)}\right] = \left[\mathbf{y}_{k+1}^{(\eta)}\right] \cap \left([\mathbf{g}]\left(\left[\mathbf{x}_{k+1}^{(\eta)}\right]\right) + \left[\boldsymbol{v}_{k+1}^{(\eta)}\right]\right)\right\}_{\eta=1}^{N_p}$

Innovation calculation: $\left\{\left[\tilde{\mathbf{y}}_{k+1}^{(\eta)}\right] = \left[\mathbf{y}_{k+1}^{(\eta)}\right] \cap \left(\color{red}{[\mathbf{y}_{k+1}]} - \left[\boldsymbol{v}_{k+1}^{(\eta)}\right]\right)\right\}_{\eta=1}^{N_p}$

Backward propagation (Overlap): $\left\{\color{blue}{\left[\tilde{\mathbf{x}}_{k+1}^{(\eta)}\right]} = \left[\mathbf{x}_{k+1}^{(\eta)}\right] \cap \mathbf{g}^{-1}\left(\left[\tilde{\mathbf{y}}_{k+1}^{(\eta)}\right]\right)\right\}_{\eta=1}^{N_p}$

- Integration of LiDAR sensor measurements (in red)
- Interval contraction to eliminate over-estimation using observation inclusion function $[\mathbf{g}]$

❹ Correction

Likelihood	$\left\{ Likelihood_k^{(\eta)} = \prod_{j=1}^{n_x} \frac{\left\| \left[\tilde{\mathbf{x}}_{k+1}^{(\eta)}(j) \right] \right\|}{\left\| \left[\mathbf{x}_{k+1}^{(\eta)}(j) \right] \right\|} \right\}_{\eta=1}^{N_p}$
Weight update	$\left\{ \omega_{k+1}^{(\eta)} = Likelihood_k^{(\eta)} \times \omega_k^{(\eta)} \right\}_{\eta=1}^{N_p}$

- Likelihood calculation based on interval volumes
- Particle weight update
- Particles well-aligned with measurements receive higher weights

IPF Algorithm Overview

Step 5: Estimation and Resampling

5 Estimation and Resampling

Weighting by box centers

$$\hat{\mathbf{x}}_{k+1} = \sum_{\eta=1}^{N_p} \omega_{k+1}^{(\eta)} \text{mid}\left(\left[\mathbf{x}_{k+1}^{(\eta)}\right]\right) = \sum_{\eta=1}^{N_p} \omega_{k+1}^{(\eta)} \mathbf{c}_{k+1}^{(\eta)}$$

If $N_{\text{eff}} < \textit{threshold}$: resampling

$$N_{\text{eff}} = \frac{1}{\sum_{\eta=1}^{N_p} \left(\omega_{k+1}^{(\eta)}\right)^2}$$

- State estimation by weighted average of centers
- Calculation of sampling efficiency
- Resampling when necessary to avoid degeneracy

Algorithm 3: Interval Particle Filter for Object Tracking

Initialization: $\left\{\left[\mathbf{x}_0^{(\eta)}\right], \omega_0^{(\eta)} = \frac{1}{N_p}\right\}_{\eta=1}^{N_p}$ — N_p initial particle boxes

for $k = 1$ **to** T **do**

  $[\mathbf{y}_k] = [[r_k]\ ,\ [\phi_k]]$ — LiDAR measurements boxes

  for $\eta = 1$ **to** N_p **do**

    Prediction: $\left[\mathbf{x}_k^{(\eta)}\right] \leftarrow [\mathbf{f}]\left(\left[\mathbf{x}_{k-1}^{(\eta)}\right], [\mathbf{u}_{k-1}], \left[\boldsymbol{w}_{k-1}^{(\eta)}\right]\right)$ — Propagation with system uncertainty

    Forward-Backward Contraction:

      $\left[\tilde{\mathbf{x}}_k^{(\eta)}\right] \leftarrow \text{Contraction}\left([\mathbf{y}_k], [\mathbf{x}_k], \left[\mathbf{y}_k^{(\eta)}\right], \left[\boldsymbol{v}_k^{(\eta)}\right]\right)$

    Correction: $\mathsf{L}_k^{(\eta)} \leftarrow \prod_{j=1}^{n_x} \frac{\left\|\left[\tilde{\mathbf{x}}_k^{(\eta)}(j)\right]\right\|}{\left\|\left[\mathbf{x}_k^{(\eta)}(j)\right]\right\|}$ and $\omega_k^{(\eta)} \leftarrow \mathsf{L}_k^{(\eta)} \times \omega_k^{(\eta)}$ — Likelihood and Weight update

  end for

  $\omega_k^{(\eta)} \leftarrow \frac{\omega_k^{(\eta)}}{\sum_{\eta=1}^{N_p} \omega_k^{(\eta)}}$ — Normalize weights

  Output $\hat{\mathbf{x}}_k \leftarrow \sum_{\eta=1}^{N_p} \omega_k^{(\eta)} \text{mid}\left(\left[\mathbf{x}_k^{(\eta)}\right]\right)$ — State estimation

  if $\frac{1}{\sum_{\eta=1}^{N_p}\left(\omega_k^{(\eta)}\right)^2} < N_{\text{thresh}}$ **then**

    Resample$\left(\left\{\left[\mathbf{x}_k^{(\eta)}\right], \omega_k^{(\eta)}\right\}_{\eta=1}^{N_p}\right)$ — Resample if needed

  end if

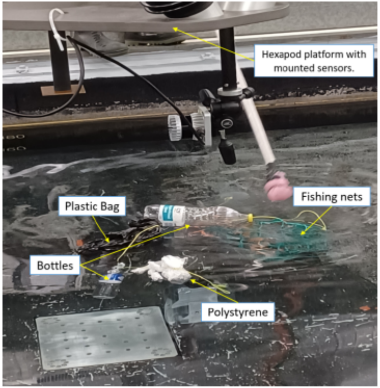
end for

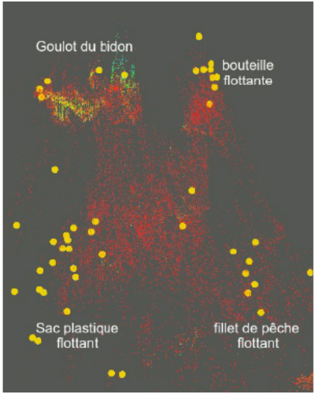
Outline

Experimental Validation: LiDAR Detection at IFREMER, France

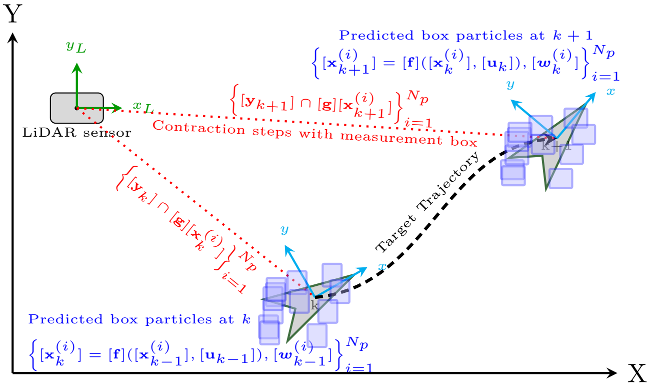
Objective: Validate LiDAR capability for floating object detection in marine environments

- Field experiments conducted using **OS1** and **VLP-16** 3D-LiDAR systems
- Multiple floating targets under various environmental conditions





LiDAR technology proves effective for marine floating object detection despite environmental challenges



- Objective: Track ASV position and heading using noisy LiDAR measurements
- Challenge: Significant sensor uncertainty and limited observations
- Solution: Interval Particle Filter for robust state estimation

System Models for ASV Tracking

Kinematic Model

State vector: $\mathbf{X}_k = [x_k, y_k, \theta_k]^T$ (position and heading)

$$\mathbf{X}_{k+1} = f(\mathbf{X}_k, \mathbf{U}_k) = \begin{bmatrix} x_k + \Delta x \cos(\theta_k) - \Delta y \sin(\theta_k) \\ y_k + \Delta x \sin(\theta_k) + \Delta y \cos(\theta_k) \\ \theta_k + \Delta\theta \end{bmatrix}$$

Control input: $\mathbf{U}_k = [\Delta x, \Delta y, \Delta\theta]^T$ (measured displacement)

LiDAR Observation Model

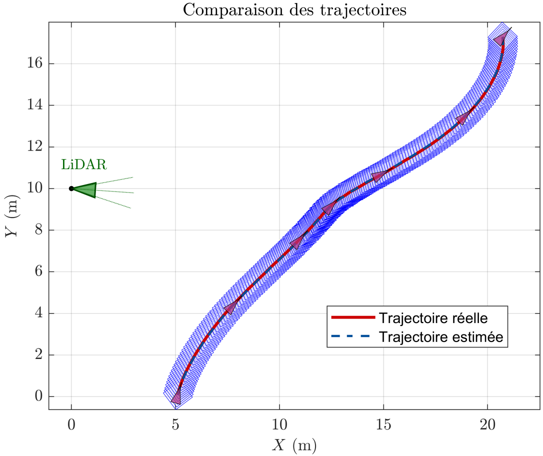
Measurements: range r_k and orientation ϕ_k to known target (x_k, y_k)

$$\mathbf{Y}_k = g(\mathbf{X}_k) = \begin{bmatrix} \sqrt{(x_k - x_L)^2 + (y_k - y_L)^2} \\ \text{atan2}(y_k - y_L, x_k - x_L) - \theta_k \end{bmatrix}$$

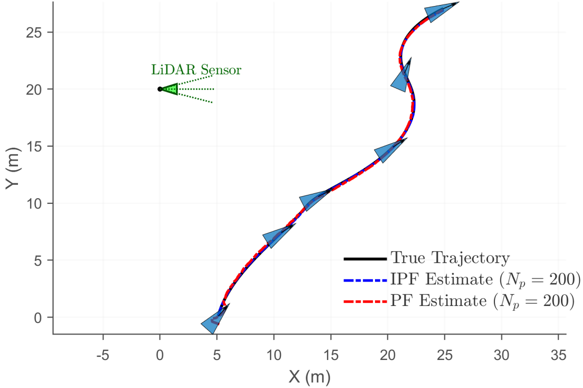
Experimental Setup - INTLAB

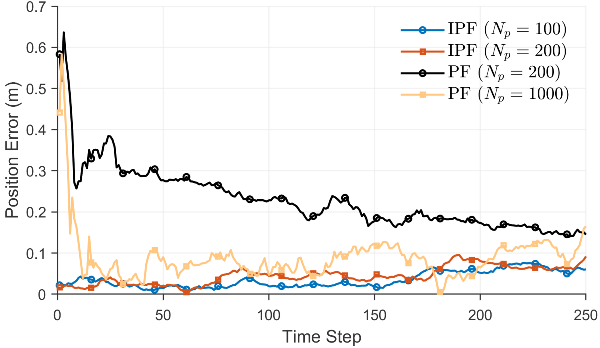
Implementation Framework

- **Implementation**: MATLAB with INTLAB toolbox
- **Comparison**: IPF vs. conventional Particle Filter (PF)
- **Application**: ASV trajectory tracking
- **Sensor**: LiDAR from static known position
- **Metrics**: RMSE and computational time



Qualitative Results: Trajectory Estimation





- IPF maintains tighter bounds around true trajectory
- Significant error reduction compared to standard PF
- Consistent performance with fewer particles

Performance Evaluation

Comparative Analysis: IPF vs. Standard PF

Method	Particles (N_p)	Time (s)	Pos. RMSE (m)	Angle RMSE (°)
IPF	**100**	0.1401	**0.0354**	**0.8327**
IPF	200	0.3409	0.0246	0.7964
PF	200	0.0213	0.2298	0.9023
PF	1000	0.0682	0.0823	0.8254

- IPF demonstrates robust performance
  - **High Accuracy**: IPF achieves lower RMSE under high-noise conditions
  - **Particle Efficiency**: Better performance with fewer particles
  - **Guarantees**: State always within computed bounds
- **Limitation**
  - Higher computational efficiency
  - Requires optimization for real-time

Outline

Conclusion and Future Work

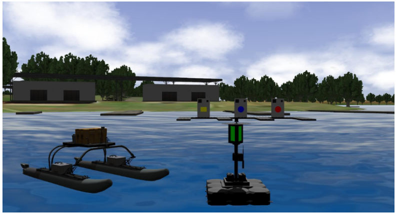
Conclusion

Interval Particle Filtering enables robust and efficient object tracking using LiDAR data:

- **Superior accuracy** with fewer particles compared to conventional methods
- **Guaranteed state enclosures** under significant uncertainty
- **Enhanced robustness** in challenging sensor conditions

Future Work

- **Advanced simulations**: Zonotope and ellipsoid PF, Mobile LiDAR on ASV, and multi-target tracking scenarios, dynamic simulations under VRX Gazebo.
- **Real-world validation**: Experimental testing in rivers, harbors, and coastal areas



VRX Gazebo Simulator

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Thank you for your attention

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Thank you for your attention

mohamed.fnadi@univ-littoral.fr